THE ELEMENTS OF

ELECTRICITY AND MAGNETISM

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ELECTRICITY AND MAGNETISM

A TEXT-BOOK FOR COLLEGES

AND TECHNICAL SCHOOLS

BY

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PREFACE AND INTRODUCTION.

"Alles Vergängliche ist nur ein Gleichniss."
(Intelligibility is only a likeness.)

The elementary theory of electricity and magnetism is essentially an extension of the science of mechanics,* and the purpose of this book is to develop the science of electricity and magnetism from this point of view.

The study of elementary physics, in one of its important phases is imaginative like the study of geometry, its purpose is to rationalize our experience of physical conditions and things, and the building up of the rational structure of physics should be the chief function of a text-book for students. This text has been prepared in accordance with this idea.

The attempt has been made throughout to bring simple practical applications into the mind of the student. It would perhaps be ridiculous in a descriptive treatise on physics for college men to consider in detail those things which are universally and perfectly known, but it is precisely such things that should be referred to in a rational treatise. If one is to rationalize, one must rationalize about something. It is a mistake, however, to shape science instruction prematurely to practical (economic) ends, but such "practical" instruction is a very different thing from the rational study of the things of everyday life. *Elementary science instruction must be made to touch upon the things of everyday life if it is to be effective.* In no other way can what is best in science be realized anew in each succeeding generation of men.

* See Art. 125 on the distinction between the mechanical theory and the atomic theory.

Special attention is called to Art. 62 on the mechanical aspect of Lenz's Law; to Chapter VI on Inductance; to Art. 89 on the mechanical analogue of the condenser; to Arts. 106, 107 and 108 on the mechanical analogies of electric doubling; and to Chapter IX on the mechanical conceptions of the electromagnetic field and of electromagnetic waves.

The authors feel that the appendices (*a*) on Terrestrial Magnetism, (*b*) on Ship's Magnetism and the Compensation of the Compass, (*c*) on Miscellaneous Phenomena, and (*d*) on Miscellaneous Practical Applications will appeal to nearly every one who has occasion to use this book. Every student should know something about these various subjects but most of this material should be omitted from a first systematic study of the Elements of Electricity and Magnetism. The appendix on Ship's Magnetism and the Compensation of the Compass especially is recommended to those who wish to gain a clear insight into the physics of this subject.

Following the plan of our *Elements of Mechanics*, we wish to include an introduction in this text. What needs to be said in introduction, however, is very brief, assuming that the student has read the introduction to our Mechanics. There seems to be among our students a general indifference towards rational physics study. What does this mean? That all students are unworthy, or that physical science is at fault? Neither. It seems to us that this indifference is due to a misunderstanding, and we believe that it may be made powerless to deter the student from a reasonable expenditure of effort in the rational study of the physical sciences if *young men be led to understand what kind of interest they may be expected to have in such study.* Gilbert Chesterton, in his essays on *Heretics*, says, very wisely, that the only spiritual or philosophical objection to steam engines is not that men pay for them or work at them or make them very ugly; or even that men are killed by them; but merely that men do not play at them. This is precisely the objection to physical science. Men do not play at it, or, when they do, it is play in the weakest and most contemptible sense of the word. Physical science in its elements is detached from the more intensely human interests, and the will alone can determine its pursuit.

THE AUTHORS.

March 22, 1908.

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CHAPTER I.

THE ELECTRIC CURRENT: ITS CHEMICAL EFFECT.

1. The electric current. — When a wire is connected to the terminals of an ordinary battery, certain phenomena are produced and an *electric current* is said to flow through the wire. A wire in which an electric current is flowing is sometimes called an *electric wire* for brevity. The production of an electric current always requires a *generator* such as a battery or a dynamo. The path of the current is usually a wire and it is termed the *electric circuit*. If the path is complete, leading out from the generator and returning to it without break or interruption, the circuit is said to be *closed;* otherwise, the circuit is said to be *open*. A steady electric current always flows in a closed circuit, that is, in a circuit which goes out from the generator and returns to it, and the current ceases to flow when the circuit is broken.

Certain substances such as metals and salt solutions may form portions of an electric circuit. Such substances are called *electrical conductors*. Other substances, such as glass, hard rubber, air, and dry wood, cannot form a portion of an electric circuit, that is, the electric current cannot flow through them to any appreciable extent. Such substances are called *insulators*.*

Energy must be supplied to an electric generator (chemical energy in the case of a battery, mechanical energy in the case of a dynamo), and this energy reappears in various parts of the electric circuit through which the current flows. Thus, energy reappears as heat in an electric lamp and as mechanical work in an electric motor.

The magnetic effect of the electric current. — When an electric wire is held above a compass and parallel to the compass needle, the compass needle is deflected. When an electric wire is

* All substances conduct the electric current more or less. See Art. 14.

stretched near one end of a magnet, as shown in Fig. 1, the wire is pushed sidewise as indicated in the figure. When an electric

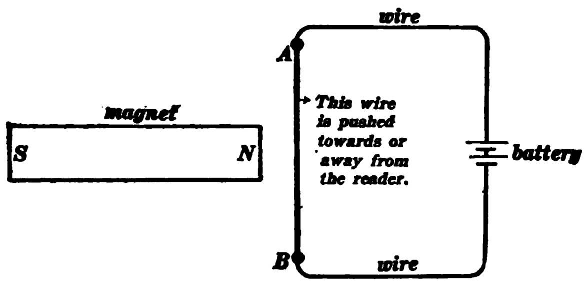


Fig. 1.

current flows through an insulated wire which is wound around an iron rod, as shown in Fig. 2, the iron rod is magnetized, as indicated by the letters *NS*. These effects constitute particular cases of what may be called in general the magnetic effect of the electric current.* The magnetic effect of the electric current, which is shown in its simplest aspect in Fig. 1, is exemplified in a common form of ammeter, the working parts

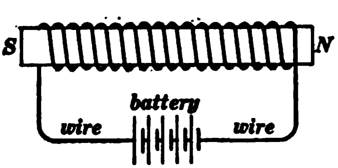
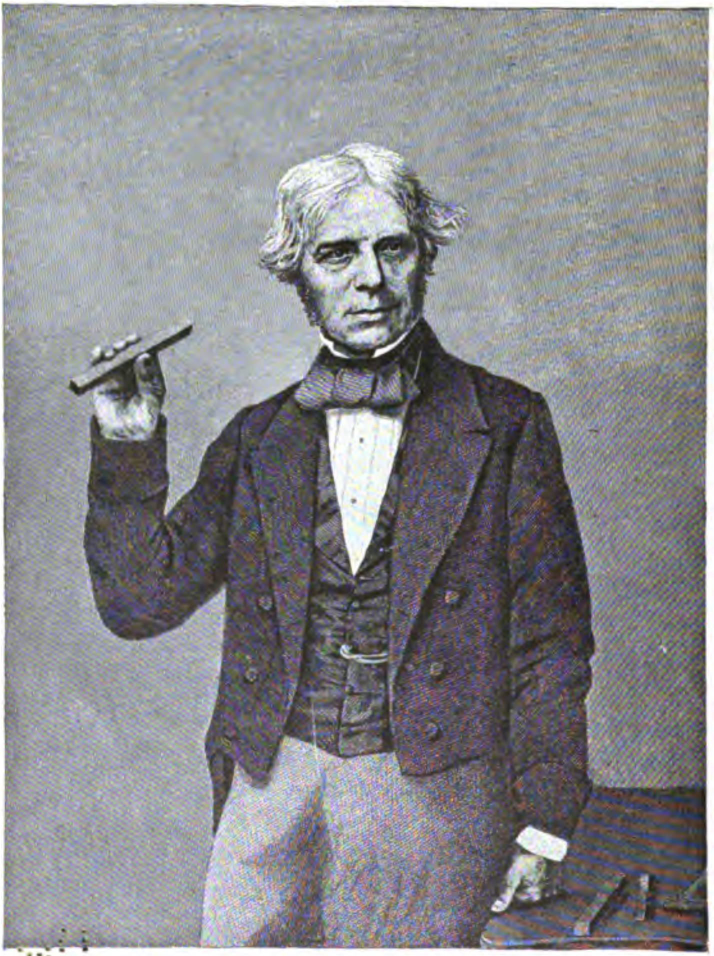


Fig. 2.

of which are shown in Figs. 3*a* and 3*b*. A horse-shoe magnet of steel is provided with soft iron pole-pieces *NN* and *SS*, between which a soft iron cylinder *C* is rigidly supported by being bolted to the brass strip *A*. In the spaces between the pole-pieces and the cylinder *C* move the sides or limbs of a small coil of wire which is delicately supported upon a pivot and which carries a pointer which plays over a divided scale. Current is led into this movable coil through the hair-spring at one end and through a very flexible conductor at the other end, and the side force which is exerted upon the limbs of the coil by the magnet

* Another aspect of the magnetic effect of the electric current, namely, the production of current in a wire when the wire is in motion near a magnet, is discussed in Art. 63.



MICHAEL FARADAY (1791-1867)

poles NN and SS turns the coil until this side force is balanced by the action of the hair-spring.

The magnetic effect of the electric current which is shown in its simplest aspect in Fig. 2, is exemplified in the Morse tele-

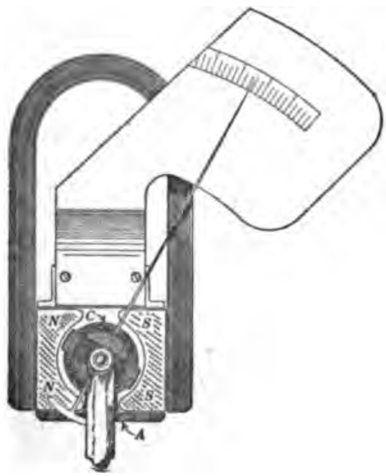


Fig. 3a.

Fig. 3b.

graph. A battery B, Fig. 4, is connected through a long line so as to send current through a wire which is wound on an iron rod RR at a distant station. A device K, called a *key*, is arranged for opening and closing the circuit through which the electric current flows, and the rod RR is magnetized every time

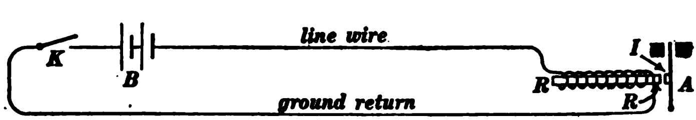


Fig. 4.

the key is closed, thus causing the rod RR to attract a small bar of iron I which is attached to a pivoted lever A; and when the key K is opened, the rod RR loses its magnetism and ceases to attract the iron I. In this way the pivoted lever A is caused to move back and forth with the opening and closing of

the key K, thus producing any desired series of signals at the distant station.*

The chemical effect of the electric current. — When a solution of a chemical compound forms a portion of an electric circuit, the compound is, in general, decomposed by the current. This chemical effect of the electric current is exemplified in the practical operation of electroplating. The essential features of an electroplating outfit are shown in Fig. 5. VV is a vessel containing, for example, a solution of copper sulphate. The metal object O to be plated is attached to one terminal of a battery B, a copper plate C is attached to the other terminal of the battery, and the current causes copper to be deposited upon the object O.

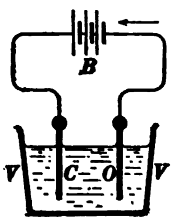


Fig. 5.

The heating effect of the electric current. — A wire, or any substance which forms a portion of an electric circuit, has heat generated in it by the current. This heating effect of the electric current is exemplified in the ordinary electric lamp, the carbon filament of which forms a portion of an electric circuit, and is heated to incandescence by the current.

Hydraulic analogue of the electric current. — The *flow* of an electric current through a circuit of wire is to some extent analogous to the flow of water through a circuit of pipe. The pump which propels the current of water is analogous to the generator which propels the electric current, and the circuit of pipe which goes out from the pump and returns to it is analogous to the circuit of wire. Energy must be supplied to the pump to produce the flow of water through the pipe, and this energy reappears as the heat which is developed by the friction of the water in the pipe or as the mechanical energy which is developed by a water motor through which the water current is forced. Similarly, energy must be supplied to an electric generator, and this energy reappears in the electric circuit as heat or as the mechanical energy

* The Morse Telegraph is described quite fully in Appendix D.

which is developed by an electric motor through which the electric current is forced.

An electric current flowing through a wire produces an influence which extends throughout the region surrounding the wire, as is evident from the fact that a compass needle is deflected when it is brought near an electric wire. There is, however, no influence exerted in the region surrounding a pipe through which water is flowing. Therefore the hydraulic analogue of the electric current is of no help in giving one a conception of the magnetic effect of the electric current. In the study of those phenomena of the electric current which depend upon its magnetic effect, the hydraulic analogue must be used with caution.

2. The chemical effect of the electric current.* — When a solution of a chemical compound forms a portion of an electric circuit, the compound is, in general, decomposed by the current, as stated above. Thus, melted salts, and acids and salts in solution are decomposed by the electric current. This chemical decomposition is called *electrolysis*, and the liquid in which electrolysis takes place is called an *electrolyte*. Electrolysis is usually carried out in a vessel provided with two flat plates of metal or carbon which serve to lead the current into and out of the electrolyte. Such an arrangement is called an *electrolytic cell*, and the plates of metal or carbon are called the *electrodes*. The electrode upon which the metallic constituent of the solution is deposited is called

* The chemical effect of the electric current is exemplified by many electrochemical processes which are now used on a large scale in various industrial establishments. See The Electrochemical Manufactures at Niagara, *Electrochemical Industry*, Vol. I, pages 11-23; The Electrolytic Refining of Copper, *Engineering and Mining Journal*, September 19, 1896, and *Electrochemical Industry*, Vol. I, page 416, August, 1903, and The Manufacture of Aluminum by Electrolysis, *Electrochemical Industry*, Vol. I, page 158, June, 1903.

Perhaps the best modern treatises on the phenomena of electrolysis are the following:

A Text-book of Electro-chemistry by LeBlanc, translated by W. R. Whitney and J. W. Brown, The Macmillan Company.

Electro-chemistry by Danneel, translated by Merriam, John Wiley & Sons.

The Theory of Electrolytic Dissociation by H. C. Jones, The Macmillan Company.

the *cathode*, and the other is called the *anode*. It is customary to speak of the current as flowing into an electrolytic cell at the anode and out of the cell at the cathode, that is, *the electric current is considered to flow in the direction in which the metallic constituent of the solution is carried in an electrolytic cell.*

Consider a solution of hydrobromic acid (HBr). When an electric current is passed through this solution, hydrogen (H) is liberated at the cathode and bromine (Br) is liberated at the anode. In general, the molecule of any dissolved salt or acid is separated into two parts by electrolysis; one part is liberated at the cathode and is called the *cathion*, and the other part is liberated at the anode and is called the *anion*. Thus, hydrogen (H) is the cathion and bromine (Br) is the anion of hydrobromic acid. In all metallic salts the metal constitutes the cathion and the acid radical or halogen constitutes the anion. In acids the hydrogen constitutes the cathion and the acid radical or halogen constitutes the anion. Thus, the cathion of copper sulphate ($CuSO_4$) is copper (Cu), and the anion is the acid radical (SO_4).

In many cases the cathion and anion are not actually liberated at the electrodes because of what are called secondary reactions. Thus, in the electrolysis of an aqueous solution of sodium chloride (NaCl), the cathion (Na), when it is liberated at the cathode, immediately reacts upon the water, forming NaOH and free hydrogen; in the electrolysis of copper sulphate between copper electrodes, the anion (SO_4) combines with the copper of the anode forming fresh $CuSO_4$ which goes into solution or is deposited as crystals on the anode if the solution is saturated; in the electrolysis of H_2SO_4 between inert electrodes such as carbon or platinum, the hydrogen is liberated at the cathode as a gas, and the anion (SO_4) reacts on the water according to the formula $SO_4 + H_2O = H_2SO_4 + O$ and the free oxygen escapes as gas. The reason for taking the unfamiliar substance hydrobromic acid in the above example is that in the electrolysis of hydrobromic acid there are no secondary reactions at the electrodes.

The chemical action which is caused by the flow of current through an electrolytic cell is confined wholly to the immediate neighborhood of the electrodes. This is exemplified by passing an electric current through a solution of lead nitrate between lead electrodes in a narrow glass vessel which can be placed before the lantern and projected on the screen. The lead is deposited upon the cathode in beautiful feather-like crystals, and the solution in the immediate neighborhood of the cathode becomes less dense as the lead is deposited out of it upon the cathode as may be seen by the upward streaming of the solution near the surface of the cathode. On the other hand, the solution near the anode is increased in density by the dissolving of the lead of the anode by the NO_3 which is liberated there by the current, as may be seen by the downward streaming of the solution in the neighborhood of the anode. The solution remains entirely unchanged throughout the region between the electrodes.*

The dissolving of the metal of the anode may be observed directly by reversing the current, thus causing the feather-like crystals of lead which have already been deposited upon one of the lead electrodes to become the anode. Under these conditions the crystals are seen to dissolve rapidly.

3. Measurement of current by its chemical effect. Definition of the ampere. — The electric current in a wire may be measured in terms of its magnetic effect, or in terms of its heating effect, or in terms of its chemical effect. Thus, it would be permissible to think of one current as being twice as strong as another if it would produce twice as much heat per second as the other current when it is allowed to flow through a given wire; † but the magnetic effect has been adopted as the basis of current measurement as fully explained in Chapter IV. The measurement of current by its chemical effect, however, is consistent with the fundamental measurement by magnetic effect, and therefore, we may for the

* Except for a slight rise of temperature due to the heating effect of the current.

† A definition of current strength on this basis would lead to a more complicated scheme of electrical theory than that at present in vogue.

present define *the strength of an electric current as proportional to the amount of a given metal deposited by the current per second in an electrolytic cell.*

The international standard ampere is defined * as that strength of current which will deposit 0.001118 gram of silver per second from an aqueous solution of pure silver nitrate. Another unit of current, the abampere or c.g.s. unit, is defined in Art. 52.

The coulombmeter. — An electrolytic cell arranged for the measurement of current by weighing the amount of metal deposited by the current in a given time is called a coulombmeter.† Thus, the copper coulombmeter consists of a glass vessel containing an aqueous solution of copper sulphate and having sheet-copper electrodes. The cathode, or gain-plate, is weighed at the beginning and again at the end of the run, and the strength of the current is calculated by dividing the observed amount of deposited copper by the amount of copper that would be deposited in the same time by one ampere.

Current density at an electrode. — The quotient of the current flowing through an electrolytic cell divided by the active area of one of the electrodes is called the current density at that electrode. The physical character of the metal which is deposited by an electric current depends very greatly upon the current density at the electrode upon which the metal is deposited. Thus, metallic copper is deposited from a solution of copper sulphate as a smooth, solid layer if the current density does not exceed 0.02 ampere per square centimeter, whereas the deposit becomes very rough with projecting crystals of the metal if the current density

* In accordance with the recommendations of the International Electrical Congress which met at Chicago in 1893. The fundamental definition of the ampere is based upon the magnetic effect of the electric current as explained in Art. 52. The value of a current in amperes (as defined by the magnetic effect) may be determined from purely mechanical measurements as explained in Art. 59. In this way the amount of silver deposited in one second by one ampere may be determined. This determination has been made a number of times with great care, the latest determination being that of H. S. Carhart and G. W. Patterson. See *Journal of the Institution of Electrical Engineers*, Vol. 34, pages 185–189, February, 1905.

† Sometimes called a voltameter.

is greatly in excess of this. The character of the chemical action which takes place at an electrode also depends upon the current density. Thus, copper alone is deposited upon a cathode from a mixed solution of zinc and copper sulphates if the current density is very small, whereas a mixture of copper and zinc is deposited upon the cathode if the current density is excessive.*

4. Faraday's laws† of electrolysis. *First law.*—The amount of a given metal which is deposited electrolytically is proportional to the strength ‡ of the current and to the time, that is,

$$M = kIt \tag{1}$$

in which M is the amount of metal in grams deposited in t seconds by a current of I amperes, and k is a constant for a given metal. This constant k is called the *electrochemical equivalent* of the given metal. Electrochemical equivalents are ordinarily specified in grams of metal deposited per ampere of current per second.

Second law.—The electrochemical equivalents of elements which can form the ions of an electrolyte, are proportional to the quotients of their atomic weights divided by their valencies. A metal which has two valencies has two values for its electrochemical equivalent. Thus one and one half times as much iron is

* The deposition of one metal instead of several from solutions of mixed salts depends more distinctly upon the electromotive-force drop between the electrode and the solution (electrode polarization) than upon current density. See Art. 22.

† The laws of physics are the *experimental facts* upon which the science is based. Thus Faraday's laws of electrolysis are the result of experiment, pure and simple; Boyle's and Gay Lussac's Laws concerning the expansion of gases are experimental facts; Newton's Laws of Motion are experimental facts; Newton's Law of Gravitation is an experimental fact; and so on. In nearly every case the so-called laws of physics are only approximately true. Thus, the product of the volume and pressure of a given amount of gas at constant temperature is not strictly constant (Boyle's Law); the amount of metal deposited by an electric current deviates in many cases from an exact proportional relationship with the current (see *Practical Physics*, Franklin, Crawford and MacNutt, Vol. II, page 136).

‡ In Faraday's experiments, which led to the formulation of this general law, the electric current was measured by a galvanometer, that is, the electric current was measured in terms of its magnetic effect.

deposited from a solution of a ferrous salt as from a solution of a ferric salt, provided that the deposition is not complicated by secondary reactions which cause the deposit to be redissolved chemically.

5. The dissociation theory of electrolysis. — The molecules of an electrolytic salt or acid when in solution, or when melted, are thought to be more or less dissociated into what are called ions. For example, the molecules of copper sulphate ($CuSO_4$) in a dilute aqueous solution are all dissociated into Cu (cathions) and SO_4 (anions); the molecules of sodium chloride (NaCl) in a dilute aqueous solution are all dissociated into Na (cathions) and Cl (anions). These ions are supposed to be electrically charged* and to wander about through the solution. When an electric current flows through the electrolyte, the positively charged ions (cathions) move towards the cathode where they part with their positive charges and are deposited as hydrogen or metal, as the case may be, and the negatively charged ions (anions) move towards the anode where they part with their negative charges. This movement of positively and negatively charged ions constitutes the electric current in the electrolyte.

Conception of Faraday's first law. — All of the ions of a given substance have the same electric charge so that the strength of the current is proportional to the number of ions deposited per second on one of the electrodes.

Conception of Faraday's second law. — All monovalent ions carry the same amount of charge, the charge on a monovalent cathion being positive and the charge on a monovalent anion being negative. For example, the cathions in the following series of chlorides are all monovalent, hydrochloric acid (HCl), potassium chloride (KCl), sodium chloride (NaCl), and cuprous chloride (CuCl), and the same number of atoms of H, K, Na, and Cu are deposited from solutions of these chlorides in a given time by a given current, so that the electrochemical equivalents

* See Art. 85 for definition of electric charge.

of these monovalent metals are directly proportional to their atomic weights.

The charge on an ion is proportional to its valency. Thus, the copper ion in a solution of cupric chloride ($CuCl_2$) has twice as much charge as the copper ion in a solution of cuprous chloride ($CuCl$), so that half as many cupric ions as cuprous ions are deposited by a given current in a given time. In general, if n is the number of monovalent ions deposited in one second by one ampere, then $n/2$ is the number of bivalent ions deposited in the same time by the same current, $n/3$ is the number of trivalent ions deposited in the same time by the same current, and so on.

Consider a series of chlorides of metals of different valencies, for example, sodium chloride ($NaCl$), cupric chloride ($CuCl_2$), ferric chloride ($FeCl_3$), and stannic chloride ($SnCl_4$). Reducing these all to a given amount, say n atoms, of chlorine, we would have n atoms of sodium (Na), $n/2$ atoms of copper (Cu), $n/3$ atoms of iron (Fe), and $n/4$ atoms of tin (Sn); so that, during the liberation of n atoms of chlorine, we would have a deposit of n atoms of sodium (Na), $n/2$ atoms of copper (Cu), $n/3$ atoms of iron (Fe), and $n/4$ atoms of tin (Sn). Therefore, the weights of these various metallic deposits would be proportional to their atomic weights divided by their respective valencies.

Let us represent each unit of charge by a plus or minus sign. Then the single, double, triple and quadruple charges on the ions of sodium, copper, iron and tin may be represented as follows: Na , Cu^{++}, Fe^{+++} and Sn^{++++}, and the single and double charges upon the monovalent and bivalent anions of chlorine and SO_4 may be represented as follows: $-Cl$ and $=SO_4$. The present hypothesis concerning chemical affinity is that it is due to the attraction of the opposite charges on the two constituents of the molecule. Thus, sodium and chlorine are held together in the molecule of sodium chloride by the attraction of the positive charge on the sodium for the negative charge on the chlorine, as may be represented thus: $Na + - Cl$.

One of the greatest difficulties in the dissociation theory of electrolysis is to account for the breaking up of such a molecule as sodium chloride, which is ordinarily very stable, into its ions. The strength of the dissociation theory, however, lies in the extent to which it correlates a wide range of experimental fact, and in this respect the dissociation theory is incomparably more useful than any other theory that has been hitherto proposed.*

6. The voltaic cell. — The chemical action that is caused by the flow of current through an electrolytic cell is usually *forced*, that is, work has to be done to bring the chemical action about or, in other words, an electric generator such as a dynamo or a battery must be used to push the current through the electrolytic cell. When, however, secondary chemical actions take place at one or both electrodes, it frequently happens that the total chemical action that is brought about by the flow of current through an electrolytic cell is a *source of energy*. In such a case the electrolytic cell itself can maintain its own current through the electrolyte from electrode to electrode and through an outside circuit of wire which connects the electrodes. Such an electrolytic cell is called a *voltaic cell*.

Example. — When a strip of clean zinc and a strip of copper or carbon are dipped into dilute sulphuric acid, no appreciable chemical action takes place. When the plates are connected together by a wire, a current immediately starts to flow through the circuit, leaving the cell at the copper or carbon electrode (the cathode) and

* Several simple applications of the dissociation theory to the interpretation of experimental results are given in *Practical Physics*, Franklin, Crawford and MacNutt, Vol. II, page 108, page 144 and pages 146 and 147. A splendid example of the application of the dissociation theory to the rationalization of a very complicated experimental result is given by E. C. Franklin and H. D. Gibbs in the *Journal of the American Chemical Society*, Vol. 29, pages 1389–1396, October, 1907.

Any student who wishes to become acquainted with the facts of electrolysis must familiarize himself with the details of the dissociation theory, and, since no other theory has ever been proposed which is to be compared in effectiveness with the dissociation theory, the student's efforts should be directed first of all to a thorough *understanding* of the theory. After he has mastered the theory its imperfections may properly be pointed out.

entering the cell at the zinc electrode (the anode). This current decomposes the sulphuric acid (H_2SO_4), the hydrogen is liberated at the copper or carbon cathode and escapes from the cell as a gas, and the sulphuric acid radical (SO_4), which is set free at the zinc anode, combines with the zinc and forms zinc sulphate ($ZnSO_4$) which goes into solution. The combination of Zn and SO_4 develops more energy than is required for the decomposition of the H_2SO_4 so that the chemical action as a whole is a source of energy.

The available energy of the reaction above described may be greatly increased by providing an oxidizing agent in the neighborhood of the cathode so that the hydrogen may be oxidized and form water (H_2O) at the moment of its liberation by the current. The energy of this oxidation is then added to the available energy of the total chemical action in the cell.*

7. Examples of voltaic cells. *The ordinary "dry cell."* — One of the most familiar types of voltaic cell is the cell in which a plate of zinc and a plate of carbon are immersed in a solution of ammonium chloride (NH_4Cl) with a mass of powdered black oxide of manganese (MnO_2) packed around the carbon electrode. When this cell delivers current, the NH_4Cl is decomposed, and chlorine is liberated at the zinc plate where it combines with the zinc to form zinc chloride. As the NH_4 ions are liberated at the carbon electrode they break up into ammonia and hydrogen ($NH_4 = NH_3 + H$), the ammonia goes into solution and the hydrogen is oxidized at the expense of the oxygen in the black oxide of manganese, forming water. The free ammonia in this type of cell may be detected by the odor after the cell has been delivering current for some time.

This type of cell is exemplified by a great variety of commercial forms of which the ordinary "dry cell" is the most familiar. In this cell the electrolyte is soaked up in a porous material such

*The student is referred to Professor H. S. Carhart's *Primary Batteries*, published by Allyn & Bacon, Boston, Mass., for full information on primary batteries (voltaic cells) and primary battery tests.

as saw-dust, the containing vessel is made of zinc and serves as the zinc electrode, and the cell is hermetically sealed so as to prevent evaporation.

The ordinary gravity cell, which is shown in Fig. 6, consists of a copper electrode in the bottom of a jar surrounded by a solution of copper sulphate, and a zinc electrode in the top of the jar surrounded by a solution of zinc sulphate. The light zinc sulphate solution floats on the heavy copper sulphate solution. When this cell delivers current, SO_4 is liberated at the zinc electrode where it combines with the zinc forming additional zinc

Fig. 6.

Fig. 7.

sulphate, and metallic copper is deposited upon the copper electrode at the bottom of the cell. When this cell is in use, the copper sulphate must be replenished occasionally by dropping fresh crystals of the salt into the cell, and a portion of the zinc sulphate solution must be occasionally drawn off and replaced by water.

The chrômic acid cell consists of a plate of amalgamated zinc and a plate of carbon dipping into a solution of a mixture of chromic acid ($H_2Cr_2O_7$) and sulphuric acid (H_2SO_4). When this cell delivers current, the flow of the current through the cell decomposes the H_2SO_4. The acid radical SO_4 is liberated at

the zinc electrode where it combines with the zinc forming zinc sulphate, and the hydrogen is liberated at the carbon electrode where it is oxidized at the expense of the oxygen in the chromic acid.

In the chromic acid cell, the zinc wastes away rapidly even when the cell is not delivering current, and it is therefore desirable to lift the zinc out of the solution when the cell is not in use. Figure 7 shows a chromic acid cell arranged so that the zinc electrode may be conveniently lifted out of the solution. In this figure the cell is shown with a zinc electrode placed between two carbon plates. The two carbon plates are connected together and constitute one electrode.

The Edison-LaLande cell consists of a zinc plate and a compact block of copper oxide (CuO) immersed in a strong solution of caustic potash (KOH). The cell shown in Fig. 8 has two zinc

Fig. 8.

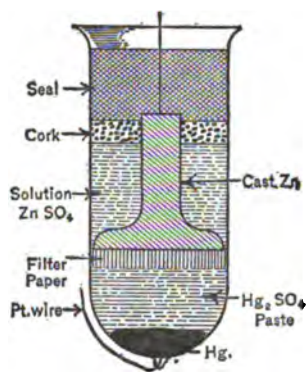


Fig. 9.

plates on opposite sides of the copper oxide plate. These two zinc plates are connected together and constitute a single electrode. When this cell delivers current, the KOH is decomposed, potassium ions are liberated at the copper oxide plate, the copper oxide is reduced to metallic copper, and the potassium is oxidized and goes into solution as KOH. At the same time hydroxyl ions (OH) are liberated at the zinc electrode where they break up into free oxygen and water ($2OH = O + H_2O$), the free oxygen combines with the zinc forming zinc oxide, and

this zinc oxide combines with the caustic potash in the solution forming potassium zincate (K_2ZnO_2).

The Clark standard cell is arranged as shown in Fig. 9. One electrode is pure mercury and the other electrode is pure zinc. When this cell delivers current, the $ZnSO_4$ in solution is decomposed, SO_4 is liberated at the surface of the zinc where it combines with the zinc forming $ZnSO_4$, and at the same time Zn is liberated at the surface of the mercury electrode where it is acted upon by the mercurous sulphate Hg_2SO_4 according to the equation

$$Zn + Hg_2SO_4 = Hg_2 + ZnSO_4$$

This cell is remarkable for the constancy of its electromotive force and it is used as a standard of electromotive force, as explained in Chapter X.

8. Voltaic action and local action. — Two kinds of chemical action are to be distinguished in a voltaic cell, namely, (*a*) the chemical action which depends upon the flow of current and does not exist when there is no current and (*b*) the chemical action which is independent of the flow of current and which takes place whether the current is flowing or not.

The chemical action which depends on the current is proportional to the current, it is essential to the operation of the voltaic cell as a generator of current, its energy is available for the maintenance of the current, and it is called voltaic action.

The chemical action in a voltaic cell which is independent of the flow of current does not help in any way to maintain the current, it represents absolute waste of materials, and it is called local action. Local action takes place more or less in every type of voltaic cell and it is especially marked in the chromic acid cell above described. It may be reduced to a minimum in a given type of voltaic cell by coating the zinc with a thin layer of metallic mercury (amalgamation).

The term, local action, originated in the following considerations: When a strip of clean zinc is immersed in sulphuric acid, no perceptible chemical action takes place.

If the zinc is connected to a carbon or copper electrode, however, or if a piece of carbon or copper touches the zinc plate in the solution, chemical action begins at once, current flows through the electrolyte from the zinc to carbon or copper and back through the metallic connection to the zinc, the sulphuric acid is decomposed, hydrogen is liberated at the carbon or copper electrode, and SO_4 is liberated at the zinc electrode where it combines with the zinc forming zinc sulphate. When a plate of impure zinc is immersed in dilute sulphuric acid, the insoluble impurities are left in the form of fine particles clinging to the surface of the zinc after the zinc is partly dissolved, and these fine particles play the part of carbon or copper cathodes, current flows through the acid from the zinc to each particle and back to the zinc through the point of attachment of the particle with the zinc plate, as indicated in Fig. 10*a*, the acid is decomposed, hydrogen is liberated at the surface of each particle, and SO_4 is liberated at the surface of the zinc plate where it combines with the zinc forming zinc sulphate. The rapid dissolving of impure zinc in sulphuric acid is no doubt due to the flow of electric currents through the minute "local circuits" as here described, and this rapid dissolving of impure zinc is therefore called local action.

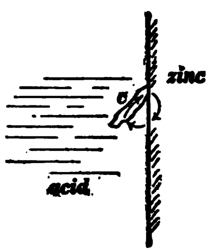


Fig. 10a.

The covering of the zinc plate with a thin layer of metallic mercury tends to produce a clean metallic surface which is free from adhering particles of the impurity which is left as the zinc wastes away, and the above described action does not take place. It is probable that in some cases chemical action (local action) takes place irrespective of the flow of electric currents in local circuits as above described. This seems to be the case, for example, in the chromic acid cell, for, as a matter of fact, more than three fourths of the zinc in such a cell is consumed independently of voltaic action, even when the zinc is thoroughly amalgamated so as to present a clean bright surface, but in the chromic acid cell the local action is very much less when the zinc is amalgamated than it is when the zinc is not amalgamated.

An essential feature of voltaic action is that it is reversed if a current is forced backwards through a voltaic cell by an outside agent, provided that no material that has played a part in the previous voltaic action has been allowed to escape from the cell. Thus in the operation of the simple voltaic cell consisting of a zinc anode and a carbon cathode in dilute sulphuric acid, the H_2SO_4 is decomposed, $ZnSO_4$ is formed at the anode, and hydrogen is liberated at the cathode. If the current is reversed so that the carbon plate becomes the anode, and the zinc plate the cathode, then the $ZnSO_4$, previously formed, will be decomposed, metallic zinc will be deposited upon the zinc cathode, and SO_4 will be liberated at the carbon anode where it will combine with

the trace of hydrogen that is clinging to the carbon plate and form H_2SO_4. In this cell, however, the greater part of the liberated hydrogen has, of course, escaped, and the reversed chemical action due to a reversed current cannot long continue. Local action, on the other hand, being independent of current, is not affected by a reversal of the current.

9. The storage cell.* — A voltaic cell which is free from local action and in which all of the materials which take part in the voltaic action are conserved in the cell, may be regenerated after use by sending through it a reversed current. This regeneration is due to the reversed chemical action that is produced by the reversed current as explained in the previous article. A voltaic cell that is adapted to be thus regenerated, that is, a voltaic cell in which there is no local action and in which all of the materials which take part in the voltaic action are conserved in the cell, is called a *storage cell*. The process of regeneration is called *charging*, and the use of the cell as an electric generator is called *discharging*. A storage cell always requires more energy to charge it than is delivered by the cell during the discharge.

The lead storage cell. — The voltaic cell which, up to the present time, has been found to be most satisfactory when used as a storage cell, is a voltaic cell having a cathode of lead peroxide (PbO_2), an anode of spongy metallic lead, and an electrolyte of dilute sulphuric acid. The lead peroxide and the spongy metallic lead are both converted into insoluble lead sulphate ($PbSO_4$) when the cell is discharged. When this cell is charged, the lead sulphate is converted back into lead peroxide and spongy lead respectively. The lead peroxide and the spongy lead are

* The description here given of the action of the lead storage cell is a simple working theory of the cell. The actions as described do, no doubt, take place, but they are complicated by more complex actions such as the formation of persulphates at the anode and of subsulphates at the cathode. See *The Theory of the Lead Accumulator*, by Friedrich Dolezalek (English translation by C. L. von Ende, published by John Wiley & Sons). A good engineering treatise on the storage battery is *Storage Battery Engineering* by Lamar Lyndon (McGraw Publishing Company).

called the *active materials* of the cell. These active materials are mechanically weak and porous and they are usually supported in the interstices of massive grids of metallic lead. These lead grids serve not only as mechanical supports for the active material, but they serve also to deliver current to or receive current from the active materials which constitute the real electrodes.

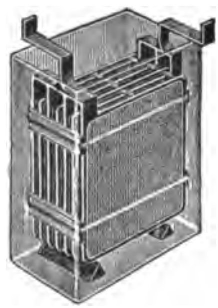


Fig. 10b.

Figure 10*b* shows a commercial form of lead storage cell. The electrodes consist of fine grids of metallic lead in the interstices of which the active material is placed. The positive electrode (out of which the current comes during discharge) consists of three grids connected together, and the negative electrode consists of four grids connected together.

Action of the cell while discharging.— When the lead storage cell delivers current, the electrolyte H_2SO_4 is split up by the current into H_2 and SO_4. The hydrogen is liberated at the cathode, where it reduces the lead peroxide to PbO, and this PbO combines with a portion of the H_2SO_4 of the electrolyte forming $PbSO_4$ and water. The SO_4 which is liberated at the anode combines with the spongy lead and forms $PbSO_4$. During this process the active material expands, because the lead sulphate is more bulky than the spongy lead and the lead peroxide; and the electrolyte grows less concentrated (and of course increases in resistance) because of the absorption of SO_4 by the active material. This decrease of concentration is especially great in the pores of the active material when the cell is discharged rapidly.

Action of the cell while being charged. — When the lead storage cell is regenerated by forcing a reversed current through it, the above-described action is reversed. The lead sulphate on one electrode is converted back to lead peroxide, the lead sulphate on the other electrode is reduced to spongy metallic lead, the electrolyte grows more dense (especially in the pores of the active material), and the active material contracts.

The following tabular arrangement gives a clear idea of the action of the lead storage cell while discharging and while being charged:

DISCHARGING.

Positive grid. $PbO_2 + H_2SO_4 + \mathbf{H_2} = 2H_2O + PbSO_4$

$\Uparrow$ Direction of current through the cell (negative to positive grid).

Negative grid. $Pb + \mathbf{SO_4} = PbSO_4$

CHARGING.

Positive grid.*	$PbSO_4 + 2H_2O + SO_4 = 2H_2SO_4 + PbO_2$	
	$\downarrow$ Direction of current through the cell (positive to negative grid).	
Negative grid.*	$PbSO_4 + H_2 = H_2SO_4 + Pb$	

10. Open-circuit cells and closed-circuit cells. — A voltaic cell in which the local action is very slight does not deteriorate appreciably when it is not called upon to deliver current. Such a cell may be left standing on open circuit in readiness for use at any moment to supply current for any purpose such as to ring an electric bell. All that is necessary is to provide a device for closing the circuit when it is desired to obtain current from the cell, and then the circuit should be opened in order to avoid deterioration of the cell by the continued flow of current. A voltaic cell which is adapted to this kind of use is called an *open-circuit cell* and perhaps the best form of open-circuit cell is the ordinary dry cell.

When an ordinary dry cell is called upon to give a steady current the electromotive force † falls off rapidly on account of what is called polarization, and the current decreases accordingly. A voltaic cell which is capable of delivering a fairly large steady current is called a *closed-circuit cell.* The gravity cell is one of the best types of closed-circuit cell. The chromic acid cell is also frequently used for delivering current more or less steadily. The Edison-LaLande cell is a fairly good open-circuit cell and it is satisfactory also for closed-circuit work.

PROBLEMS.

1. The anode of an electrolytic cell consists of a copper rod 3 centimeters in diameter, and the cathode consists of a hollow

* It is the usual practice among electrical engineers to call that terminal of an electric generator out of which current flows, the positive terminal, and that terminal into which current flows, the negative terminal. In conformity with this usage, that electrode of a storage cell which is cathode during discharge is called the *positive grid* and the other the *negative grid.* The positive grids are of a pale salmon color and the negative grids are a neutral gray.

† See Art. 22.

copper cylinder of which the inside diameter is 12 centimeters. 15 centimeters of length of anode and cathode are submerged in the electrolyte, and a current of 25 amperes is passed through the cell. (*a*) Find the current density at the cathode, and (*b*) find the current density at the anode. Ans. (*a*) 0.044 ampere per square centimeter; (*b*) 0.177 ampere per square centimeter.

2. An electric current is sent through an ammeter and through a silver coulombmeter. The current gives a steady reading of 1.068 amperes on the ammeter, and the amount of silver deposited in 1 hour and 20 minutes is found by weighing to be 5.635 grams. Find the error of the ammeter reading. Ans. 0.018 ampere too high.

Note. — The silver coulombmeter is usually arranged as shown in Fig. 11. The silver nitrate solution is contained in a clean platinum bowl which serves as the cathode on the interior of which the silver is deposited, and the anode consists of a plate of pure silver surrounded by a covering of filter paper to prevent detached particles from falling to the bottom of the platinum bowl.

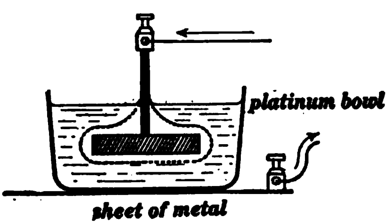


Fig. 11.

3. Calculate the electro-chemical equivalents of the following: (*a*) Cuprous copper; (*b*) cupric copper; (*c*) zinc; (*d*) hydrogen; (*e*) aluminum; and (*f*) ferric iron. The valencies of the respective metals may be inferred from the following formulæ of their chlorides: (*a*) $CuCl$; (*b*) $CuCl_2$; (*c*) $ZnCl_2$; (*d*) HCl; (*e*) $AlCl_3$; (*f*) $FeCl_3$ Ans. (*a*) 0.0006587 gram per ampere per second; (*b*) 0.0003293 gram per ampere per second; (*c*) 0.000339 gram per ampere per second; (*d*) 0.00001046 gram per ampere per second; (*e*) 0.0000936 gram per ampere per second; (*f*) 0.0001929 gram per ampere per second.

4. A current which gives a steady reading of 10 amperes on an ammeter is found to deposit 8.24 grams of copper in 40 minutes from a solution of $CuSO_4$. What is the error of the ammeter reading? Ans. 0.42 ampere too low.

5. Find the time required for 10 amperes of current to generate 2 cubic feet of hydrogen and 1 cubic foot of oxygen, both gases being measured at 730 millimeter pressure and at a temperature of 20° C.; 22 millimeters of the pressure in each case being due to the water vapor which is present. Ans. 11.8 hours.

Note. — The amount of hydrogen or oxygen generated in one second by one ampere may be found from the electrochemical equivalent of silver and the atomic weights of hydrogen, oxygen and silver, or the data given in the note to problem 6 may be used.

6. A current which produces a steady reading of 5 amperes on an ammeter generates 186.6 cubic centimeters of a mixture of oxygen and hydrogen in 3 minutes, the mixed gases being measured at a net pressure of 710 millimeters and at a temperature of 25° C. Find the error of the ammeter reading. Ans. 0.1 ampere too low.

Note. — By net pressure in this problem is meant the pressure due to the gas alone after correction has been made for the part of the pressure which is due to the water vapor that is present.

The water coulombmeter is frequently used for quickly standardizing an ammeter, and it is convenient to note that one ampere in one second generates 0.174 cubic centimeter of mixed hydrogen and oxygen, the gases being measured dry, at 760 millimeters pressure, and at a temperature of 0° C.

7. A voltaic cell which is free from local action gives a current of 1.5 amperes for 50 hours. Calculate the number of grams of zinc consumed. Ans. 91.5 grams.

Note. — The zinc consumed in a voltaic cell by voltaic action is equal to the amount of zinc that would be deposited in an electrolytic cell by the current which the cell delivers.

8. A single chromic acid cell consumes 125 grams of zinc during the time that the current from the cell is depositing 25 grams of copper from a solution of cupric sulphate ($CuSO_4$). What portion of the zinc is consumed by local action? Ans. 79.4 per cent.

9. A gravity cell is used to give a steady current of 0.1 ampere continuously, night and day, for 30 days. During this time

1668.6 grams of copper sulphate crystals are used. Find: (*a*) The amount of copper sulphate crystals which is consumed by voltaic action, and (*b*) the amount of copper sulphate crystals which is consumed by local action. Ans. (*a*) 504.6 grams usefully consumed in voltaic action and (*b*) 1,164 grams wasted in local action.

Note. — Copper sulphate crystals contain 12 molecules of water of crystallization, that is to say, the formula for copper sulphate crystals is $CuSO_4 + 12H_2O$ so that 375.9 grams of copper sulphate crystals contain 63.6 grams of copper.

10. An ordinary dry cell was connected to a circuit, the current at the start was 5.00 amperes, and the current was observed at intervals of 10 minutes, giving the following values in amperes in order: 4.20, 3.92, 3.70, 3.55, 3.40, 3.28, 3.16, 3.02, 2.90, 2.81, 2.72, 2.60, 2.54, 2.48, 2.43, 2.37, 2.30, 2.24, 2.16. Plot a curve of which the abscissas represent elapsed times and of which the ordinates represent the decreasing values of the current delivered by this cell.

11. A lead storage cell delivers 10 amperes for 8 hours. Find the increase of weight of each electrode. Ans. The positive electrode or grid gains 95.5 grams, and the negative grid gains 143.3 grams.

12. The storage cell specified in problem 11 contains 4,000 cubic centimeters of dilute sulphuric acid of which the density at 18° C. is 1.1700 grams per cubic centimeter when the cell is fully charged. Find the density of the electrolyte after the cell has delivered 10 amperes for 8 hours. Ans. 1.1286 grams per cubic centimeter.

Data Required in the Above Problems.

Atomic Weights.

Silver	107.93	Sodium	23.05
Aluminum	27.1	Oxygen	16.00
Copper	63.6	Lead	206.91
Iron	55.88	Sulphur	32.06
Hydrogen	1.01	Zinc	65.40

Density of dry hydrogen at 0° C. and 760 mm. pressure, 0.0000896 gram per cubic centimeter.

Density of dry oxygen at 0° C. and 760 mm. pressure, 0.001429 gram per cubic centimeter.

DENSITY OF DILUTE SULPHURIC ACID IN GRAMS PER CUBIC CENTIMETER AT 18° C.

0 per cent.	H_2SO_4	0.9986
10 per cent.	H_2SO_4	1.0673
20 per cent.	H_2SO_4	1.1414
30 per cent.	H_2SO_4	1.221

Per cent. of H_2SO_4 in this table means the number of grams of H_2SO_4 in 100 grams of the solution.

The electrochemical equivalent of silver is 0.001118 gram per ampere per second.

CHAPTER II.

RESISTANCE AND ELECTROMOTIVE FORCE.

HEATING EFFECT OF THE ELECTRIC CURRENT.

11. Electrical resistance.—When a pump forces water through a circuit of pipe, a part of the work expended in driving the pump reappears as heat in the various parts of the circuit of pipe because of the resistance which the pipe offers to the flow of water. Similarly, when an electric generator produces an electric current in a circuit, a part of the work expended in driving the generator reappears as heat in the various parts of the circuit. The current seems to be opposed by a kind of *resistance* * more or less analogous to the resistance which a pipe offers to the flow of water, and a portion of an electrical circuit is said to have more or less *electrical resistance* according as more or less heat is generated in it by a given current.

12. The heating effect of the electric current. Joule's law.—*The amount of heat which is generated in a given wire is proportional to the square of the current that is flowing in the wire and to the time that the current continues to flow*, that is,

$$H = RI^2t \tag{2}$$

in which H is the amount of heat generated in a wire in t seconds by a current of I amperes, and R is a constant for a given wire. The value of this factor R is used as a numerical measure of the electrical resistance of the wire.

Practical applications of the heating effect.—The heating effect of the electric current is utilized in the various forms of electric lamps in which a filament of carbon or refractory metal is heated to brilliant incandescence by the electric current. The heating

* An exact mechanical analogue of electrical resistance is given in Art. 62.

effect of the electric current is also utilized in a variety of electric furnaces.*

Definition of the ohm. — If H in equation (2) is expressed in joules,† I in amperes, and t in seconds, then R is expressed in terms of a unit which is called the ohm, that is, a wire has one ohm of resistance when one joule of heat is generated in it in one second by one ampere of current. The meaning of the factor R in equation (2) may be made clear by solving this equation for R, which gives $R = H/I^2t$. According to this equation, the resistance of a wire in ohms is equal to the joules of heat generated in it per ampere squared per second, or in other words, an ohm is one joule-per-ampere-squared-per-second. The abohm is defined in Art. 52.

The international standard ohm. — The resistance of a wire or other portion of an electrical circuit can be measured with great ease in terms of a known resistance, whereas a fundamental measurement of resistance requires elaborate arrangements, and it is very tedious if a moderate degree of accuracy is desired. Therefore, for practical purposes, the ohm has been legally defined ‡ as the resistance at the temperature of melting ice of a column of pure mercury 106.3 centimeters long, of uniform cross-sectional area, and weighing 14.4521 grams.

Measurement of resistance. — A direct method for measuring the resistance of a wire is to send a known current I through the wire for a known length of time t and to determine the amount of heat generated in the wire by means of a water calorimeter. This direct method for measuring the resistance of a wire in

* See Calcium Carbide Manufacture at Niagara, *Electrochemical Industry*, Vol. I, page 22, and Carborundum Manufacture at Niagara, *Electrochemical Industry*, Vol. I, page 50. See report of Canadian Commission on Electrothermic Processes for the Smelting of Iron and Steel, by Eugene Haanel.

† Ordinarily heat is expressed in terms of the calorie but it is desirable in the present instance to express heat in joules, one joule of heat being an amount of heat which is equivalent to one joule of work. One calorie is equal to 4.2 joules. One joule of work is the amount of work done in one second by an agent which does work at the rate of one watt. One watt is equal to 1/746 of a horse-power.

‡ In accordance with the recommendations of the International Electrical Congress which met at Chicago in 1893.

ohms is never used because it is tedious and inaccurate. Practical methods for measuring resistance are described in Chapter X.

13. Power required to maintain a current in a circuit, expressed in terms of resistance and current. — When all of the energy which is delivered to an electrical circuit by a generator reappears in the circuit as heat, then the rate at which work is delivered to the circuit by the generator is equal to the rate at which energy reappears in the circuit as heat. Equation (2) expresses the amount of heat in joules which appears in a circuit of wire in t seconds; dividing this amount of heat by the time t, gives the rate at which heat appears in the circuit in joules per second (watts), and this is equal to RI^2. Therefore the power P, in watts, required to maintain a current of I amperes in a circuit of which the resistance is R ohms, is

$$P = RI^2 \tag{3}$$

14. Dependence of resistance upon length and size of a wire. — The resistance R of a wire of given material is directly proportional to the length l of the wire and inversely proportional to the sectional area s of the wire; that is,

$$R = k\frac{l}{s} \tag{4}$$

in which k is a constant for a given material; it is called the *resistivity* * of the material. The exact meaning of the factor k may be made apparent by considering a wire of unit length ($l = 1$) and unit sectional area ($s = 1$). In this case k is numerically equal to R, that is to say, the resistivity of a material is numerically equal to the resistance of a wire of that material of unit length and unit sectional area. Electrical engineers nearly always express lengths of wires in feet and sectional areas in circular mils.† If equation (4) is to be used to calculate

* Sometimes called *specific resistance*. The reciprocal of the resistivity of a substance is called its *conductivity*.

† One mil is a thousandth of an inch. One circular mil is the area of a circle of which the diameter is one mil. The area of any circle in circular mils is equal to the square of the diameter of the circle in mils.

the resistance of a wire in ohms when the length of the wire is expressed in feet and the sectional area in circular mils, then the value of k must be the resistance of a wire of the given material one foot long and one circular mil in sectional area; for example, the resistance of a copper wire one foot long and one circular mil in sectional area is about 10.4 ohms at 20° C.

TABLE.—RESISTIVITIES AND TEMPERATURE COEFFICIENTS.

	a	*b*	*c*
Aluminum wire (annealed) at 20° C.	27.4×10^{-7}	16.5	+0.0039
Copper wire (annealed) at 20° C.	17.24×10^{-7}	10.4	+0.0040
Iron wire (pure annealed) at 20° C.	95×10^{-7}	58.0	+0.0045
Steel telegraph wire at 20° C.	150×10^{-7}	91‡	+0.0043‡
Steel rails at 20° C.	120×10^{-7}	72‡	+0.0035‡
Mercury at 0° C.	943.4×10^{-7}	—	+0.00088
Platinum wire at 0° C.	89.8×10^{-7}	54.0	+0.00354
German-silver wire at 20° C.	212×10^{-7}	127‡	+0.00025‡
Manganin wire (Cu 84, Ni 12, Mn 4) at 20° C.	475×10^{-7}	286	—*
"Ia Ia" metal wire, hard (copper-nickel alloy) at 20° C	500×10^{-7}	300‡	—0.00001‡
"Climax" or "Superior" metal (nickel-steel alloy) at 20° C.	800×10^{-7}	480‡	+0.00067‡
Arc-lamp carbon at ordinary room temperature	0.005		—0.0003‡
Sulphuric acid, 5 per cent. solution at 18° C.	4.8 ohms		—0.0120†
Ordinary glass at 0° C. (density 2.54)	10^{15} ohms‡		
Ordinary glass at 60° C.	10^{12} ohms‡		
Ordinary glass at 200° C.	10^{8} ohms‡		

a = resistance in ohms of a bar 1 centimeter long and 1 square centimeter sectional area.

b = resistance in ohms of a wire 1 foot long and 0.001 inch in diameter.

c = temperature coefficient of resistance per degree centigrade (mean value between 0° C. and 100° C.).

* See temperature-resistance curve, Fig. 15.

† Between 18° C. and 19° C.

‡ These values differ greatly with different samples.

15. Resistivities of alloys.—The ordinates of the three curves in Fig. 12 represent the resistivities at a given temperature of alloys of zinc and tin, of silver and gold, and of silver and platinum, respectively, and the abscissas represent the percentages of the constituent metals. The zinc-tin line, marked Zn + Sn, is sensibly straight; that is, the change of resistance from pure zinc to pure tin is proportional to the percentage of tin in the alloy.

The silver-platinum line marked Ag + Pt, and the silver-gold line, marked Ag + Au, are not straight. In particular, it is to be noticed that a very small percentage of platinum added to pure silver increases the resistance of the metal very greatly indeed.

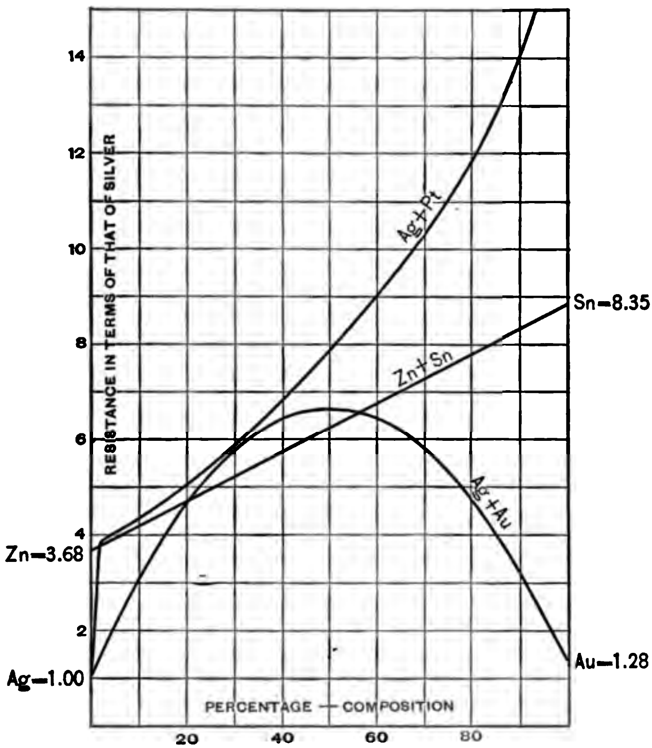


Fig. 12.

In respect to electrical resistance, the alloys of tin, lead, cadmium and zinc are similar to the alloys of zinc and tin, that is to say, the resistivity varies in proportion to the percentage of one of the metals in the alloy. Alloys of most other metals are more or less similar to the alloys of silver and gold and of silver and platinum, and, in general, the addition of a very small percentage of one metal to another increases the resistivity greatly. The exact opposite to this is true of many non-metallic substances, a

pure substance has a very high resistance and the admixture of a very small quantity of another substance reduces its resistance very greatly indeed. Thus a beaker of freshly distilled water (free from air) in which are placed two clean platinum electrodes has a resistance of, say, 25,000 ohms and the addition of one one-thousandth of one per cent. of sulphuric acid reduces the resistance to a few hundreds of ohms.

16. The rheostat. — An arrangement for inserting more or less resistance into an electrical circuit at will is called a *rheostat.* Figure 13 shows the usual arrangement of a rheostat. A number of resistances *rrrr* are connected to terminal blocks of metal *bbbbb* and a contact finger *f* of metal, broad enough to bridge over the space between the adjacent blocks *bb*, is arranged so that it can

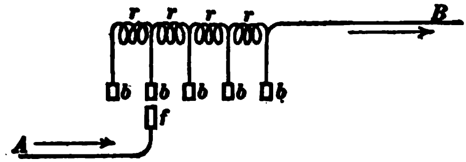


Fig. 13.

be moved sidewise, thus connecting any number of resistances *r* in circuit between the terminals *A* and *B* of the rheostat. The resistances *rrrr*, Fig. 13, are usually made of metal of high specific resistance so that the wire may be of moderate length and yet large enough to be mechanically strong and to have sufficient area to radiate the heat which is generated in it by the current. One of the most satisfactory of these high resistance metals is a nickel-steel alloy which is known in commerce under the name of "Climax" metal or "Superior" metal.

The so-called water rheostat which is frequently used consists of two electrodes dipping into a vessel, or tank, containing a weak solution of common salt. The current enters at one electrode, flows through the salt solution, and leaves it at the other electrode, and the resistance can be adjusted by varying the amount of salt in solution or by moving the electrodes.

17. Variation of resistance with temperature. — The electrical resistance of a wire, or of a liquid column which forms a portion of an electrical circuit, varies with temperature. Consider, for example, (*a*) an iron wire, (*b*) a copper wire, (*c*) a platinum wire, (*d*) a german-silver wire, (*e*) a carbon rod, and (*f*) a column of dilute sulphuric acid, each of which has a resistance of 100 ohms

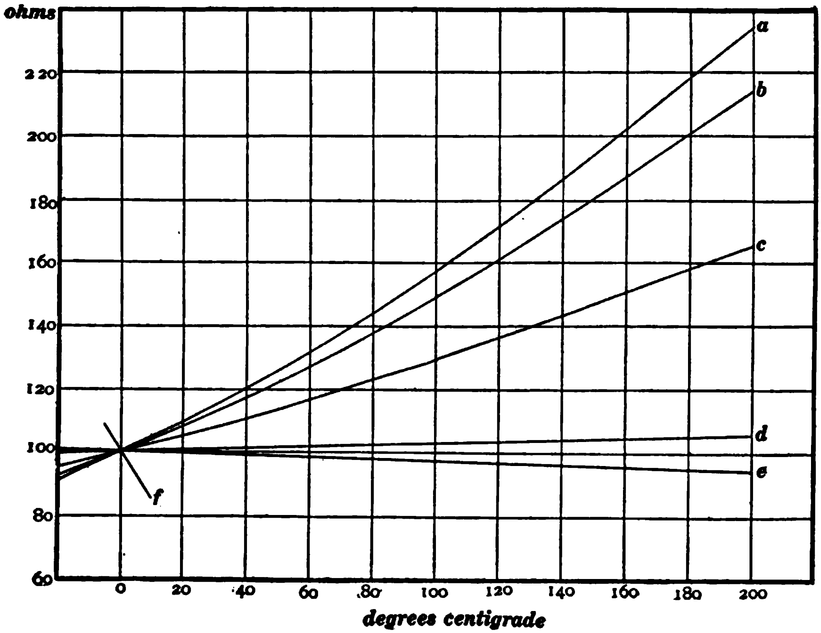


Fig. 14.

at 0° C. The values of the resistance of (*a*), (*b*), (*c*), (*d*), (*e*) and (*f*) at other temperatures are shown by the ordinates of the curves in Fig. 14. It is evident from Fig. 14 that iron and copper increase very greatly in resistance with rise of temperature, and that german silver increases slightly, whereas the carbon and sulphuric acid decrease in resistance with rise of temperature. All pure metals increase in resistance with rise of temperature in approximately the same ratio, alloys usually increase in resistance with rise of temperature but to a much smaller extent than pure

metals, and all acids and salt solutions decrease in resistance with rise of temperature.

A rod of a substance like glass or porcelain has, at ordinary room temperature, a resistance which is expressed in millions of millions of ohms, but the resistance decreases rapidly with rise of temperature. Both glass and porcelain become fairly good conductors at a low red heat. This is strikingly shown by the following experiment: Fine copper wire is wound around the ends of a thin-walled glass tube about 20 centimeters long, and these copper wires are connected through a fairly high metallic resistance to the terminals of a 1,000-volt transformer. The side of the tube is then heated with a blast lamp. At a low red heat a sufficient amount of current begins to flow to develop a very considerable amount of heat, and the glass tube becomes still hotter, which permits still more current to flow, which makes the glass tube still hotter, and so on, until the glass tube melts down because of the heat which is generated in it by the flow of current.

Alloys which change but little in resistance with change of temperature are especially suitable for resistance standards and resistance boxes.* Wires of manganin † are now almost universally employed for this purpose. Figure 15 ‡ shows the change of resistance of a manganin wire with temperature. A manganin wire which has a resistance of 100 ohms at 15° C. has a resistance of 100.01 ohms at 20° C.; a german-silver wire which has a resistance of 100 ohms at 15° C., has about 100.2 ohms resistance at 20° C.; and a copper wire which has resistance of 100 ohms at 15° C., has about 102 ohms resistance at 20° C.; that is, for the specified rise of temperature the change of resistance of the manganin wire is only 0.01 per cent., the change of resistance of the german-silver wire is 0.2 per cent., and the change of resistance of the copper wire is 2 per cent.

* See Chapter X.

† Manganin is an alloy of 84 parts by weight of copper, 12 parts by weight of nickel, and 4 parts by weight of manganese.

‡ From the results of Dr. Lindeck. See the *Proceedings of the International Electrical Congress*, Chicago, 1893, page 165.

Temperature coefficient of resistance. — The curves a, b and c in Fig. 14 are approximately straight lines; the same is true of the temperature-resistance curves of all pure metals and of many alloys. Therefore, the increase of resistance of a wire from a standard temperature, say, 0° C., to any other temperature t° C.

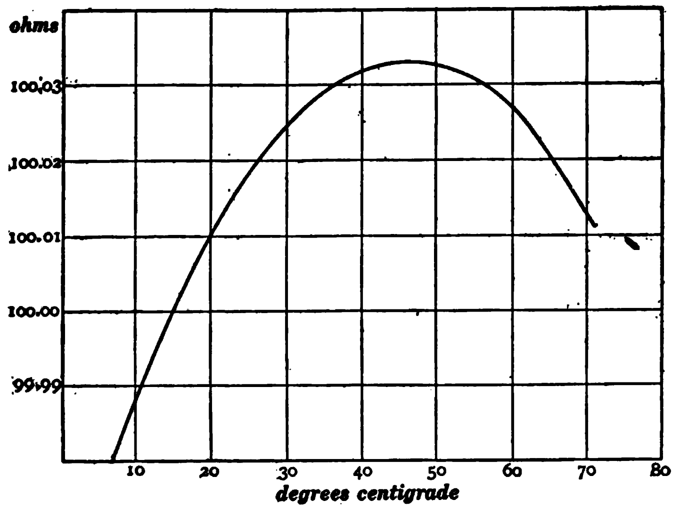


Fig. 15.

is approximately proportional to t, and in every case the increase of resistance is exactly proportional * to the resistance of the wire at the standard temperature. Therefore the increase of resistance from 0° C. to t° C. may be expressed as $\beta R_0 t$, where R_0 is the resistance of the wire at 0° C. and β is a factor which is approximately constant for a given metal. The resistance of the wire at t° C. is equal to $R_0 + \beta R_0 t$, so that, writing R_t for the resistance of the wire at t° C., we have

$$R_t = R_0(1 + \beta t) \tag{5}$$

* This is analogous to the fact that the increase of length of a metal bar due to a given rise of temperature is exactly proportional to the initial length of the bar. Consider for example, a bar 10 feet long at 0° C. When the temperature is increased, each foot of the bar increases its length by a certain fractional part of a foot, and the entire bar increases its length by the same fractional part of its total initial length.

The factor β is called the *temperature coefficient of resistance* of the given metal. It is equal to the increase of resistance of the metal for one degree rise in temperature expressed as a fractional part of the resistance of the metal at 0° C. Its value for pure metals is approximately 0.0037 per degree centigrade. For pure commercial copper wire its value is about 0.004 per degree centigrade.

It is to be remembered that equation (5) is based on the assumption that the temperature-resistance curve is a straight line. If the actual resistances of any wire or substance at 0° C. and at t° C. are substituted in equation (5) for R_0 and R_t, respectively, the value of β may be calculated. The value of β so calculated is called the *mean* temperature coefficient of resistance of the given substance for the given range of temperature.

The value of the temperature coefficient of a substance depends upon the choice of the standard temperature in a way that may be most easily explained by considering the thermal expansion of a gas. A gas at constant pressure undergoes a certain definite increment of volume for one degree rise of temperature. Thus, a gas at constant pressure undergoes the same increment of volume when heated from 10° C. to 11° C., or when heated from 50° C. to 51° C., or when heated from 200° C. to 201° C. This increment of volume per degree rise of temperature is equal to $\frac{1}{273}$ of the volume of the gas at 0° C., to $\frac{1}{274}$ of the volume of the gas at 1° C., to $\frac{1}{373}$ of the volume of the gas at 100° C., and so on, and this fraction is the temperature coefficient of expansion of the gas. In order to avoid ambiguity, the increment of volume of a gas for 1° rise of temperature is always expressed as a fractional part of the volume of the gas at 0° C., and the coefficient of expansion of a gas at constant pressure is therefore equal to $\frac{1}{273}$ (equals 0.00366). Similarly, the temperature coefficient ot resistance of a metal should always refer to a definite standard temperature, say, 0° C. It is interesting to note that the temperature coefficient of resistance of most pure metals is very nearly the same in value as the temperature coefficient of expansion of a

gas at constant pressure. That is to say, the resistance of a wire made of pure metal is approximately proportional to the absolute temperature.

Electromotive Force.

18. Power delivered by an electric generator. Definition of electromotive force. — From Faraday's laws of electrolysis it is evident that the amount of zinc consumed per second in a voltaic cell by voltaic action is proportional to the strength of the current. Therefore the available * energy developed per second by the chemical action in the cell is proportional to the strength of the current, or in other words, the electrical energy developed per second by a given type of voltaic cell in the maintenance of a current is equal to a constant multiplied by the current. That is,

$$P = EI \tag{6}$$

in which P is the electrical energy developed per second by a voltaic cell, I is the current produced by the cell, and E is a constant for the given type of cell. This constant E is called the *electromotive force* of the cell. This definition of electromotive force applies to any form of electric generator. Imagine a dynamo driven at constant speed, and having a field magnet of which the strength is invariable. Ignoring friction, the only opposition to the motion of the dynamo is that which is due to the current flowing through the armature wires. Therefore to double the current output of such a dynamo would double the force required to drive it,† and therefore double the rate at which work would be expended in driving it, its speed being constant; but the work which would be expended in driving such a dynamo would all go to maintain the current, so that the rate at which

* Available, that is, for the production of current. In some voltaic cells the whole of the energy developed by the voltaic action goes to maintain the current; but, in general, a definite fractional part only of this energy is available for the production of an electric current. See *Physical Chemistry*, H. C. Jones, pages 376–405. See also papers by H. S. Carhart "On the Thermodynamics of the Voltaic Cell," *Physical Review*, Vol. XI, p. 1, Vol. XVI, p. 248, and Vol. XXVI, p. 209, March, 1908.

† See Art. 52, on the side push of a magnetic field on an electric wire.

work is expended in maintaining the current is proportional to the current, according to equation (6).

Hydraulic analogue of electromotive force. — An electric generator, such as a voltaic cell or a dynamo, is analogous to a centrifugal pump, or fan blower, which develops a definite difference of pressure between its inlet and outlet. Imagine a fan blower connected to a circuit of pipe which goes out from the outlet and returns to the inlet. The volume of air per second forced through this pipe may be called the *strength* of the air current, and the rate at which the fan delivers energy in the maintenance of this air current is equal to the product of the strength of the air current and the pressure difference between inlet and outlet of the fan. Let I be the strength of the air current (volume of air flowing per second) and let E be the pressure difference between inlet and outlet. The power developed by the fan in maintaining the flow of air is

$$P = EI$$

This equation is identical to equation (6), and the pressure difference between inlet and outlet of the fan blower is exactly analogous to what is called the electromotive force of an electric generator.

Note. — The power delivered by a fan to a circuit of pipe is not strictly proportional to the volume of air delivered per second because an increased flow of air usually causes a slight decrease in the speed of the fan. Similarly, the power delivered to a circuit of wire by a voltaic cell or dynamo is not strictly proportional to the strength of the current because an increase of current usually causes a decrease in the electromotive force of the cell or generator. This decrease of electromotive force of a voltaic cell is called *polarization* and it is discussed in Art. 22. The decrease of electromotive force of a dynamo due to increase of current output is generally due to a slight decrease of speed or to a weakening of the field magnet, or to both.

Definition of the volt. — When P in equation (6) is expressed in watts (joules of work per second) and I in amperes, then E is expressed in terms of a unit which is called the *volt*. That is to say, the electromotive force of an electric generator in volts is equal to the power in watts delivered by the generator divided by the current in amperes, or in other words, the power delivered by a generator in watts is equal to the current delivered by the generator in amperes multiplied by the electromotive force of

the generator in volts. The abvolt or c.g.s. unit of electromotive force is defined in Art. 52.

Unsatisfactory character of the fundamental definition of electromotive force. — The definition of any physical quantity consists, in every case, of a concise statement of the fundamental method of measuring that quantity, and when this fundamental method of measuring a quantity involves operations which are not feasible under ordinary conditions of practical work, the definition seems more or less unsatisfactory. Thus, the above definition of electromotive force as units-of-work-per-second-per-ampere (P/I) assumes that the rate of doing work in a pushing current through a circuit is to be measured directly in mechanical units, and no method is specified for doing this. The simplest definition of electromotive force is based on Ohm's Law as explained in the following article.

19. Ohm's Law. — The current produced by a voltaic cell, or, in general, by any electric generator, is inversely proportional to the resistance of the circuit.* This relation was discovered by G. S. Ohm in 1827 and it is called Ohm's Law. A complete statement of Ohm's Law together with a clear specification of the conditions under which the law applies may be derived as follows: The power output of an electric generator is equal to EI according to equation (6). **If the whole of this power is used to heat the circuit in accordance with Joule's Law,** then we must have

$$EI = RI^2$$

according to equation (3). Therefore we have

$$E = RI \tag{7a}$$

or

$$I = \frac{E}{R} \tag{7b}$$

*This statement and the statement given in the previous article to the effect that the power output of a generator is proportional to the current, are not exactly true, because of the fact that the electromotive force of a generator usually falls off in value, to some extent, when the generator is called upon to give an increased current.

Definition of the volt on the basis of Ohm's Law. — According to equation (7*a*), the electromotive force required to force a current through a circuit is equal to the product of the resistance of the circuit and the current. When the resistance is expressed in ohms and the current in amperes, this equation gives the value of the electromotive force in volts. That is to say, a voltaic cell, or any electric generator (assumed, for the sake of simplicity of statement to have no internal resistance and to be unaffected by those secondary influences which cause a decrease of electromotive force with delivery of current), has an electromotive force of one volt if it produces one ampere of current in a circuit of which the resistance is one ohm.

20. Application of equations (2), (6) and (7) to a portion of an electrical circuit. — Equation (2) expresses the heat which is generated in a portion of the electrical circuit, R being the resistance of that portion. Equation (6) expresses the power which is delivered to a portion of an electrical circuit, E being the electromotive force across the terminals of that portion. Equation (7) expresses the relationship between the current in an electrical circuit, the electromotive force across any given portion of the circuit, and the resistance of that portion.

The current produced by a voltaic cell not only flows through the wire which is connected to the terminals of the cell, but it flows also through the electrolyte in the cell. Let E_tI represent the total rate at which work is supplied by the voltaic cell in the maintenance of the current, let R_x be the resistance of the external circuit of wire, and let R_a be the resistance of the electrolyte and electrodes in the cell. Then the rate at which heat is generated in the entire circuit is $(R_a + R_x)I^2$, and this is equal to E_tI, so that

$$E_t = R_xI + R_aI$$

whence

$$R_xI = E_t - R_aI \tag{8}$$

but R_xI is the electromotive force which is required to force the current I through the external resistance R_x; that is, R_xI is

the *actual electromotive force between the terminals of the cell while it is delivering current.* Therefore, we have

$$E_x = E_t - R_a I \tag{9}$$

in which E_x is the electromotive force across the terminals of the cell while it is delivering current, and, inasmuch as $E_x = R_x I$, and $E_x I = R_x I^2$, we may write:

$$I = \frac{E_x}{R_x} \tag{10}$$

and

$$P_x = E_x I \tag{11}$$

in which P_x is the power delivered by the cell to the external circuit. In these equations E_t is the total electromotive force of the voltaic cell (or generator), $R_a I$ is the portion of this total electromotive force which is used to overcome the resistance of the cell (or generator), $E_x = (E_t - R_a I)$ is the electromotive force between the terminals of the cell (or generator), and P_x is the power delivered to the external circuit which does not include the power developed in heating the cell (or generator).

Equations (6) and (7) are nearly always used in practice in their application to a portion of a circuit. Thus, Fig. 16 shows a battery B supplying current to a lamp L, the electromotive force between the terminals of the lamp is E, the current flowing in the circuit is I, the power delivered to the lamp is EI, and the current is equal to the electromotive force between the terminals of the lamp divided by the resistance of the lamp, according to equation (7).

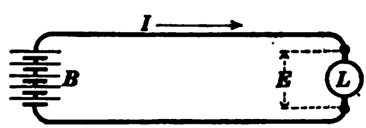


Fig. 16.

Voltage drop in a generator.—The electromotive force $R_a I$ required to overcome the resistance of the generator (or voltaic cell) in the above discussion is subtracted from the total electromotive force of the generator to give the electromotive force between the generator terminals, as indicated in equation (8). This electromotive force $R_a I$ which is used to overcome the resistance

of a generator is called the *electromotive force drop* or *voltage drop* in the generator. It is analogous to the decrease of pressure-difference between the terminals of a fan blower due to the resistance which is encountered by the stream of air in passing through the fan chamber.

Voltage drop in a transmission line. — A current of I amperes is delivered to a distant lamp or motor over a pair of wires the combined resistance of which is R ohms. Let E_0 be the electromotive force across the terminals of the generator, and let E_1 be the electromotive force across the terminals of the distant lamp.

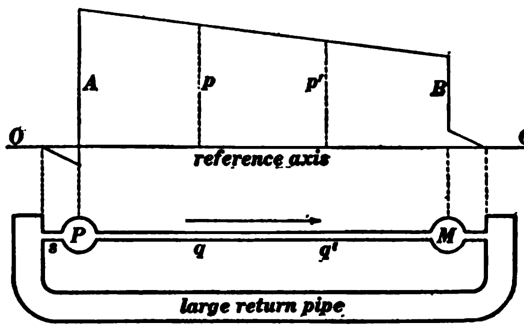


Fig. 17a.

The difference between the voltage across the terminals of the generator and the voltage across the terminals of the lamp, namely, $E_0 - E_1$ is equal to the electromotive force which is used to overcome the resistance of both wires, namely, RI volts. This loss of electromotive force over a transmission line is called the *voltage drop over the line.*

Example. — The electromotive force across the terminals of a generator is 115 volts. The generator supplies 100 amperes of current to a motor at a distance of 1,000 feet, and the wire (2,000 feet) used for the transmission has a total resistance of 0.05 ohm. The voltage drop over the line is 100 amperes × 0.05 ohm, or 5 volts, and therefore the voltage across the terminals of the motor is 115 volts − 5 volts = 110 volts.

Hydraulic analogue of voltage drop. Definition of potential

difference. — Figure 17*a* represents a pump P forcing water through a small pipe and through a distant water motor M, the water being returned to the pump through a very large and approximately frictionless pipe. The motor may be most conveniently thought of as an ordinary pump with a piston, but driven as a motor by the water which is forced through it by P. Choosing the pressure in the large pipe as zero or reference pressure, the pressure at any other point in the system is to be specified by giving its value above or below the pressure in the large pipe. The pump draws water through the supply pipe s, and the pressure in this small pipe falls below the zero line or axis OO. At the pump there is a sudden rise of pressure which is represented by the ordinate A, and the friction of the long pipe causes a steady drop of pressure until the motor M is reached. There is a sudden drop of pressure at the motor which is represented by the ordinate B, and then a slow drop of pressure along the remaining portion of the small pipe. In the diagram OO, the pump and motor are supposed to be located at definite *points* so that the rise of pressure in the pump and the drop of pressure in the motor are represented by the vertical ordinates A and B.

Figure 17*b* represents an electric generator G forcing an electric current through a small conductor and through a distant electric motor M, the current being returned to the generator through a very large conductor of negligible resistance. Choosing the line OO as a reference axis, the electromotive force between the point P and any other point in the system may be represented by an ordinate measured upwards or downwards from the reference axis. In the diagram OO the generator and motor are supposed to be located at definite *points* so that the propelling electromotive force of the generator is represented by a vertical ordinate A, and the opposing electromotive force of the motor is represented by the vertical ordinate B.

When one has chosen a reference point, like P, Fig. 17b, in an electrical system, the electromotive force between that point and any

other point in the system is called the electric potential at the other point. — Thus, the ordinates p and p' in Fig. 17*b* represent the

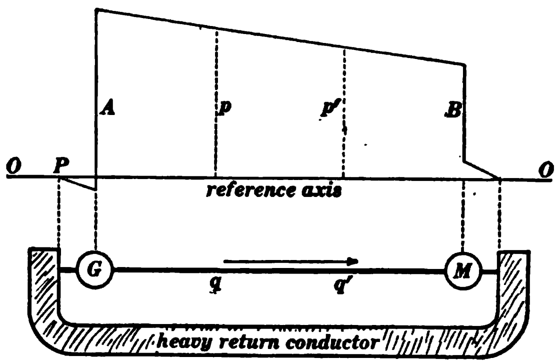


Fig. 17b.

values of the electric potential at the two points q and q' on the wire in the same way that the ordinates p and p' in Fig. 17*a* represent the values of the hydrostatic pressure at the points q and q' on the small pipe, that is to say, the potential at a point in an electrical system is analogous to the hydrostatic pressure at a point in hydraulics; and the electromotive force between two points in an electrical system which by definition is equal to the difference of potential between those points, is analogous to the difference of pressure between two points in a hydraulic system

21. Voltmeters * and ammeters. — Figure 18*a* shows an ammeter A arranged to measure the current delivered by a gener-

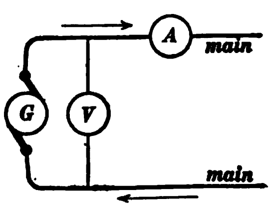


Fig. 18a.

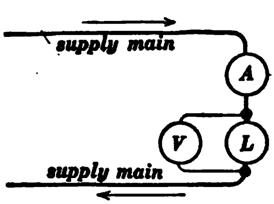


Fig. 18b.

* The voltmeter is essentially a high-resistance ammeter except in the case of the electrostatic voltmeter which is seldom used. Thus, an ammeter gives a definite deflection with a certain current I flowing through it, and the electromotive force be-

ator G, and a voltmeter V connected so as to indicate the electromotive force between the terminals of the generator. Figure 18*b* shows an ammeter A and a voltmeter V arranged to measure the power delivered to a lamp L.

An ammeter must have a very low resistance in order that it may not obstruct the flow of current in a circuit in which it is placed. A voltmeter must have a high resistance in order that it may not take sufficient current to disturb the system to which it is connected. Thus, the well-known voltmeter of the Weston Electric Company having a scale ranging from zero to 150 volts has a resistance of about 15,000 ohms, so that it takes about 0.01 ampere when it is connected to a 150-volt generator. When an ammeter and a voltmeter are arranged to measure the power delivered to a lamp, as shown in Fig. 18*b*, the ammeter reading should be taken when the voltmeter circuit is open in order that the ammeter reading may indicate the true current flowing through the lamp..

22. Polarization * of the voltaic cell. — When a voltaic cell delivers current, the chemical action in the immediate neighborhood of the electrodes exhausts the electrolyte, and the electromotive force of the cell falls off greatly. Thus, the ordinates of the curve AA in Fig. 19 represent the values of the electromotive force of a dry cell after it has been delivering a fairly large current for one minute, for two minutes, for three minutes,

tween the terminals of the instrument is equal to RI, where R is the resistance of the instrument. If the instrument is to be used as an ammeter the position of the pointer is marked with the number which gives the value of I in amperes, if the instrument is to be used as a voltmeter the position of the pointer is marked with the number which gives the value of RI in volts.

The instrument described in Art. 1 and shown in Fig. 3 may be considered to be a voltmeter if it has a high resistance.

* The word polarization has two distinct meanings in its application to electrolysis. The *polarization of a voltaic cell* means the decrease of electromotive force of the cell due chiefly to changes of concentration of the electrolyte in the neighborhood of the electrodes of the cell as the cell delivers current; and the *polarization of an electrode*, as this term is generally used in scientific writings, means the total electromotive force between the electrode and the electrolyte. See *Practical Physics*, Franklin, Crawford and MacNutt, Vol. II, pages 136–147.

and so on, the electromotive force being measured in each case on open circuit (the cell being disconnected from the circuit to which it delivers current and connected to a voltmeter for a moment when it is desired to read its electromotive force).

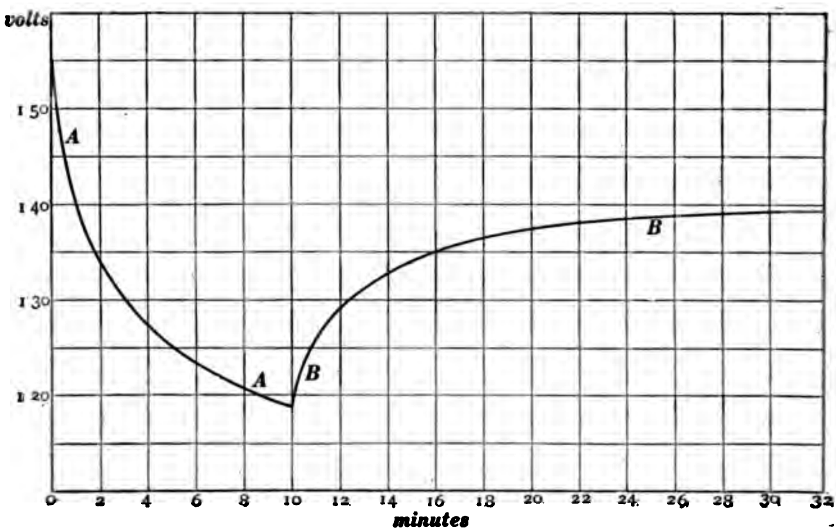


Fig. 19.

When a voltaic cell has been polarized by delivering current for some time, its electromotive force rises slowly when it is left standing on open circuit. This recovery of a voltaic cell from polarization is due chiefly to the refreshing of the electrolyte in the neighborhood of the electrodes by the slow diffusion of the acid or salt from distant portions of the electrolyte to the surfaces of the electrodes. The ordinates of the curve *BB* in Fig. 19 show the increasing values of the electromotive force of a dry cell standing on open circuit after it has been allowed to deliver current for some time.

Branched Circuits.

23. Series and parallel connections.—When two portions of an electric circuit are so connected that the entire current in the circuit passes through both portions, the portions are said to be

onnected in *series.* When two portions of an electrical circuit re so connected that the current in the circuit divides and part f it flows through each portion, the portions are said to be connected in *parallel.* Thus, Fig. 20 shows two lamps L and L'

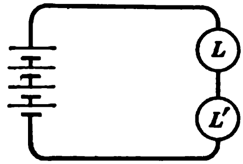


Fig. 20.

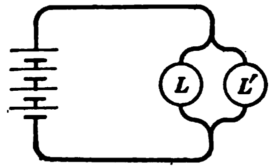


Fig. 21.

connected in series, and Fig. 21 shows two lamps connected in parallel.

The ordinary arc lamps which are used to light city streets are connected in series, and the entire current delivered by the lighting generator flows through each lamp. On the other hand, if the electromotive force of the generator is, say, 2,000 volts and if there are 40 similar * lamps in series, the electromotive force between the terminals of each lamp will be 50 volts. *The electromotive force of a generator is subdivided among a number of lamps or other units connected in series.*

The ordinary glow lamps which are used for house-lighting are connected in parallel between copper mains which lead out from the terminals of the generator, and, except for a slight drop of electromotive force in the mains, the full electromotive force of the generator acts upon each lamp. On the other hand, if the generator delivers, say, 1,000 amperes and if there are 2,000 similar * lamps connected between the mains, the current in each lamp will be one half ampere. *The current delivered by a generator is subdivided among a number of lamps or other units connected in parallel.*

Voltaic cells are often connected in series. When this is done the electromotive force which is available for the maintenance of current is equal to the sum of the electromotive forces of the indi-

* Having the same resistance.

vidual cells. Figure 22 is a top view of three dry cells connected in series and delivering current to a circuit R.

A number of voltaic cells of the same kind are often connected in parallel. When this is done the total current delivered by the set is equal to the sum of the currents delivered by the individual

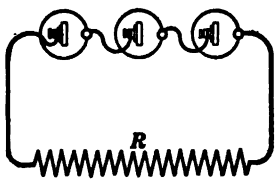


Fig. 22.

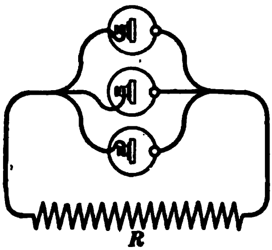


Fig. 23.

cells, and the electromotive force of the set is the same as the electromotive force of a single cell. Figure 23 is a top view of three dry cells connected in parallel and delivering current to a circuit R.

Sometimes it is desirable to connect a number of cells in groups, each group containing a number of cells in series, and to connect

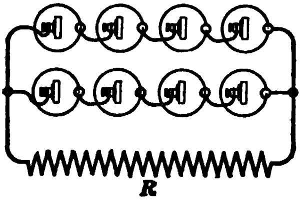


Fig. 24.

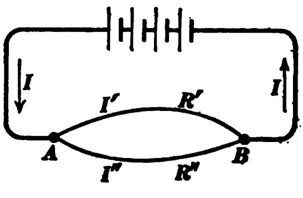


Fig. 25.

these groups of cells in parallel. Figure 24 is a top view showing two groups of dry cells connected in parallel, each group consisting of four cells connected in series.

24. Discussion of the division of current in two branches of a circuit. — Figure 25 shows a battery delivering current to a circuit which branches at the points A and B. Let I be the current in the main circuit, I' the current in the upper branch,

I'' the current in the lower branch, R' the resistance of the upper branch, and R'' the resistance of the lower branch. The product $R'I'$ is the electromotive force between the points A and B, the product $R''I''$ is also equal to the electromotive force between the points A and B, and therefore we have

$$R'I' = R''I'' \tag{12}^*$$

The current in the main part of the circuit is equal to the sum of the currents in the various branches into which the circuit divides. Therefore we have the equation

$$I = I' + I'' \tag{13}^*$$

It is an easy matter to determine the values of I' and I'' [with the help of equations (12) and (13)] in terms of the total current I and the resistances R' and R'' of the respective branches. It is important to note that a *a definite fractional part of the total current flows through each branch, and equation* (12) *shows that the currents I' and I'' are inversely proportional to the resistances R' and R'' respectively.* Thus, if R' is nine times as large as R'', then I'' is nine times as large as I', so that I'' must be equal to nine tenths of I, and I' must be equal to one tenth of I.

25. Combined resistance of a number of branches of a circuit. — (*a*) The combined resistance of a number of lamps or other units connected in series is equal to the sum of the resistances of the individual lamps. (*b*) The combined resistance of a number of

* Equations (12) and (13) express two principles which were first enunciated by Kirchhoff and which are usually called Kirchhoff's laws, as follows:.

(*a*) Equation (12) may be written $R'I' - R''I'' = 0$, which means that the *sum* of the RI drops *taken in a chosen direction* around the mesh formed by the two branches of the circuit is equal to zero. This relation is true of a mesh of any network of conductors. If one side of the mesh contains a voltaic cell of which the electromotive force is E, then the sum of the RI drops around the mesh is equal to E.

(*b*) Equation (13) may be written $I - I' - I'' = 0$, which means that the *sum* of the currents flowing *towards* one of the branch points A or B is equal to zero. This relation may be generalized as follows: The sum of the currents flowing towards a branch point in any network of conductors is equal to zero.

lamps or other units connected in parallel is equal to the reciprocal of the sum of the reciprocals of the respective resistances. The proposition (*a*) is almost self-evident. Proposition (*b*) may be established as follows: Let E be the electromotive force between the points A and B where the circuit divides into a number of branches. Then, according to Ohm's Law, we have

$$I' = \frac{E}{R'} \tag{i}$$

$$I'' = \frac{E}{R''} \tag{ii}$$

$$I''' = \frac{E}{R'''} \tag{iii}$$

where R', R'' and R''' are the resistances of the respective branches, and I', I'' and I''' are the currents flowing in the respective branches.

Let I be the total current flowing in the circuit ($=I'+I''+I'''$). The combined resistance of the branches is defined as the resistance through which the electromotive force E between the branch points would be able to force the total current I. That is, the combined resistance is defined by the equation

$$I = \frac{E}{R_c} \tag{iv}$$

in which R_c is the combined resistance. Adding equations (i), (ii) and (iii), member by member, and substituting E/R_c for $I' + I'' + I'''$, we have

$$\frac{E}{R_c} = \frac{E}{R'} + \frac{E}{R''} + \frac{E}{R'''} \tag{v}$$

whence

$$R_c = \frac{1}{\frac{1}{R'} + \frac{1}{R''} + \frac{1}{R'''}} \tag{14}$$

26. Typical problem in branched circuits. — The battery in Fig. 25 has an electromotive force of 15 volts; the battery and the wires which connect the battery to the points A and B have a total resistance of 2 ohms; the upper branch has a resistance of 3 ohms ($R' = 3$) and the lower branch has a resistance of 4 ohms ($R'' = 4$), and it is required to find: (*a*) the combined resistance of the two branches and total resistance of the circuit, (*b*) the total current, (*c*) the electromotive force between the branch points, (*d*) the current in the upper branch, and (*e*) the current in the lower branch.

(*a*) The combined resistance of the two branches is the reciprocal of $(\frac{1}{3} + \frac{1}{4})$, or $\frac{12}{7}$ of an ohm. Therefore the total resistance of the circuit through which the battery sends current is $3\frac{5}{7}$ ohms.

(*b*) The total current is found by dividing the electromotive force of the battery by the resistance of the circuit, which gives $4\frac{1}{26}$ amperes.

(*c*) The electromotive force between the branch points is equal to the product of the total current by the combined resistance of the two branches or to $4\frac{1}{26}$ amperes times $1\frac{5}{7}$ ohms, which gives $6\frac{12}{13}$ volts.

(*d*) The current in the upper branch is found by dividing the electromotive force between the branch points by the resistance of the upper branch, which gives $2\frac{4}{13}$ amperes.

(*e*) The current in the lower branch is found by dividing the electromotive force between the branch points by the resistance of the lower branch, which gives $1\frac{19}{26}$ amperes.

27. The use of shunts with galvanometers and ammeters. — In the use of a galvanometer, or other current-measuring instrument, it is frequently not desirable to send the whole of the current which is to be measured through the instrument. In such a case a definite fractional part of the current may be diverted by making the instrument one of two branches of the circuit, as shown in Fig. 26*a*, in which A represents the galvanometer or ammeter

and s represents the auxiliary branch. This auxiliary branch is called a *shunt.*

Example. — A galvanometer (or ammeter) of which the resistance is R ohms is shunted by a resistance of $R/99$ ohms. In this case 99 times as much current flows through the shunt as through the galvanometer, that is, $\frac{1}{100}$ of the total current flows through the galvanometer and $\frac{99}{100}$ of the total current flows through the shunt.

28. Use of voltmeter multiplying coils. — Suppose one has a voltmeter which is capable of indicating the value of any electromotive force up to a limit of 10 volts (more than 10 volts throws the pointer off the scale, and much more than 10 volts may damage the instrument). Let R be the resistance of the

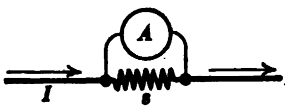


Fig. 26a.

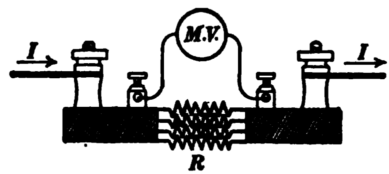


Fig. 26b.

instrument. Let an auxiliary resistance equal to $(n - 1)R$ be connected in series with the instrument, let the combination be connected to an electromotive force which is to be measured, and let E be the reading of the instrument; then the value of the electromotive force is equal to nE. This is evident if we consider that a definite deflection on the voltmeter means a definite current flowing through the instrument. Let this current be represented by I. Then the electromotive force between the terminals of the instrument is RI and this is the electromotive force which is indicated by the instrument reading, whereas the electromotive force between the terminals of the combination is equal to the product of the current times the resistance of the combination, or nRI.

29. Use of a standard shunt and a millivoltmeter, combined, as an ammeter. — A millivoltmeter is a voltmeter for reading very

small electromotive forces, and it is called a millivoltmeter because its scale reading indicates the value of an electromotive force in millivolts (one millivolt equals one one-thousandth of a volt). The current I to be measured flows through a known low resistance R, and the electromotive force between the terminals of this resistance is measured by means of a millivoltmeter as indicated in Fig. 26*b*. If the value of R is one one-thousandth of an ohm, then the reading of the millivoltmeter in millivolts is the value of the current in amperes. If the value of R is one one-hundredth of an ohm, then the reading of the millivoltmeter in millivolts must be divided by 10 to give the value of the current in amperes. If the value of R is one tenth of an ohm, then the reading of the millivoltmeter in millivolts must be divided by 100 to give the value of the current in amperes. It is evident from the connections shown in Fig. 26*b* that the total current is equal to the current in the known resistance R plus the current flowing through the millivoltmeter; but inasmuch as the resistance of the millivoltmeter is always quite large, the current which flows through it is very small and is always negligible in comparison with the current which flows through R. The resistance R in Fig. 26*b* forms a shunt to the millivoltmeter and the combination exemplifies the matter which is discussed in Art. 27.

PROBLEMS.

13. A current of 0.5 ampere flowing through a glow lamp generates 150 calories of heat in 10 seconds. (*a*) Required the resistance of the lamp in ohms. (*b*) What power is expended in the lamp? Express in watts and in horse-power. Ans. (*a*) 252 ohms; (*b*) 63 watts or 0.0844 horse-power.

14. A wire having a resistance of 250 ohms is coiled in a vessel containing 2,000 grams of oil of which the specific heat is 0.60. The vessel itself weighs 200 grams and its specific heat is 0.095. A current of 1.5 amperes is passed through the coil of wire. How long will it take to raise the temperature of the oil and the vessel one centigrade degree? Ans. 9.11 seconds.

15. The field coil of a dynamo contains 25 pounds of copper (specific heat 0.094), weight of cotton insulation negligible. The resistance of the coil is 100 ohms. (*a*) At what rate does the temperature of the coil begin to rise when a current of 0.5 ampere is started in the coil? (*b*) How long would it take for the temperature of the coil to rise to 20° C. if no heat were given off from the coil by radiation? Ans. (*a*) 0.02345 centigrade degree per second; (*b*) 14.2 minutes.

16. A given spool wound full of copper wire 60 mils in diameter has a resistance of 3.2 ohms. An exactly similar spool is wound full of copper wire 120 mils in diameter. What is its resistance? Ans. 0.2 ohm.

17. What is the resistance at 20° C. of 2 miles of commercial copper wire 300 mils in diameter? Ans. 1.22 ohms.

18. What is the resistance at 20° C. of one mile of a conductor consisting of seven copper wires each 40 mils in diameter? Ans. 4.9 ohms.

19. Find the resistance at 20° C. of a copper conductor 100 feet long, having a rectangular section 0.5 × 0.25 inch. Ans. 0.00653 ohm.

20. A sample of commercial copper wire 3 feet long and 120 mils in diameter is found, by test, to have at the same temperature a resistance equal to that of 26.2 inches of pure copper wire 100 mils in diameter. Find the ratio of the specific resistance of the sample to the specific resistance of pure copper. Ans. 1.048.

21. What is the resistance at 20° C. of a steel rail 30 feet long weighing 900 pounds? The specific gravity of the steel is 7.8. Ans. 0.000191 ohm.

22. What is the resistance at 20° C. of an iron pipe 120 feet long having 1 inch inside diameter, and 1¼ inches outside diameter? The pipe has seven joints, and each joint is assumed to have the resistance of one foot of pipe. Specific resistance assumed to be the same as rail steel. Ans. 0.01536 ohm.

23. A pure copper wire, 2,000 feet long weighs 125 pounds. What is its resistance at 20° C. How will its resistance be changed by doubling the length without changing its weight? The specific gravity of copper is 8.9. Ans. 1.01 ohms.

24. The specific resistance of carbon such as used for arc lamps is about 2,400 times as great as that of pure copper. Find the watts lost, that is, find Ri^2, in the two carbons of an arc lamp, 8 inches of each carbon being in circuit, the carbon being $\frac{1}{2}$ inch in diameter, and the current passing through the lamp being 9.6 amperes. Ans. 12.3 watts.

25. A column of a 15 per cent. solution of $CuSO_4$, 1 meter long, having one square millimeter section, has a resistance of 260,000 ohms. An electrolytic cell of this solution has two flat electrodes, 30 × 30 centimeters, 2.5 centimeters apart. Calculate the current due to 2 volts between electrodes, allowing 0.2 volt for polarization. Ans. 27.7 amperes.

26. A copper transmission line has a resistance of 5 ohms at 20° C. What is its resistance at 90° C.? Ans. 6.297 ohms.

Note. — The difference between the temperature coefficient of resistance of a metal expressed as a fraction of its resistance at 0° C. and its temperature coefficient expressed as a fraction of its resistance at any other temperature not greatly different from 0° C. is less than the variations of the temperature coefficient for different samples of the same (commercial) metal, and therefore it is ridiculous to insist on the refined calculations which grow out of the above-mentioned difference. The answers to all temperature-resistance problems in this collection are, however, found by the correct (arithmetically correct!) method. The formula is

$$R_{t'} = R_t\left(\frac{1+\beta t'}{1+\beta t}\right)$$

27. A wire has a resistance of 164.8 ohms at 20° C. and a resistance of 186.2 ohms at 70° C. What is the mean temperature coefficient? Ans. 0.002739.

28. The field coil of a dynamo has a resistance of 42.6 ohms after the dynamo has stood for a long time in a room at 20° C. After several hours' running the resistance of the coil is 51.6 ohms. What is its temperature? Ans. 76.5°.

29. A platinum wire has 254 ohms resistance at 0° C. When

placed in a furnace its resistance is 1,630 ohms. What is the temperature of the furnace? Ans. 1,531°.

30. A platinum wire which has 254 ohms resistance at 0° C. has a resistance of 81 ohms when placed in a bath of liquid air. What is the temperature of the liquid air? Ans. — 192°.5.

31. A glow lamp has a resistance of 220 ohms at a temperature of 1,000° C. (a bright red heat). At 20° C. its resistance is 277 ohms. What is the mean temperature coefficient of the carbon filament? Ans. — 0.00021.

32. The temperature coefficient of a given metal is 0.004 per degree centigrade when expressed in terms of the resistance of the metal at 0° C. Find the temperature coefficient per degree Fahrenheit expressed in terms of the resistance at 0° F. Ans. 0.00239 per degree F.

Note. — Assume a wire of the given metal of which the resistance at 0° C. is one ohm and calculate its resistance R at — 17.78° C. (equals 0° F.). The temperature coefficient per degree centigrade expressed in terms of the resistance at — 17.78° C. is greater than the temperature coefficient per degree centigrade expressed in terms of the resistance at 0° C. in the ratio of R to unity and this result must be divided by 1.8 to get the coefficient per degree Fahrenheit in terms of the resistance at 0° F.

33. Practically all of the energy of the chemical action which takes place in the gravity cells goes to maintain the current produced by the cell. When one gram of powdered zinc is stirred into a solution of copper sulphate 756 calories of heat are generated. Calculate the electromotive force of the Daniell cell. Ans. 1.07 volts.

Note. — Assume the current of one ampere and find the fraction of a gram z of zinc which would be deposited by this current per second. This is the amount of zinc which is consumed per second by voltaic action. Find the number of calories of heat represented by the reaction of z grams of zinc with copper sulphate, and reduce this result to joules. We thus find the number of joules per second developed by the voltaic action which is produced when one ampere flows through the cell and this is equal to the desired electromotive force in volts.

34. A fan blower develops between its inlet and outlet a pressure-difference of three fourths pound per square inch. When the outlet is open the fan delivers 20 cubic feet of air per second.

At what rate does the fan do work in delivering this air? Ans. 2,160 foot pounds per second.

Note. — Reduce the pressure-difference to pounds per square foot and then the unit in terms of which the result is expressed will be at once evident.

35. When a certain electric generator is giving out no current it takes 1.75 horse-power to drive it. When the generator delivers a current of 150 amperes it takes 25 horse-power to drive it. Assuming that the increased power is all used in the maintenance of the 150 amperes of current, find the electromotive force of the generator. Ans. 115.7 volts.

36. An incandescent lamp takes 0.6 ampere when the electromotive force between its terminals is 110 volts. Find the power delivered to the lamp in watts and in horse-power. Ans. 66 watts or 0.0884 horse-power.

37. In the electrolytic refining of copper an electromotive force of 0.3 of a volt suffices to send the current through the electrolytic cell in which the pure copper is deposited. Calculate the number of kilowatt-hours required to deposit a ton of pure copper. Ans. 230 kilowatt-hours.

Note. — See data on pages 23 and 24.

38. In the electrolytic manufacture of aluminum by electrolysis an electromotive force of 5.5 volts suffices to send the current through the electrolytic cell in which the metallic aluminum is deposited. Find the number of kilowatt-hours required for the production of one ton of aluminum. Ans. 14,810 kilowatt-hours.

Note. — One kilowatt continuously for one year costs from $20 to $40 when developed on a large scale by water power. See data on pages 23 and 24.

39. When electrical energy costs 15 cents per kilowatt-hour how much does it cost to operate, for 10 hours, a glow lamp which takes ½ an ampere from 110-volt mains? Ans. 8¼ cents.

40. An electric motor which delivers 5 horse-power at its belt has an efficiency of 85 per cent. This motor is supplied with current from 110-volt mains. What current does it take? Ans. 39.89 amperes.

41. A fine copper wire wound in one layer upon a pane of glass is submerged in an oil-bath and a measured current I is allowed to flow through the wire causing the temperature of the bath to rise slowly. A voltmeter is connected across the terminals of the coil of copper wire and simultaneous readings of current I in the coil, electromotive force E across the terminals of the coil and temperature T of the bath were taken as follows:

T	E	I	T	E	I
20° C.	55.80 volts.	4.74 amp.	55° C.	56.00 volts.	4.21 amp.
25	55.80	4.65	60	56.15	4.15
30	55.80	4.50	65	56.20	4.05
35	55.80	4.48	70	56.25	4.025
40	55.85	4.41	75	56.30	3.96
45	55.95	4.35	80	56.35	3.91
50	55.95	4.28	85	56.40	3.85
			90	56.40	3.79

Calculate the resistance of the wire at each observed temperature, and plot a curve of which the abscissas represent observed temperatures and of which the ordinates represent the calculated values of the resistance of the wire.

42. An electrolytic cell, consisting of a one per cent. solution of sulphuric acid between lead electrodes, was connected to supply mains, and the following values of current I flowing through the cell, electromotive force E between the electrodes, and temperature T of the solution wete observed. Calculate the resistance of the cell at each temperature, and plot a curve of which the abscissas represent temperatures, and of which the ordinates represent the corresponding calculated resistances of the cell.

I	E	T	I	E	T
3.3 amp.	56.1 volts.	22.95° C.	5.06 amp.	52.6 volts.	60° C.
3.73	54.0	30	5.47	53.1	70
4.35	54.6	40	5.75	52.9	80
4.68	53.0	50	6.00	52.5	90

Note. — The resistance of the cell in this problem is to be calculated by means of Ohm's Law. Ohm's Law, however, is not strictly applicable in this case, because a portion of the work which is done on the cell is used to produce chemical action, whereas Ohm's Law is true only in case all the work delivered to a circuit is spent in

heating the circuit in accordance with Joule's Law. This matter may be stated in another way, as follows: A certain portion of the observed electromotive force is used to produce chemical action, and the remainder is used to overcome the resistance of the electrolyte in accordance with Ohm's Law. The portion of the electromotive force which is used to produce chemical action is about 2 or 2½ volts, so that but little error is introduced by ignoring this effect and assuming that the whole of the electromotive force is used to overcome resistance in accordance with Ohm's Law.

43. A coil of which the resistance is to be determined is connected in series with an ammeter across 110-volt mains, and the current is observed to be 26 amperes. What is the resistance of the coil? Ans. 4.23 ohms.

44. A wire of which the resistance is 150 ohms is connected to the terminals of a 110-volt dynamo, and a point on this wire is grounded, the resistance between the positive terminal of the dynamo and the grounded point being 60 ohms. Choosing the ground as the region of zero potential, find the potential of each terminal of the dynamo. Ans. The potential of the positive terminal is + 40 volts, and the potential of the negative terminal is — 60 volts.

45. A voltaic cell of which the electromotive force is 1.07 volts and the resistance is 2.1 ohms is connected to a coil of 5 ohms resistance. (*a*) What current is produced? (*b*) What is the electromotive force drop in the cell? (*c*) What is the electromotive force between the terminals of the cell? Ans. (*a*) 0.15 ampere, (*b*) 0.32 volt, (*c*) 0.75 volt.

46. A storage battery consisting of 54 cells connected in series has a resistance of 0.0002 ohm per cell, and an electromotive force per cell which ranges from 2 volts at the beginning to 1.85 volts at the end of the discharge. The battery supplies current to 100 glow lamps (each having 220 ohms resistance) connected in parallel between copper wires 0.325 inch in diameter at a distance of 200 feet from the battery. Find the electromotive force between the terminals of the group of lamps at the beginning and at the end of the discharge of the storage battery. Ans. 105.6 volts and 97.7 volts.

47. The electromotive force of a battery is 15 volts (measured on open circuit). The battery terminals are connected by a wire, when it is observed that a current of 1.5 amperes is produced and the electromotive force between the battery terminals is 9 volts. Find the resistance of the wire and the apparent resistance of the battery. Ans. 6 ohms and 4 ohms.

Note. — When a voltaic cell is called upon to give current, the terminal voltage of the cell falls off, not only on account of the ri drop in the cell, but also on account of what is called polarization. This problem is to be solved on the assumption that the whole of the decrease in terminal voltage is due to ri drop. The value of the resistance as calculated on this assumption is greater than the true resistance of the battery.

48. Find the total electromotive force that must be induced in a dynamo armature to send a charging current of 100 amperes through a storage battery consisting of 54 cells connected in series. Each cell has an average counter electromotive force of 2.3 volts, the resistance of each cell is 0.0004 ohm, the resistance of the dynamo armature is 0.02 ohm, and the resistance of the leads is 0.03 ohm. Ans. 131.36 volts.

49. A dynamo having an electromotive force of 115 volts between its terminals delivers 200 amperes to a group of glow lamps 1,000 feet distant from the generator. Find: (*a*) the size of copper wire for the mains in order that 95 per cent. of the power output of the generator may be delivered to the lamps; (*b*) the electromotive force between the mains at the lamps. Ans. (*a*) 792,300 circular mils; (*b*) 109.25 volts.

50. What size of copper wire is required to deliver current at 110 volts to a 10-horse-power motor of 85 per cent. efficiency, the motor being 2,000 feet from the generator, and the electromotive force between the generator terminals being 125 volts. Ans. 221,200 circular mils.

51. A motor receiving 100 kilowatts of power is at a distance of 15 miles from the generator. Line wires 200 mils in diameter are to be used. The line loss is to be 10 per cent. of the generator output. Find: (*a*) the current; (*b*) the voltage at the generator, and (*c*) the voltage at the motor. Ans. (*a*) 16.44 amperes; (*b*) 6,763 volts; (*c*) 6,087 volts.

Note. — High-voltage direct-current power transmission is not much used in American practice.

52. A motor using 100 kilowatts of power is 10 miles from the generator. Line wires 200 mils in diameter are to be used. What electromotive force is required at the generator in order that the line loss may be only 5 per cent. of the output of the generator? Ans. 7,602 volts.

53. A direct-reading voltmeter V, Fig. 27, having 16,000 ohms resistance, is connected from main A to earth. The voltmeter gives a reading of 2.6 volts and the electromotive force between the mains is 110 volts. Find the insulation resistance between main B and the earth on the assumption that the insulation resistance of main A is: (a) infinite; (b) the same as that of main B; (c) one tenth of that of main B. Ans. (a) 660,900 ohms; (b) 644,900 ohms; (c) 500,900 ohms.

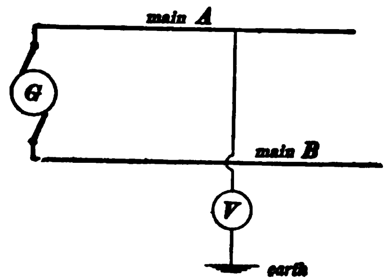


Fig. 27.

54. Three resistances A, B and C of which the values are 500 ohms, 200 ohms and 1.2 ohms, respectively, are connected to a battery of negligible resistance, the electromotive force of the battery being 2 volts. The connections are made so that the whole current produced by the battery flows through A, then divides and passes through B and C in parallel, and returns to the battery. Calculate the total resistance of the circuit, the total current, the current in B, the current in C, the electromotive force between the terminals of A, and the electromotive force between the terminals of B (or C). Ans. 501.19 ohms, 0.00399 ampere, 0.000024 ampere, 0.003966 ampere, 1.995 volts, 0.005 volt, in order.

55. Three resistances of 4, 4 and 2 ohms respectively are connected in parallel; and two resistances of 6 and 3 ohms in parallel. The first combination is connected in series with the second, and

with a battery of three volts electromotive force and negligible resistance. What is the current in the 2-ohm and 3-ohm resistances? Ans. 0.5 ampere, 0.66 ampere.

56. Six voltaic cells, each having a resistance of 2 ohms, are connected to a coil of which the resistance is 5 ohms. What is the total resistance of the circuit: (*a*) When the 6 cells are connected in series, (*b*) when the 6 cells are connected 2 in parallel by 3 in series; (*c*) when the 6 cells are connected 3 in parallel by 2 in series; and (*d*) when the 6 cells are connected in parallel? Ans. (*a*) 17 ohms, (*b*) 8 ohms, (*c*) $6\frac{1}{3}$ ohms, and (*d*) $5\frac{1}{3}$ ohms.

The electromotive force of each cell is 1.6 volts. What current is produced in the coil in each of the above cases? Ans. (*a*) 0.57 ampere, (*b*) 0.6 ampere, (*c*) 0.5 ampere, (*d*) 0.3 ampere.

Note. — When n voltaic cells are connected in series, their combined electromotive force is nE, where E is the electromotive force of one cell.

57. A direct-reading ammeter has a resistance of 0.05 ohm. The instrument is provided with a shunt so that the total current passing through the instrument and shunt is 10 times the ammeter reading. What is the resistance of the shunt? Would it be practicable to construct such a shunt, measure its resistance by a Wheatstone's bridge, and connect it to the ammeter terminals? If not, how could such a shunt be accurately adjusted? Ans. 0.00556 ohm.

58. A millivoltmeter has a resistance of 15.4 ohms. What resistance must be connected in series with the instrument so that the scale reading may give volts instead of millivolts? Ans. 15,384.6 ohms.

59. The scale of a direct-reading millivoltmeter has 100 divisions, each division corresponding to one millivolt between the terminals of the instrument. This instrument is connected to the terminals of a low resistance shunt and each division of the scale corresponds to 0.25 ampere in the shunt. What is the resistance of the shunt? Ans. 0.004 ohm.

CHAPTER III.

THE MAGNETISM OF IRON.

30. Ferromagnetism and electromagnetism. — There are two distinct groups of magnetic phenomena, namely, (*a*) the phenomena of *ferromagnetism*, that is to say, the phenomena which are associated with magnetized iron and steel, and (*b*) the phenomena of *electromagnetism*, that is to say, the magnetic phenomena which are exhibited by the electric current in the absence of iron and steel. In developing the subject of magnetism it is necessary to study ferromagnetism first because the phenomena of ferromagnetism are much more familiar than the phenomena of electromagnetism; in fact, the phenomena of electromagnetism are comparatively obscure, and, in many cases, almost imperceptible, except when they are enhanced by the presence of iron. Thus, a dynamo or an induction coil would operate if all its iron parts were removed, but the effects produced would be so slight as to be almost imperceptible. Practically, therefore, the phenomena of ferromagnetism and the phenomena of electromagnetism are inextricably associated with each other. In the rational study of magnetism, however, a consideration of the phenomena of the magnetism of iron leads to the all-important conception of the magnetic field, and the subject of electromagnetism is then developed on the basis of this conception as exemplified in Chapters IV, V, and VI.

The magnet. — The name magnet was originally applied to the lodestone, a mineral composed of iron oxide, which, in its native state, possesses the power of attracting iron.

The electromagnet. — One aspect of the magnetic effect of the electric current, as described in Art. 1 and as shown in Fig. 2, is that an iron rod which is wound with an insulated wire becomes a magnet when an electric current is sent through the wire.

Such an iron rod with its winding of wire is called an electromagnet. The iron rod is called the *core,* the coil of wire is called the *winding,* and the electric current which flows through the coil is called the *exciting current* of the electromagnet.

The permanent magnet. — When the core of an electromagnet is made of soft iron it loses * its magnetism very quickly and almost completely when the exciting current ceases to flow. When the core of an electromagnet is made of hardened steel, however, it retains its magnetized condition very persistently after the exciting current has ceased to flow, and of course such a bar of magnetized steel may be removed from the magnetizing winding. A steel bar magnetized in this way is called a permanent magnet. A permanent magnet, so-called, loses its magnetism more or less rapidly when it is subjected to mechanical shocks or temperature changes.

Fig. 28.

Poles of a magnet. Compass. Naming the poles.— Certain parts only of a magnet possess the power of attracting iron. These parts of a magnet are called its poles. The poles of a bar magnet, for example, are usually at its ends. Thus, Fig. 28 shows a bar magnet with iron filings clinging to its ends. A horizontal magnet which is free to turn about a vertical axis places itself, at most places on the earth approximately north and south. This behavior of a magnet is exemplified in the ordinary magnetic compass which consists of a pivoted magnet playing over a divided circle. The terms *magnetic north, magnetic east,* etc., are occasionally used in referring to the cardinal points of the compass as indicated by the compass needle.

The north pointing pole of a magnet is called its *north pole,* and the south pointing pole of a magnet is called its *south pole.*

Mutual force action of two magnets. — When a magnet is suddenly brought near to a compass, the compass needle is set more

* Except when the core is long and slim, or when the core is part of a complete iron circuit.

or less violently into motion (coming quickly to rest) because of the force which the magnet exerts on the compass needle. In general, any two adjacent magnets exert forces on each other, and this mutual force action is always resolvable into four parts, namely, the force with which each of the poles of one magnet acts upon each of the poles of the other magnet. The north pole of each magnet attracts the south pole of the other magnet, the north poles of both magnets repel each other, and the south poles of both magnets repel each other. Unlike magnetic poles attract each other, like magnetic poles repel each other.

31. Distributed and concentrated magnet poles. — The poles of a magnet, that is, the seats of the attracting and repelling forces above described, are distributed over considerable portions of the bar, generally the end portions. This is especially the case with short thick bars. In the case of long slim magnets, however, the poles are ordinarily more nearly concentrated at the ends of the bar. In the first case we have what are called *distributed poles*, and in the second case we have what are called *concentrated poles*. The laws of attraction and repulsion of magnets are quite simple for long slim magnets with concentrated poles, and the *ideal slim magnet with concentrated poles* will be made use of in the following development of the fundamental ideas relating to the magnetism of iron and to the magnetic action of the electric current.

32. Strength of a magnet pole. — The poles of a magnet may attract iron with greater or less force according to the size of the magnet and according to the thoroughness with which the magnet has been magnetized. The poles of a magnet are said to be *strong* when they attract iron or steel with a relatively great force. Consider a pair of long slim magnets *a*, Fig. 29, another pair *b*, another pair *c*, another pair *d*, and so on, the two magnets of each pair being exactly

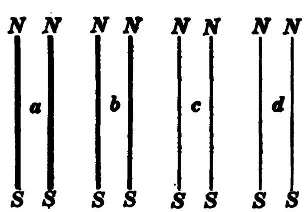


Fig. 29.

alike. *It is possible to select, from such a set, two similar magnets of which the two north poles, for example, repel each other with a force of one dyne when they are one centimeter apart. Each of the poles is then said to be of one unit strength, and the strength* **m** *of any other pole is equal to the force in dynes with which this other pole is acted upon by a unit pole at a distance of one centimeter.* The force with which two poles, of strengths m' and m'', respectively, attract each other when they are at a distance of one centimeter apart is $m'm''$ dynes. This is evident when we consider that each of the m' unit poles, which may be thought of as being combined to give the pole m', attracts each of the m'' unit poles which may be thought of as being combined to give the pole m'', with a force of one dyne.

33. Coulomb's law.—The force of attraction or repulsion of two magnet poles is inversely proportional to the square of the distance between them. This fact was discovered in 1800 by Coulomb, who measured the force of attraction of two magnet poles at different distances apart and found the force to vary inversely with the square of the distance. A long slim magnet was suspended horizontally by a wire, thus forming a torsion pendulum. One of the poles of another long slim magnet was brought near to one of the poles of the suspended magnet, the force action between the two poles produced a twist in the suspending wire, and the value of the force was determined from the observed amount of twist.

Complete expression for the force of attraction of two magnet poles.—According to the previous article, two poles attract or repel each other with a force of $m'm''$ dynes when they are one centimeter apart, therefore, according to Coulomb's Law, the poles attract or repel each other with a force of $m'm''/r^2$ dynes when they are r centimeters apart; that is,

$$F = \frac{m'm''}{r^2} \qquad (15)$$

in which m' and m'' are the respective strengths of the two

magnet poles, r is their distance apart in centimeters, and F is the force in dynes with which they attract or repel each other.

Algebraic sign of magnet pole.—The poles m' and m'' are alike in sign when both are north poles or when both are south poles. On the other hand, m' and m'' are unlike in sign when one is a north pole and the other is a south pole. It is customary to consider a north pole as positive and the south pole as negative. The force in equation (15) is considered as positive when it is a repulsion.

Two poles of a magnet always equal in strength and opposite in sign.—The behavior of a magnet in what is called a uniform magnetic field, as described in Art. 41, shows that the poles of a magnet are always equal in strength and opposite in sign. A bar of steel may be irregularly magnetized so as to have one or more north poles and one or more south poles, but the sum total of the north polarity is equal to the sum total of the south polarity. When a magnet is broken in two, each piece is found to be a complete magnet with a north pole and a south pole. *It is often convenient, nevertheless, to speak of an isolated magnet pole, meaning one pole of a very long magnet,* the other pole being so far away as to be negligible in its effects.

34. Magnetic figures. The magnetic field. — When iron filings are dusted upon a pane of glass which is placed over a magnet, the filings tend to arrange themselves in regular filaments. Slight tapping of the glass facilitates the arrangement of the filings. Figure 30 is a photographic reproduction of a magnetic figure obtained in this way. This magnetic figure conveys the idea that something emanates from one end of the magnet, traverses the surrounding region in beautifully curved lines, and enters the other end of the magnet. In fact, the entire region surrounding a magnet is in a peculiar physical condition as is shown by the behavior of a compass needle when the compass is brought into the neighborhood of a large magnet.

Wherever a compass may be placed in the neighborhood of a magnet, the compass needle points in a definite direction, the

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same direction, indeed, as would be taken by filaments of iron filings at that place. *Any region in which a compass needle tends to point in a definite direction is called a magnetic field, and*

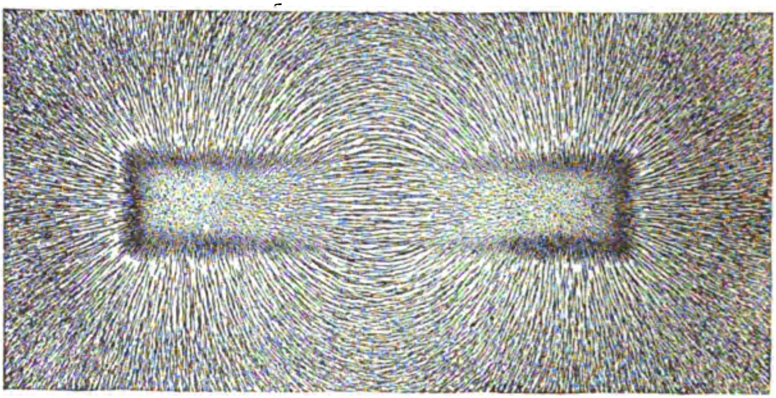

Fig. 30.

the direction of the compass needle (arrow-head thought of as being at the north-pointing end of the needle) is the direction of the field at the place where the compass is located.

The filaments of iron filings in a magnetic figure shown in Fig. 30 indicate the trend of what are called the *lines of force* of the magnetic field. A line of force is at each point in the direction of the field at that point.

Example. — The fine parallel lines in Fig. 31 represent the lines of force of a magnetic field in which a bar magnet *NS* is placed, and the heavy arrows *FF* represent the forces with which the field acts on the two poles of the magnet tending to turn it into the direction of the field.

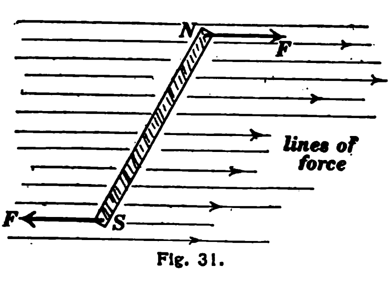

Fig. 31.

35. Intensity of a magnetic field at a point. — A magnetic field has been defined as a region in which a compass needle tends to

point in a definite direction, and the tendency of the needle to point in a definite direction is due to the fact that equal * and opposite forces *FF*, Fig. 31, are exerted on the two poles of a magnet by the magnetic field, that is to say, a magnetic field is a region in which a magnet pole is pulled in a definite direction (the *direction of the field* in the case of a north pole, the opposite direction in the case of a south pole). The force H, in dynes, which acts upon a unit magnet pole when it is placed at a given point in a magnetic field is adopted as the numerical measure of the intensity of the field at the point. This **force-per-unit-pole** H is hereafter spoken of simply as the intensity of the field. The unit of magnetic field intensity (one-dyne-per-unit pole) is called the *gauss*.

Complete expression for the force with which a magnetic field acts on a magnet pole. — The force with which a magnetic field acts upon a magnet pole of m units strength is m times as great as the force H with which the field acts upon a unit pole placed at the same point. Therefore

$$F = mH \tag{16}$$

in which F is the force in dynes which acts upon a magnet pole of strength m when it is placed in a magnetic field of which the intensity is H gausses.

Uniform and non-uniform fields. — A magnetic field is said to be *uniform* or *homogeneous* when it has at every point the same direction and intensity, otherwise, it is said to be *non-uniform* or *non-homogeneous*. The earth's magnetic field is in many places sensibly uniform throughout a room. The magnetic field surrounding a magnet is non-uniform. The magnetic field surrounding an electric wire is non-uniform.

36. Direction and intensity of the magnetic field surrounding an isolated magnet pole. — Consider the poles of two magnets of which the strengths are M and m, respectively, as shown in

* This statement refers to the case in which the field is uniform, as will be seen later.

Fig. 32. The force F with which the pole M repels the pole m is given by equation (15), namely, $F = Mm/r^2$, but the force which acts upon the pole m is equal to mH where H is the

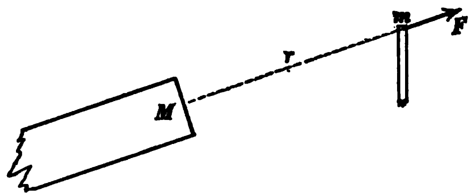

Fig. 32.

intensity AT m of the magnetic field which is due to the agent which is exerting the force on m, that is, where H is the intensity of the field at m due to M. Therefore $mH = F = Mm/r^2$, or

$$H = \frac{M}{r^2} \tag{17}$$

in which H is the intensity in gausses of the magnetic field at a point distant r centimeters from an isolated magnet pole of which the strength is M. In the neighborhood of a north pole the magnetic field is directed away from the pole and in the neighborhood of a south pole the magnetic field is directed towards the pole. This is evident when we consider that the direction of the field is indicated by the direction in which a compass needle would point, arrow-head being supposed to be on the north-pointing pole of the needle.

37. Representation of magnetic field intensity at a point by means of a line. — The magnetic field intensity at a point, like the velocity of a fluid at a point, may be represented by a line drawn in the direction of the field at the point, the length of the line being such as to represent the intensity of the field at the point to a convenient scale.

Composition of magnetic fields. — Consider two agents which acting *singly* produce magnetic fields whose respective directions and intensities at a point p are represented by the lines 1 and 2

in Fig. 33*a*. These two agents acting *together* produce a magnetic field at p which is represented by the line 3 which is the resultant of 1 and 2.

Resolution of a magnetic field into components. — Consider a magnetic field whose direction and intensity at a point p, Fig.

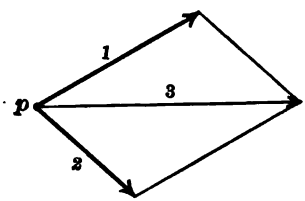

Fig. 33a.

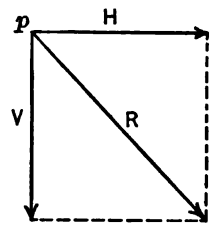

Fig. 33b.

33*b*, is represented by the line R. It is often convenient to consider that part of the field which acts in a given direction; thus, H, Fig. 33*b*, is called the horizontal component of R, and V is called the vertical component of R.

38. Magnetic flux. — Let a (expressed in square centimeters) be an area at right angles to the velocity of a moving liquid and let v (expressed in centimeters per second) be the velocity of the liquid. Then av is the flux of the liquid across the area in cubic centimeters per second. Thus, if v is the velocity of liquid in a pipe and a is the sectional area of the pipe, then av is the number of cubic centimeters of liquid discharged per second by the pipe.

Similarly, the product of the intensity H of a magnetic field and an area a at right angles to H is called the *magnetic flux* across the area; that is,

$$\Phi = aH \tag{18}$$

in which Φ is the magnetic flux across an area of a square centimeters which is at right angles to a magnetic field of which the intensity is H gausses.

Representation of the magnetic flux across an area by the number of lines of force which pass through the area. Imagine a surface

drawn across a magnetic field, the surface being at each point at right angles to the field. Of course, this chosen surface will be curved if the lines of force are not parallel straight lines, which, in general, they are not. Imagine lines of force drawn through the field so that the number of lines which pass through each square centimeter of this surface is equal to the intensity of the magnetic field at that part of the surface. Then the magnetic flux passing through any area anywhere in the field will be equal to the number of these lines that cross that area. The unit of flux (that is, the flux across a square centimeter at right angles to a field of which the intensity is one gauss) is therefore called the *line of force* or simply the *line*, and a magnetic flux is usually specified as so many lines. The name *maxwell* has, however, been internationally adopted as the name for the unit of magnetic flux.

39. Total magnetic flux emanating from a magnet pole of strength M. *Proposition.* — The number of lines of force (the number of maxwells of flux) which emanate from a magnet pole of strength M is

$$\Phi = 4\pi M \tag{19}$$

Proof. — Imagine a spherical surface of radius r drawn with the pole M at its center, as represented by the dotted line in Fig. 34. The area of this spherical surface is $4\pi r^2$ (neglecting the small portion of the sphere which falls inside of the material of the slim magnet at the point b); the magnetic field at the spherical surface due to the pole M is everywhere at right angles to the surface, and its intensity is everywhere equal to M/r^2, according to equation (17). Therefore, according to equation (18), the magnetic flux Φ across the spherical surface is equal to $4\pi r^2$ times M/r^2, which is equal to $4\pi M$.

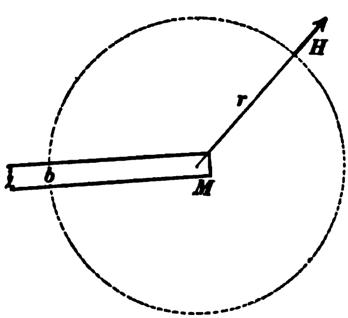

Fig. 34.

General relation between pole strength and flux.—A magnet pole may be defined as a place where magnetic lines of force pass from iron into air (north pole) or from air into iron (south pole). A piece of iron may be magnetized so that the magnetic flux does not pass out of the iron. In such a case, there are no magnetic poles. Thus, the iron ring shown in Fig. 35 has no magnetic poles when it is magnetized by a current flowing through the winding of wire.

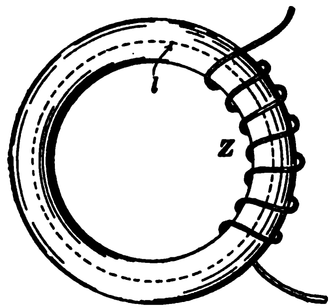

Fig. 35.

The relation between pole strength and magnetic flux which is given in equation (19) is entirely general; *$4\pi m$ lines of force emanate from any north pole of which the strength is m, whatever the shape and size of the pole may be; and $4\pi m$ lines of force converge upon any south pole of which the strength is m.*

40. Magnetic field in the neighborhood of a long slim pole.—Consider a long slim magnet having one of its poles spread uniformly over l centimeters of its end, as indicated by the shaded area in Fig. 36.

The lines of force emanate from this uniformly distributed pole in planes at right angles to the axis of the rod as indicated by

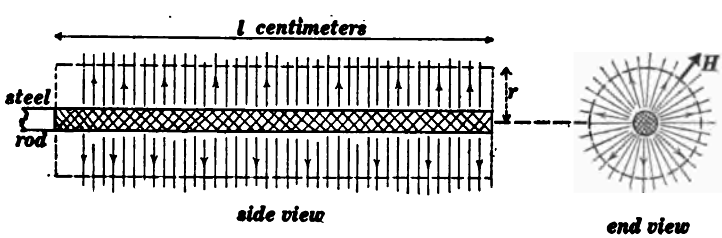

Fig. 36.

the fine lines in Fig. 36, and the intensity H of this field at a point distant r centimeters from the axis of the rod is

$$H = \frac{2\left(\frac{m}{l}\right)}{r} \tag{20}$$

in which m/l is the pole strength per unit length of the shaded area in Fig. 36, and H is the field intensity in gausses at a point r centimeters from the axis of the rod.

Proof.—Imagine a cylindrical surface of radius r to be drawn with its axis coincident with the axis of the rod, as shown by the dotted lines in Fig. 36. The area of this cylindrical surface is equal to $2\pi rl$, and, inasmuch as the magnetic field at this cylindrical surface is everywhere at right angles to it and everywhere the same value, the magnetic flux through this cylindrical surface is equal to $2\pi rl \times H$ according to equation (18). This is the total magnetic flux emanating from the pole, and it must be equal to $4\pi m$ according to equation (19), so that we have $2\pi rlH = 4\pi m$ whence $H = 2m/rl$. In this discussion the non-uniformity of the magnetic field near the ends of the long slim pole is ignored; in fact, the effect of this non-uniformity is negligible if r is small in comparison with the length l of the slim pole.

The above formula expressing the field intensity at a distance from a long slim pole applies also to the case of the pole which is distributed along the edge of a steel ribbon which is magnetized crosswise as shown in Fig. 37. In this case, however, the

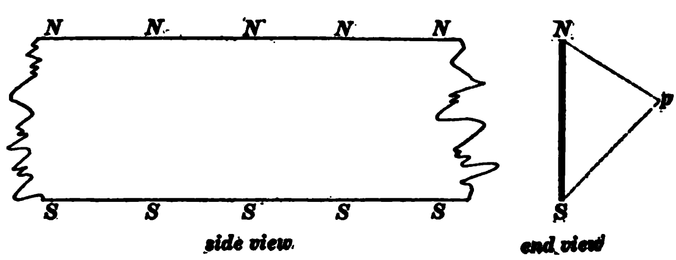

Fig. 37.

actual magnetic field at any given point p is the resultant of the fields due to the poles along both edges of the strip.

41. Behavior of a magnet in a uniform magnetic field.—A bar of steel weighs the same before and after being magnetized (earth's field being uniform), and the fiber by which a magnet is suspended hangs vertically (earth's field being uniform). Any force tending to produce translatory motion of a magnet would cause it to weigh more or less after magnetization than before, or would tend to cause a suspending fiber to be out of plumb. Therefore the forces with which the uniform magnetic field of the earth acts upon a magnet do not tend to produce translatory motion, the force which acts on the north pole of the magnet is equal in value and opposite in direction to the force which acts upon the south pole of the magnet, as indicated in Fig. 38, and therefore the poles of the magnet are equal in strength and opposite in sign.

Consider a magnet of length l placed in a uniform magnetic field of intensity H, the angle between the axis of the magnet and the direction of the field being θ, as shown in Fig. 38. The poles of the magnet are acted upon by the forces $+mH$ and $-mH$, respectively, the moment of each of these forces about the center of the magnet is equal to $mH \times l/2 \times \sin\theta$, and both of these moments tend to turn the magnet in the same direction. Therefore the total torque T tending to turn the magnet into the direction of the field is

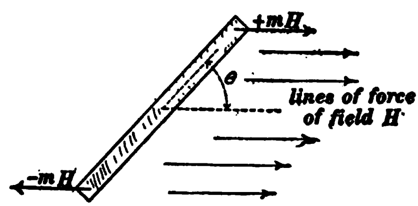

Fig. 38.

$$T = -mlH \sin\theta \tag{21}$$

The negative sign is chosen simply for the reason that the torque tends to reduce θ which may be considered as a positive angle. This equation expresses the torque in dyne-centimeters. When the angle θ is equal to zero or $180°$, the torque T is zero and the forces $+mH$, $-mH$ have no tendency to turn the magnet,

that is to say, the magnet is in equilibrium. This equilibrium is stable when the north pole of the magnet points in the direction of the magnetic field (θ equal to zero), and it is unstable when the south pole of the magnet points in the direction of the magnetic field (θ equal to 180°).

If the angle θ is never large in value then θ (in radians) may be written for $\sin\theta$ in equation (21) giving

$$T = -mlH \cdot \theta \tag{22}$$

This equation shows * that a suspended magnet when started will perform harmonic vibrations about its axis of suspension in such a manner that

$$\frac{4\pi^2 K}{t^2} = mlH \tag{23}$$

in which K is the moment of inertia of the magnet about the axis of suspension, and t is the period of one complete vibration. This equation is not even approximately true if θ reaches large values, that is, if the amplitude of the oscillations of the magnet is large.

42. Gauss's method for measuring the horizontal component of the earth's magnetic field. — A method was devised by Gauss in 1850 for determining the value of the horizontal component of the earth's magnetic field. The details of this method are described in Chapter X.

43. Behavior of a magnet in a non-uniform magnetic field. — The forces which act upon the poles of a magnet in a non-uniform magnetic field tend in general to turn the magnet and also to impart to it a motion of translation, because the force which acts on the north pole of the magnet is in general not opposite in direction and not equal in value to the force which acts on the south pole of the magnet; that is, the field at the north pole of the magnet is in general different in intensity and in direction from the field at the south pole of the magnet. This is shown

* See discussion of harmonic motion in any good treatise on elementary mechanics.

in Fig. 39 where a small magnet is placed in the non-uniform field near the pole of a large magnet. The forces F and F' are different in value and not opposite in direction.

The *attraction* of a particle of iron by a magnet depends in the first place upon the magnetization of the particle of iron and in the second place upon the *non-uniformity* of the magnetic field in which the magnetized particle finds itself, that is to say, the particle of iron becomes a magnet and its two poles are acted upon by unequal forces on account of the non-uniformity of the field.

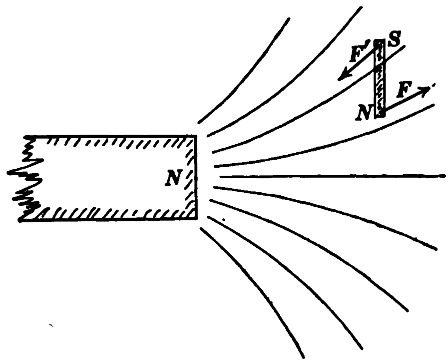

Fig. 39.

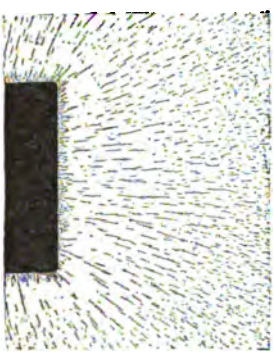

Fig. 40.

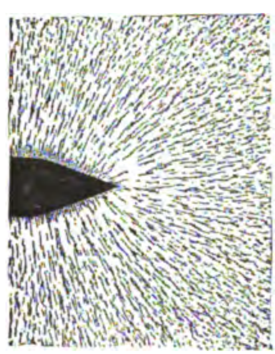

Fig. 41.

The magnetic field near a flat-ended magnet pole is approximately uniform (lines of force parallel straight lines) as shown in Fig. 40; near the sharp corners of the pole, however, the field is distinctly non-uniform (lines of force diverge strongly). Therefore particles of iron are not perceptibly attracted by the flat-face of the pole whereas the sharp corners of the pole attract particles of iron

very strongly. This is shown very strikingly by passing the flat end of a magnet pole over a table on which a very few iron filings have been placed, the filings are all caught by the corners of the pole.

The lines of force in the neighborhood of a sharp-pointed magnet pole diverge very greatly indeed as shown in Fig. 41, that is to say, the magnetic field in the neighborhood of the point is non-uniform to a high degree, and such a magnet pole has a strong attraction for small particles of magnetic material. A pointed magnet pole is an essential feature of the magnetic ore separator, the action of which is shown in Fig. 42. The crushed ore falls in a thin stream before a pointed, or wedge-shaped, magnet pole. The particles of magnetic material are attracted by the pointed pole and thus deflected, while the non-magnetic material falls straight downwards.

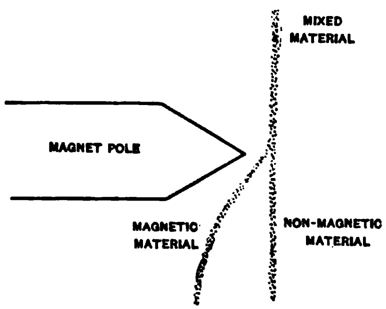

Fig. 42.

Surgeons sometimes make use of a pointed magnet for removing particles of iron or steel from the eye.

44. Tension and energy of the magnetic field. — Consider the opposite poles of two magnets as shown in Fig. 43. Their force of attraction is due to the tension of the magnetic field, the tension of the lines of force as it is sometimes called. The lines of force of the magnetic field also push each other apart sidewise. This sidewise push of the lines of force on each other is evident if we consider that the lines of force in Fig. 43 are curved so that they must exert a side force if they are under tension.

When the two magnet poles in Fig. 43 are allowed to move nearer together, their force of attraction does mechanical work, and the mechanical work thus obtained comes from the magnetic field; that is to say, a magnetic field represents a store of energy,

and when a magnetic field is reduced* in extent (volume) or in intensity, a portion of its energy is transformed.

A simple discussion of the tension and energy of the magnetic field cannot be based on an arrangement like Fig. 43 because of the non-uniformity of the field. Consider one end of a very broad flat strip of magnetized steel, as shown in Fig. 44, and *let*

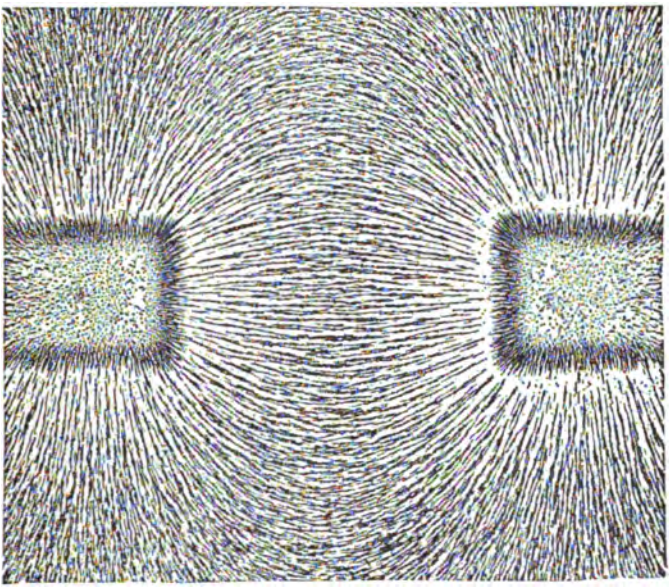

Fig. 43.

us assume that the total pole strength m *is spread uniformly over the end of the strip*† as indicated by the shading in the figure.

* When the two poles in Fig. 43 move nearer together the intensity of the intervening field is increased in some parts and decreased in other parts.

† The fundamental relations involved in the study of electricity and magnetism may be established in a comparatively simple way by assuming simply geometrical forms and distributions. Thus, the formula expressing the magnetic field intensity in the neighborhood of a magnet pole is extremely complicated unless the pole be assumed to be concentrated at a point, or to be spread uniformly over a certain length of a rod, or to be spread uniformly over a certain plane area. The formula expressing the intensity of a magnetic field in the neighborhood of a wire carrying an electric current is extremely complicated unless the wire be simple in shape. Thus, the formula expressing the intensity of a magnetic field in the neighborhood of a long straight wire is very

Magnetic lines of force emanate from both faces of the polar area s as shown in the edge view in Fig. 44, and the magnetic field on each side of the flat pole is a uniform field (except, of course, near

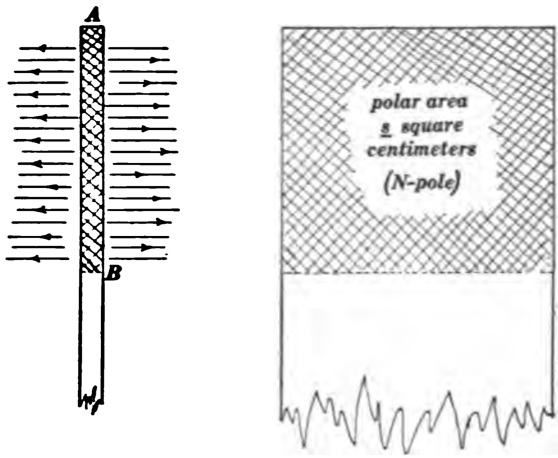

Fig. 44.

the edges, but the polar area is assumed to be so large that the edge complications may be ignored). Let H_1 be the intensity of this field. Then H_1s* is the magnetic flux passing out from the polar area on each side, $2H_1s$ is the total flux emanating from the pole, and this must be equal to $4\pi m$ according to Art. 39, so that we find:

$$H_1 = \frac{2\pi m}{s} \tag{i}$$

Consider two similar flat magnet poles AB and $A'B'$ placed side by side as shown in Fig. 45, one being a north pole and the other a south poie, as indicated in the figure. Consider the mag-

simple. These simple modes of distribution of magnet pole, and long straight wires carrying electric currents are never met with as actual facts, but they are possible and therefore legitimate as starting points for the development of simple mathematical theory.

* This expression ignores the non-uniformity of the field near the edges of the flat pole.

netic field which is due to AB, its intensity is equal to $2\pi m/s$ throughout the whole region occupied by the pole $A'B'$, according to equation (i), and therefore the force which is exerted upon $A'B'$ is equal to the product of the strength of $A'B'$ and the intensity of the field due to AB, whence we find:

$$F = \frac{2\pi m^2}{s} \tag{24}$$

in which F is the force in dynes with which the two poles in Fig. 45 attract each other. It is noteworthy that this force is independent of the distance d, provided the distance d is small in comparison with the length and breadth of the polar areas AB and $A'B'$.

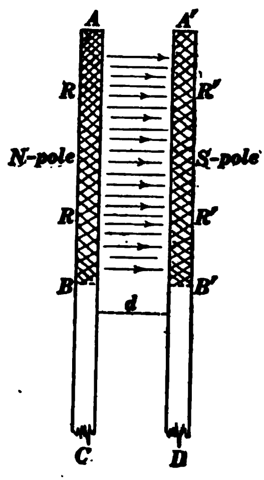

Fig. 45.

To find the intensity of the field in the region between the flat poles in Fig. 45.—The north pole AB, Fig. 45, tends to produce in the region RR a uniform magnetic field directed towards the *left*, of which the intensity is $2\pi m/s$, whereas the south pole $A'B'$ tends to produce in the region RR a uniform magnetic field directed towards the *right*, of which the intensity is $2\pi m/s$, and the net result is that the magnetic field intensity in the region RR is zero, or, in other words, no lines of force traverse the region RR. In a similar manner it can be shown that no lines of force traverse the region $R'R'$. In the region between AB and $A'B'$ each magnet pole tends to produce a magnetic field towards the *right* of which the intensity is $2\pi m/s$, so that the actual intensity H of the field between AB and $A'B'$ is

$$H = \frac{4\pi m}{s} \tag{25}$$

The arrangement in Fig. 45 is equivalent to the arrangement shown in Fig. 46.—In the arrangement shown in Fig. 45 the magnetic flux which crosses from AB to $A'B'$ comes up through the steel at C and goes down through the steel at D. Figure 46 shows the flat ends of two massive steel or iron bars which are magnetized so that the face of one bar is a north pole

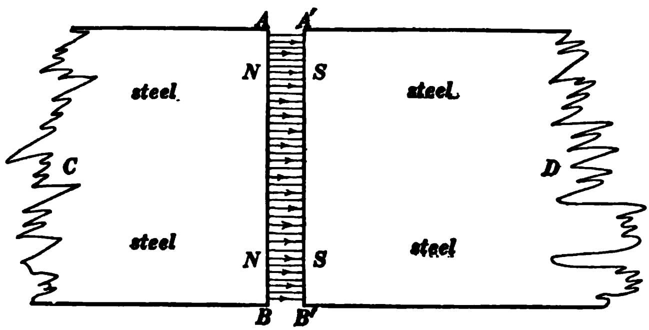

Fig. 46.

and the face of the other bar is a south pole as indicated. In this case, the magnetic flux comes up to the polar areas through the steel at C and D, Fig. 46. The two magnet poles in Fig. 46 act on each other in the same way as the two magnet poles in Fig. 45, and equations (24) and (25) apply to both figures.

Tension of the lines of force.—The force attraction of the two poles in Figs. 45 and 46 is due to the tension of the lines of force. It is desirable to express this tension in terms of the field intensity, and for this purpose the force of attraction of the two poles must be expressed in terms of the field intensity between them, instead of being expressed in terms of the strengths of the two poles, as in equation (24). The strength of each pole may be expressed in terms of the intensity of the field between the poles by solving equation (25) for m. This value of m may then be substituted in equation (24), giving

$$F = \frac{sH^2}{8\pi} \tag{ii}$$

Dividing both members of this equation by the sectional area of the region between poles in Figs. 45 and 46, we get *the force per unit area* which is transmitted across the region, or in other words, the *tension* of the magnetic field. Therefore

$$\left.\begin{matrix}\text{Tension of a magnetic field in}\\ \text{dynes per square centimeter}\end{matrix}\right\} = \frac{H^2}{8\pi} \tag{26}$$

in which H is the intensity of the field in gausses.

Energy of the magnetic field.—If the magnet poles in Fig. 45 or 46 are allowed to move together, their force of attraction will do an amount of work, $W = Fd$, where d is the initial distance apart of the two pole faces, and the mechanical work thus gained comes from the magnetic field that existed in the air space. Therefore, using the value of F from equation (ii) we have

$$W = Fd = H^2sd/8\pi$$

but sd is the volume of the region between the poles, so that

$$\left.\begin{matrix}\text{Energy of a magnetic field in}\\ \text{ergs per cubic centimeter}\end{matrix}\right\} = \frac{H^2}{8\pi} \tag{27}$$

45. The magnetization of iron.*—When a piece of iron or other magnetic substance, such as cobalt or nickel, is placed in a magnetic field, it becomes a magnet. For example, a neutral or unmagnetized bar of iron or steel when held in the direction of the earth's magnetic field shows north polarity at one end and south polarity at the other end (the polarity of the bar may be indicated by a compass needle). If the bar is turned end for end its magnetism is reversed. A sharp blow with a hammer renders the bar more susceptible to the influence of the weak magnetic field of the earth. This action of a magnetic field upon iron is called *magnetization.*

When a piece of iron is placed in a magnetic field the trend of

* For a full discussion of the theory of the magnetization of iron the student is referred to Franklin and Esty's *Elements of Electrical Engineering*, Vol. I, Appendix A; to J. A. Ewing's *Magnetic Induction in Iron and Other Metals*, London, 1900; and to H. DuBois' *Magnetic Circuit in Theory and Practice*, translated by Atkinson, New York, 1896.

the lines of force in the field is greatly altered; in fact, the field becomes the resultant of two fields, namely, the original field and

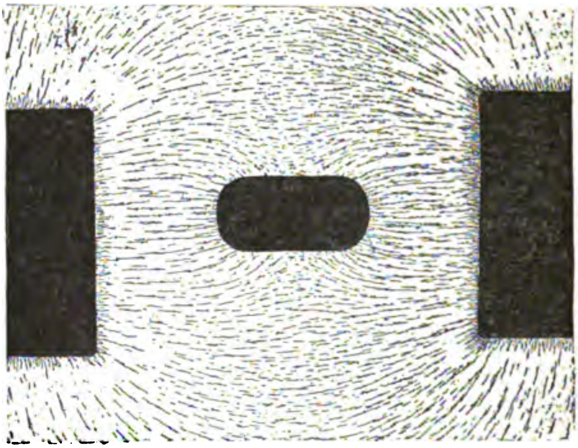

Fig. 47.

the field due to the piece of iron which has become a magnet. Thus, Fig. 47 shows the effect of a small piece of iron upon the

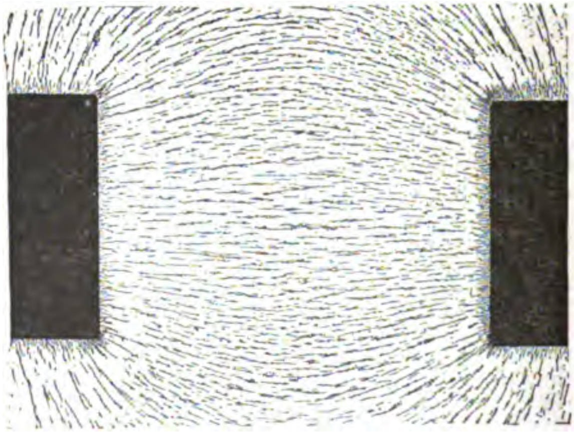

Fig. 48.

magnetic field between two flat-ended magnet poles. In the absence of the iron the field is as shown in Fig. 48. The effect of

a piece of iron in a magnetic field is always such as to suggest that "iron is a better carrier of lines of force than air." The lines of force tend to converge into the iron and pass through it.

Magnetic screening. — A shell of soft iron forms a very effective screen which protects the region inside of the shell from the action of outside magnetic influences. The lines of force, which would pass through the region occupied by the shell if the shell were not present, pass into the iron and tend to flow around through the shell and pass out on the other side without crossing the region inside of the shell. This screening effect has been used for the protection of watches against magnetic disturbances by providing the watch with a thick case of soft iron.

Note. — The region surrounding a magnet is a magnetic field, it magnetizes any piece of iron in the neighborhood, and the piece of iron is then attracted by the magnet.

46. Residual magnetism. Permanent magnets. — An iron rod retains much of its magnetism when it is removed from a magnetic field in which it has been magnetized; or in case of an electromagnet, when the magnetizing current is reduced to zero. Long slim bars retain a greater portion of their magnetism than short thick bars, because of the fact that in short bars the poles of the magnet are closer together and produce of themselves a strong demagnetizing field along the bar. The magnetism which is thus left in a bar of iron or in an electromagnet is called *residual magnetism.* Long slim bars of annealed wrought iron may retain in this way as much as 90 per cent. of their magnetism, but a very weak demagnetizing field or a very slight mechanical shock is sufficient to cause such a bar to lose its residual magnetism almost completely. Cast iron, hard drawn iron wire and mild steel retain a smaller portion of their magnetism but with greater persistence, and hardened steel bars retain a portion of their magnetism very persistently even when roughly handled. Magnetized bars of hardened steel are called permanent magnets.

Aging of permanent magnets. — A freshly magnetized bar of hardened steel loses a portion of its residual magnetism rapidly

when it is subjected to mechanical shocks or to changes of temperature. After the residual magnetism has been reduced in this way, a remainder is left which decreases but little with repeated mechanical shocks and changes of temperature, and the magnet is said to be aged. Permanent magnets for use in electrical measuring instruments are always subjected to an aging process which consists, usually, in placing the magnet repeatedly in hot and then in cold water, and in subjecting it to a series of slight mechanical shocks.

Demagnetization. — When iron is heated to bright redness it loses its magnetic properties. Thus, red hot iron is not attracted by a magnet. When a magnetized bar of steel is heated to bright redness its magnetization disappears and the bar, upon cooling, is found to be completely demagnetized.

? Any piece of iron or steel may be completely demagnetized by the following operation: Place the piece of iron or steel in a coil of wire through which a strong electric current is flowing. Reverse the current repeatedly and at the same time slowly reduce its value to zero. This operation is called *demagnetization by reversals.* A watch which has been disturbed by a strong magnetic field is usually demagnetized by this process.

47. Intensity of magnetization. Magnetic saturation. — Let m be the strength of the magnetic pole at the end of an iron rod of which the sectional area is s square centimeters. The ratio m/s is called the *intensity of magnetization* of the rod. When an iron rod is subjected to a stronger and stronger magnetizing field, its magnetization becomes more and more intense and approaches a definite limiting value beyond which it cannot be magnetized however strong the magnetizing field may be. The iron rod is said to approach *magnetic saturation* as it approaches this limiting intensity of magnetization. The limiting value of m/s is about 1,730 units of pole per square centimeter of section for wrought iron, about 1,600 for mild steel, about 1,310 for cobalt, and about 540 for nickel. Permanent magnets of hardened steel

have at the utmost about 800 units pole per square centimeter of section.

48. The molecular theory of the magnetization of iron. — When a magnet is broken in pieces, each piece is found to be a complete magnet having a north pole and a south pole. This fact suggests the possibility that each molecule of iron may be a magnet. Indeed, the hypothesis that each molecule of iron, or any substance capable of being magnetized, is a permanent magnet leads to a very useful conception of what takes place in a bar of iron when it is magnetized.

Explanation of magnetization. — In unmagnetized iron or steel the molecular magnets are thought of as pointing at random in all directions, thus neutralizing each other. When the iron or steel is placed in an intense magnetic field, the molecular magnets are turned with their axes parallel to the field, their north poles all in one direction, and the iron or steel is completely magnetized or saturated. If the magnetizing field is weak the molecular magnets are only partially turned and the iron is only partially magnetized.

Explanation of retention of magnetization. — A bar of iron which is strongly magnetized, does not return to its original state when the magnetizing field ceases to act. This is analogous to the production of a *permanent set* when an imperfectly elastic substance is greatly distorted. This persistence of a portion of the magnetization in a strongly magnetized bar may be ascribed to a friction-like opposition to the rotation of the molecular magnets. In annealed iron this friction is small, in hard drawn iron wire it is greater, and in hardened steel it is very great. Mechanical vibration and rise of temperature both act as if to decrease this frictional resistance, thus enabling a given magnetizing field to produce more intense magnetization and causing the residual magnetism to disappear.

Behavior of iron and steel when subjected to slight changes of magnetization. — When a bar of iron or steel is placed in a weak

magnetizing field, it returns almost completely to its initial condition when the weak field ceases to act. A bar of iron or steel, which is placed in a strong magnetizing field, returns almost completely to its initial condition when the field is slightly increased and then decreased again. That is, a bar of iron or steel exhibits a kind of magnetic elasticity. This action is especially prominent in hardened steel. Thus, a small magnet *ns*, Fig. 49, is repelled by the strong north pole *N* of another magnet. But when the small magnet is brought very near to *N*, as shown in Fig. 50, its magnetism is reversed and

Fig. 49.

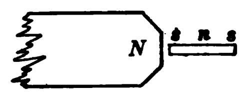

Fig. 50.

it is attracted by *N*, and then, if the reversal of magnetization of *ns* has not been carried too far, it will be found to be again repelled by *N* when it is removed to the position shown in Fig. 49. This shows that the magnetism of *ns* after having been actually reversed by the field near *N* returns approximately to its initial value when this reversing field ceases to act. This is analogous to the following: A flat steel spring is fixed in a vise and bent sufficiently to give it a permanent set to the left, a force is then exerted on the rod bending it to a slight extent to the right and when this force ceases to act, the rod again takes on its "permanent" bend to the left.

Ewing's theory. — The apparent frictional and elastic opposition to the turning of molecular magnets may both be ascribed to the mutual action of these molecules as magnets. This was first pointed out by Ewing* who constructed a model consisting of a large number of small magnets supported on jewels and pivots and arranged on a board. When this system of magnets is subjected to the action of a weak magnetic field, each magnet is slightly turned, and every magnet returns to its initial position

* See *Philosophical Magazine*, series 5, Vol. 30, page 205.

when the field ceases to act. If the field is increased in intensity more and more the magnets turn more and more until the configuration of the system becomes unstable, when the magnets suddenly fall, as it were, into a new configuration.* If now the field is slowly reduced in intensity the magnets tend to persist in their new configuration.

49. Paramagnetic substances and diamagnetic substances.—Cobalt and nickel are similar to iron in their magnetic properties except that the limit or saturation value of their intensity of magnetization is not so great. Many other substances, such as manganese, chromium, platinum, and oxygen, show similar properties but to a lesser degree. Such substances are said to be *paramagnetic*, or simply *magnetic*. On the other hand, substances such as bismuth, antimony, zinc and lead, when they are near a magnet, are magnetized in such a way as to be repelled † by the magnet. Such substances are said to be *diamagnetic*.

Paramagnetic substances are better carriers of lines of force than air and diamagnetic substances are poorer carriers of magnetic lines of force than air, that is to say, when a paramagnetic substance is placed in a magnetic field the lines of force converge towards it and pass through it, and when a diamagnetic substance is placed in a magnetic field the lines of force tend to spread out and go round it.

A paramagnetic substance when placed in a non-uniform magnetic field is drawn towards the region where the field is most intense, whereas a diamagnetic substance when placed in a non-uniform magnetic field is drawn towards the region where the field is least intense. This behavior of a diamagnetic substance in a non-uniform field may be shown by suspending a very small bar of bismuth between the pointed poles of a strong electromagnet. If the suspending fiber is sufficiently flexible the bar of bismuth sets itself at right angles to the lines joining the two pointed poles.‡

* A group of magnets mounted on pivots may be in equilibrium in a great variety of configurations.

† See note in Art. 45, page 83.

‡ A bar of bismuth tends to place itself parallel to the lines of force in the uniform magnetic field the same as a bar of iron. This apparently similar property of bismuth and iron may be explained as follows: If a bar of iron (or bismuth) were magnetized to the same degree irrespective of its direction in a uniform field, it would stand indifferently in any position, but as a matter of fact, an iron rod is more strongly magnetized when it is parallel to a magnetic field than when it is at right angles to the field, because of the demagnetizing action of the free poles on the rod, and the result is that the rod takes up the position in which it is most strongly magnetized. On the other hand, the effect of the free magnetic poles on a rod of a diamagnetic substance is to increase the negative magnetization, so that the negative magnetization of a rod of bismuth is least when it is parallel to the magnetic field in which it is placed. A rod of iron tends to place itself in the direction in which it is *most strongly magnetized* by the field, and a rod of bismuth tends to place itself in the direction in which it is *least strongly magnetized* by the field, and in each case this position is parallel to the lines of force if the field is uniform.

Weber's theory of diamagnetism. — A mass of copper near the end of an iron rod has electric currents induced in it when the iron rod is suddenly magnetized, and as long as this current continues to flow in the copper, the copper is strongly repelled by the magnet, the lines of force from the magnet tend to spread out and pass around the copper. The electrical resistance of the copper, however, very soon stops the induced current and then the strong repulsion ceases. The diamagnetic property of a substance has been explained by Weber on the hypothesis that the molecules of the substance are perfect electrical conductors so that permanent electrical currents are induced in the molecules when the substance is brought near a magnet.

PROBLEMS.

60. Two permanent magnets 1 centimeter × ½ centimeter × 30 centimeters long are magnetized to an intensity of 700 units pole per square centimeter of sectional area. (*a*) Calculate the strength of each pole. (*b*) Calculate the force with which the north pole of one rod attracts the south pole of the other rod when the poles are at an approximate distance of 10 centimeters from each other. Ans. (*a*) 350 units pole. (*b*) 1,225 dynes.

Note. — In this and the succeeding problems assume the poles of the magnet to be concentrated at the center of the ends of the bars. The intensity of magnetization of an iron rod is the strength of pole on one end divided by the sectional area of the rod. See Art. 47.

61. The two magnets specified in problem 60 are arranged as shown in Fig. 51. Find the total force with which one magnet acts upon the other. Ans. — 227.39 dynes (attraction).

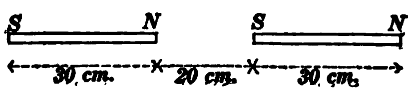

Fig. 51.

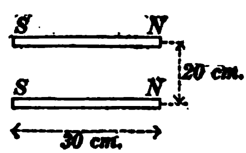

Fig. 52.

62. The two magnets specified in problem 60 are arranged as shown in Fig. 52. Find the total force with which one magnet acts on the other. Ans. + 507.8 dynes (repulsion).

63. A magnet 1 by ½ by 40 centimeters long having 800 units pole per square centimeter of sectional area is laid across one of the magnets specified in problem 60, as shown in Fig. 53. Find

the total force with which one magnet acts on the other. Ans. 5,376 dyne-centimeters of torque tending to turn magnets as shown by arrows in Fig. 53.

64. The two magnets specified in problem 60 are hung from a balance beam as indicated in Fig. 54. Assuming that the magnets exactly balance each other before they are magnetized, find the number of grams which must be added to one pan to balance the magnets after they are magnetized and specify to which pan it must be added. Ans. 0.715 grams must be added to the left pan.

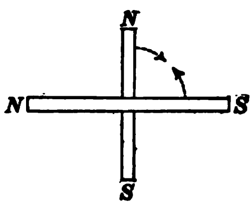

Fig. 53.

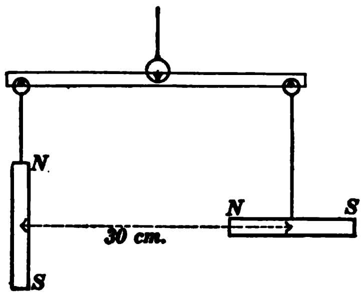

Fig. 54.

65. Determine the intensity H of the magnetic field at a point p distant 18 centimeters from one pole and 24 centimeters from the other pole of one of the magnets specified in problem 60, and determine the value of the angle θ, as shown in Fig. 55. Ans. $H = 1.24$ gausses, $\theta = 203° 46'.5$.

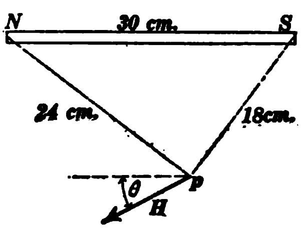

Fig. 55.

66. The intensity of the earth's magnetic field at Washington is 0.58 gauss and its dip is 62°. Find its horizontal and vertical components. Ans. $H = 0.272$ gauss, $V = 0.512$ gauss.

67. Find the direction and intensity of the resultant magnetic field at a point 30 centimeters due magnetic north of an isolated

north pole of 600 units strength at Washington. Ans. 1.07 gausses, north, and dipping at $\tan^{-1} 0.5452$ (28° 36′) below the horizontal.

68. Find the distance and direction from the magnet pole specified in problem 67 to the point at which the resultant field is zero. Ans. 32.16 centimeters, south, and elevated 62° above the horizontal.

69. A room 6 meters long by 5 meters wide by 3 meters high has its longest dimension magnetic north and south. The intensity of the earth's field in the room is 0.62 gauss and the dip is 72°. Find the number of lines of magnetic flux across each of the walls, the ceiling, and floor of the room and specify in each case whether the flux is passing out of the room or into the room. Ans. East wall, 0; west wall, 0; north wall, 28,740 maxwells out; south wall, 28,740 maxwells in; ceiling, 176,900 maxwells in; floor, 176,900 maxwells out.

70. The pole face of the field magnet of a dynamo has an area 20 centimeters by 30 centimeters. The magnetic field between the pole faces and the armature core is perpendicular to the pole face at each point and its intensity is 6,000 gausses. Calculate the number of lines of force which pass from the pole face into the armature core. Ans. 3,600,000 maxwells.

71. Calculate the number of lines of force which emanate from the north pole of one of the magnets specified in problem 60. Ans. 4,400 maxwells.

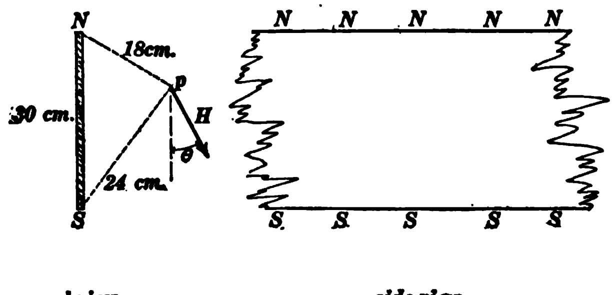

Fig. 56.

72. A very long steel ribbon of which the thickness is 0.1 centimeter and the width is 30 centimeters is magnetized so that one edge becomes a north pole and the other edge becomes a south pole, as shown in Fig. 56, the intensity of magnetization being 800 units pole for each square centimeter of section of the steel (80 units pole for each centimeter length of edge). Find the intensity of the magnetic field due to the north polar edge of the strip at a point distant 18 centimeters from the edge and specify its direction. Ans. 8.89 gausses.

73. Find the intensity of the resultant field at the point p in Fig. 56, and determine the value of the angle θ, using the data given in problem 72. Ans. $H = 11.11$ gausses, $\theta = 16° 16'$.

74. One of the magnets specified in problem 90 is balanced horizontally on a knife edge at Washington. The magnet weighs 120 grams. Find the horizontal distance from the knife edge to the center of the bar taking the acceleration of gravity to be 980 centimeters per second per second. Use the data specified in problem 66. Ans. 0.046 centimeter.

75. The moment of inertia of one of the magnets specified in problem 60 is 9,000 gr.-cm². Calculate the time of one complete oscillation of this magnet when it is suspended horizontally at Washington. Ans. 11.15 seconds.

76. A magnet makes one complete oscillation per second in a magnetic field of which the intensity is 0.2 gauss. Another magnet is twice as long, twice as wide, and twice as thick, it is magnetized to twice the intensity (units pole per units sectional area) and it is suspended in a field of which the intensity is 0.1 gauss. What is its period of oscillation? Ans. 2 seconds.

Note. — The moment of inertia of a rotating body is equal to the product of the mass of the body into the square of its radius of gyration. Given two bodies of exactly the same shape, their radii of gyration are proportional to their linear dimensions whereas their masses are proportional to their volumes.

77. A suspended magnet makes 20 oscillations in 184.5 seconds at one place, and 20 oscillations in 215.8 seconds at another

place. What is the ratio of the intensities of the horizontal component of the earth's magnetic field at the two places, and at which place is it the more intense? Ans. 1.367. Field more intense at first place.

78. Two flat-ended poles arranged as shown in Fig. 46 are observed to pull towards each other with a force of 1,500 pounds. The steel rods are round with a diameter of 3 inches. (*a*) Find the intensity of the magnetic field in the region between the flat poles in gausses. (*b*) Find the total strength of each pole. Ans. (*a*) 19,170 gausses, (*b*) 69,570 units pole.

Note. — Equation (*i*) on page 78 expresses that part of the force action between the two poles in Fig. 46 which depends upon the polarity of the rods alone. If the field between the ends of the rods is due in part to the direct action of the magnetizing coil, then the force of attraction between the two rods becomes $s/8\pi \times (H_1^2 + 2H_1H_2 + H_2^2)$, where H_1 is the field due to the magnetic polarity on the ends of the rods, and H_2 is the field due to the direct action of the magnetizing coils. Therefore, this total force consists of three parts, namely, $sH_1^2/8\pi$, $2sH_1H_2/8\pi$, and $sH_2^2/8\pi$. The first of these three parts is the force of attraction of the magnetic poles on the ends of the rods, and the second and third parts are forces which act in part upon the iron and in part upon the coils of wire which are wound upon the iron.

79. Find the total magnetic energy in the room specified in problem 69. Ans. 1,377,000 ergs.

CHAPTER IV.

MAGNETIC EFFECT OF THE ELECTRIC CURRENT.*

50. The magnetic field due to an electric wire. — The behavior of a compass needle in the neighborhood of an electric wire shows that the region surrounding an electric wire is a magnetic field. The lines of force of this magnetic field encircle the wire. Thus Fig. 57 shows the way in which iron filings arrange themselves

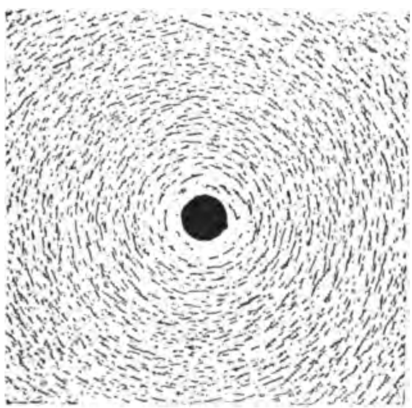

Fig. 57.

in filaments around a long straight electric wire, the black circle at the center of the figure represents a section of the wire, which is perpendicular to the plane of the figure.

The lines of force of the magnetic field due to a circular loop or coil of wire are shown in Fig. 58. In general, the lines of force of the magnetic field produced by a coil of wire trend inwards toward the opening of the coil at one end, pass through the opening of the coil, and spread out at the other end. Thus,

* Chapter V on Induced Electromotive Force, and Chapter VI on Inductance constitute continuations of this general subject, the Magnetic Effect of the Electric Current.

a long coil of wire is exactly equivalent to a magnet in so far as its relation to surrounding objects is concerned, lines of magnetic force flow out of one end of the coil through the surrounding region and into the other end of the coil in the same way that lines of force flow out from the north pole of a magnet through the surrounding region and in towards the south pole of the magnet.

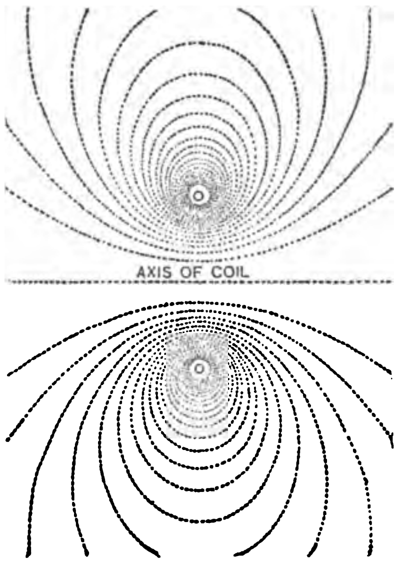

Fig. 58.

The behavior of a magnet in the neighborhood of a long straight electric wire.—The small circles in Figs. 59 and 60 represent the section of a long straight wire in which current is flowing towards* the reader. Figure 59 shows the forces *FF* with which the magnetic field due to the wire acts on the poles of a moderately long magnet, and Fig. 60 shows the forces *FF* with which the magnetic field of the wire acts upon the poles of a very short magnet. Thus, a long magnet is drawn towards the wire, although the forces acting on each pole are at right angles to the dotted lines in Fig. 59, whereas a very short magnet is not perceptibly attracted by the wire because the two forces *FF* in Fig. 60 are very nearly opposite to each other in direction. The north pole of a magnet tends to move around the wire in one direction and the south pole of a magnet tends to move around the wire in the opposite direction. Thus, the north pole tends to

* In representing a flow of current towards the reader in the section of a wire, a dot is used as if one were looking at the point of an arrow, and, when representing a flow of current away from the reader, a cross is used as if one were looking at the feathered end of an arrow; thus, ⊙ represents a flow of current towards the reader and ⊕ represents a flow of current away from the reader.

move around the wire in a counter-clockwise direction in Figs. 59 and 60. The direction of a current in a wire may be determined by means of the compass, as follows: Bring the compass

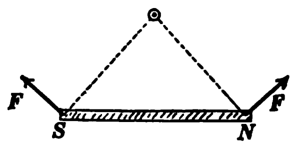

Fig. 59.

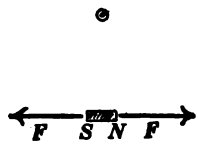

Fig. 60.

near the wire, and, knowing that the forces which act on the two poles of the compass needle are at right angles to lines drawn from the poles to the wire, infer, from the observed movements of the needle, the direction in which the north-pointing pole of the needle tends to move around the wire. *The direction of the current in the wire* * *is the direction in which a right-handed screw with its axis parallel to the wire would travel if the screw is turned in the direction in which the north-pointing pole tends to move around the wire.*

51. The composite magnetic field which is produced when a straight electric wire is stretched across a region which, but for the

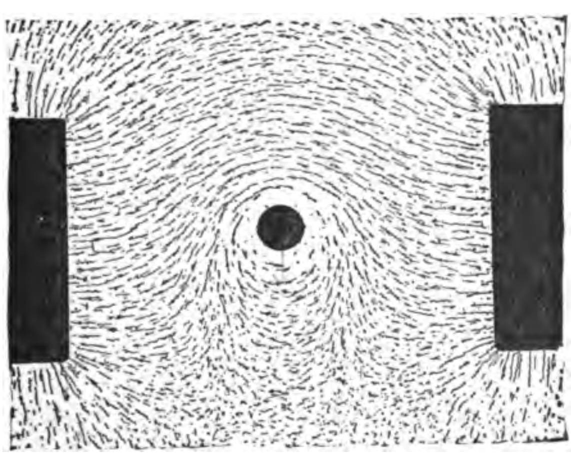

Fig. 61.

* See Art. 2.

presence of the electric wire, would be a uniform field. — The magnetic field between the flat-ended magnet poles in Fig. 48 is sensibly uniform. Figure 61 shows the same field modified by the presence of a straight electric wire. The small black circle in Fig. 61 represents the section of the wire and the wire is perpendicular to the plane of the figure. The magnetic field in Fig. 61 is due to two distinct causes, namely, (*a*) the two flat-ended magnet poles, and (*b*) the electric wire, and it may therefore be called a composite field. If the field were due to the wire alone its lines of

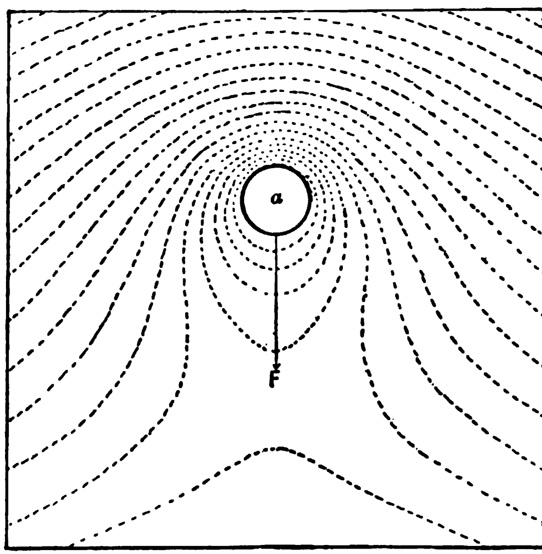

Fig. 62.

force would be circles as shown in Fig. 57, and if the field were due to the flat-ended poles alone its lines of force would be as shown in Fig. 48. The trend of the lines of force in Fig. 61 in the immediate neighborhood of the wire are more clearly shown in Fig. 62 which is from a drawing.

Side push on an electric wire which is stretched across a uniform magnetic field. — The wire shown in Figs. 61 and 62 is pushed sidewise by the magnetic field * as indicated by the arrow *F* in

* Strictly, one should perhaps speak of the side force on the wire in Figs. 61 and 62 as due to the two magnet poles, because the two magnet poles constitute the actual

Fig. 62. This side force is at right angles to the wire and to the magnetic field (Fig. 48) which is acting on the wire. The side force which acts upon the wire in Fig. 61 may be ascribed to the tension of the lines of force.

Examples. — The simple example of the magnetic effect of the electric current which is cited in Art. 1 and represented in Fig. 1 illustrates the side push of a magnetic field on a wire inasmuch as the magnetic field which emanates from the north pole of the magnet in Fig. 1 is partly, at least, at right angles to the wire *AB*. The side push on an electric wire in a magnetic field is also exemplified in the electric motor. A cylindrical mass of iron *A*, Fig. 63, has wires arranged on its surface parallel to its axis, and

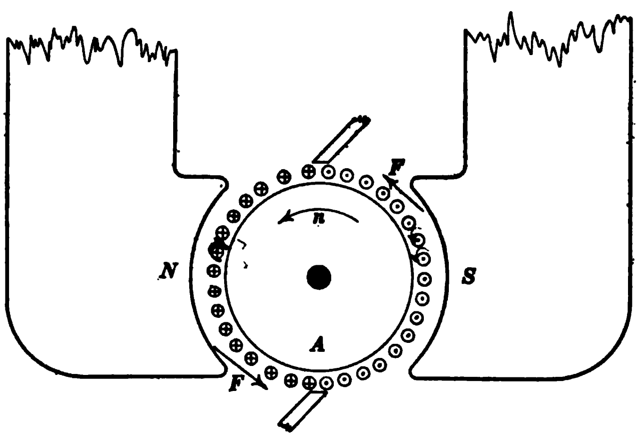

Fig. 63.

the whole is placed between two magnet poles *NS*, as shown. The narrow region between the cylinder *A* and the poles *N* and *S* is a strong magnetic field, the lines of force of which are radial. An electric current is made to flow through the wires in the directions indicated by the dots and crosses, and the result is that the

visible agent which is acting on the wire. It is very important, however, that the student become familiar with the idea of a magnetic field as a physical reality, and to ascribe the side force in Figs. 61 and 62 *to the field which is produced by the two magnet poles* puts the whole matter in the most intelligible form.

wires are pushed sidewise by the magnetic field causing the cylinder A to rotate in the direction of the curved arrow.

52. Strength of current magnetically defined. — Consider a straight electric wire stretched across a uniform magnetic field of which the intensity is one gauss, the wire being at right angles to the field as described in the foregoing article. The force in dynes with which the field pushes sidewise on one centimeter of this wire has been adopted as the fundamental measure of the strength of the current in the wire. This **force-per-unit-length-of-wire-per-unit-field-intensity** is called simply *the strength of the current in the wire;* let it be represented by I. The force pushing sidewise on l centimeters of the wire is lI, and, if the field intensity is H gausses instead of one gauss, the force is H times as great, or IlH; that is,

$$F = IlH \tag{28}$$

in which F is the force in dynes pushing sidewise upon l centimeters of wire at right angles to a uniform magnetic field of which the intensity is H gausses, and I is the strength of the current in the wire.

Definition of the abampere. — A wire is said to carry a current of one abampere when one centimeter of the wire is pushed sidewise with a force of one dyne when the wire is stretched across a magnetic field of which the intensity is one gauss. That is, F in equation (28) is expressed in dynes when l is expressed in centimeters, H in gausses, and I in abamperes.

Definition of the ampere.—The ampere is one tenth of an abampere.

In the early days of the development of the theory of electricity and magnetism, a great variety of arbitrary units was used. Thus, the resistance of a particular piece of wire would be used as a unit of resistance, the electromotive force of a particular voltaic cell would be used as a unit of electromotive force, and current values were often specified in terms of the deflections of a particular galvanometer. The introduction of a uniform system of units was due chiefly to Weber and Gauss in Germany and to Maxwell and Kelvin in England. This uniform system of units was based on the units already in use in mechanics, the centimeter, the gram, and the second, and the units of this c.g.s. system were called *absolute units*.

The electrical units which are now almost universally employed, the ampere, the ohm, the volt, the coulomb, the henry, and the farad are, however, not the original c.g.s. units, but multiples or submultiples of them The original c.g s. units as a rule have no names. Therefore in this text the c.g.s. units (of the so-called "electromagnetic" system) which correspond to the ampere, the ohm, the volt, etc., are designated by the prefix *ab*. Thus, we have the abampere, the abohm, the abvolt, etc.

Definition of the abohm.—A wire has a resistance of one abohm when one erg of heat is generated in it in one second by a current of one abampere. When H in equation (2), Art. 12, is expressed in ergs, t in seconds, and I in abamperes, then R is expressed in abohms.

Definition of the abvolt. — An electric generator has an electromotive force of one abvolt when it delivers one erg per second of power with a current output of one abampere [see equation (6), Art. 18].

The abvolt may be defined, on the basis of Ohm's Law, as an electromotive force which is capable of producing a current of one abampere through a circuit of which the resistance is one abohm.

Definition of the ohm. — A wire has a resistance of one ohm when one joule of heat is generated in it in one second by a current of one ampere. The ohm is equal to 10^9 abohms.

Definition of the volt.—An electric generator has an electromotive force of one volt when it delivers one joule per second (one watt) of power with a current output of one ampere. The volt is equal to 10^8 abvolts.

The volt may be defined, on the basis of Ohm's Law, as an electromotive force which is capable of producing a current of one ampere through a circuit of which the resistance is one ohm.

Side force on an electric wire which is not at right angles to a magnetic field.—When an electric wire is parallel to a magnetic field, no force acts on the wire. If the angle between the wire and the direction of the field is θ, then the field may be resolved into two components $H \sin \theta$ and $H \cos \theta$, perpendicular to and parallel to the wire, respectively; the latter component has no action on the wire and the former component produces the side force

$$F = IlH \sin \theta \tag{29}$$

If the wire is not straight, or if the field is not uniform, then one must consider the force action on an element of the wire, and equation (29) becomes

$$\Delta F = IH \sin \theta \cdot \Delta l \tag{30}$$

in which Δl is a short portion, or *element*, of the wire, H is the intensity of the field at the element, θ is the angle between H and Δl, I is the strength of the current in the wire in abamperes, and ΔF is the force pushing on Δl. This force is perpendicular both to H and to Δl.

53. Contribution to the magnetic field at a given point by one element of an electric wire. — The region surrounding an electric circuit is a magnetic field and each element of the wire which constitutes the circuit may be considered as contributing its share to the field intensity at each point. Imagine a magnet pole of strength m to be placed at the point at which it is desired to find the field intensity ΔH which is produced by a given element Δl of the wire.

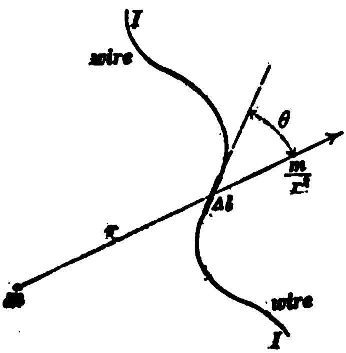

Fig. 64.

Let r be the distance from m to Δl and let θ be the angle between r and Δl, as shown in Fig. 64. The field intensity at the element due to the pole is m/r^2 according to equation (17). The component of this field which is at right angles to the element is $m/r^2 \times \sin\theta$, and this component of the field pushes sidewise on the wire with a force which is given by

$$\Delta F = I\frac{m}{r^2}\sin\theta\cdot\Delta l$$

according to equation (30). This is the force with which the pole m acts on the element, and therefore it is also the force (disregarding sign) with which the element acts upon the pole. But the force with which the element acts upon the pole must be equal to the product of the strength of the pole and the field intensity at the pole due to the element, that is,

$$\Delta F = m\cdot\Delta H$$

whence

$$\Delta H = \frac{I\sin\theta}{r^2}\cdot\Delta l \qquad (31)$$

in which ΔH is the field intensity at the point m in Fig. 64 due to the element Δl. This field ΔH is perpendicular to r and to Δl.

Note. — It is evident from the above discussion that the magnetic field at a given point in the neighborhood of a given coil of wire, or a circuit of any form, in which an electric current is flowing is proportional to the strength of the current, and that its direction is fixed. That is to say, if the strength of the current is doubled the field intensity is doubled everywhere, but the direction of the field is everywhere unaltered. The trend of the lines of force of the magnetic field due to a given coil or circuit depends only upon the shape and size of the coil.

54. The intensity of the magnetic field at the center of a circular loop of wire. — If we can calculate the force with which a circular loop of wire with given current acts on a magnet pole of given strength placed at the center of the circular loop, we can derive an expression for the intensity of the field at the center of the loop due to the current, because the force exerted on the

pole by the loop of wire must be equal to the intensity of the field at the pole due to the loop multiplied by the strength of the pole according to equation (16). Consider therefore a magnet pole of strength m placed at the center of the circular loop as shown in Fig. 65. This pole produces a magnetic field of which the intensity *at the wire* is m/r^2, and which is everywhere at right angles to the wire. Therefore the force with which the wire is pushed sidewise (perpendicular to the plane of the paper in Fig. 65) is equal to the product of the length of the wire, the intensity of the field (m/r^2), and the strength of the current I in the wire in abamperes; but the length of the wire is $2\pi rZ$ where Z is the number of turns of wire in the loop, so that $2\pi rZ \times m/r^2 \times I$ is the force with which the wire is pushed sidewise by the pole m. But, disregarding sign, this is equal to the force mH with which the loop of wire pushes on the pole. Therefore we have

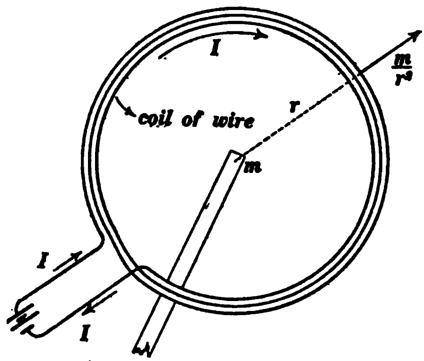

Fig. 65.

$$mH = 2\pi rZ \times \frac{m}{r^2} \times I$$

from which we obtain

$$H = \frac{2\pi ZI}{r} \tag{32}$$

55. Magnetic field in the neighborhood of a long straight electric wire.— The lines of force of the magnetic field surrounding a long straight electric wire are circles with their planes at right angles to the wire and their centers on the axis of the wire, as explained in Art. 50 and as shown in Fig. 57. To derive an expression for the intensity of this field at a point distant r centimeters from the axis of the wire, proceed as follows: A long straight wire AB carries a current of I abamperes, and a long magnetized steel strip is placed with its north polar edge parallel to AB and at a distance of r centimeters from AB as shown in Fig 66. The magnetic field due to AB has the same value all the way along the polar edge $NNNN$, as is evident from considerations of symmetry, the wire being indefinitely long. Consider

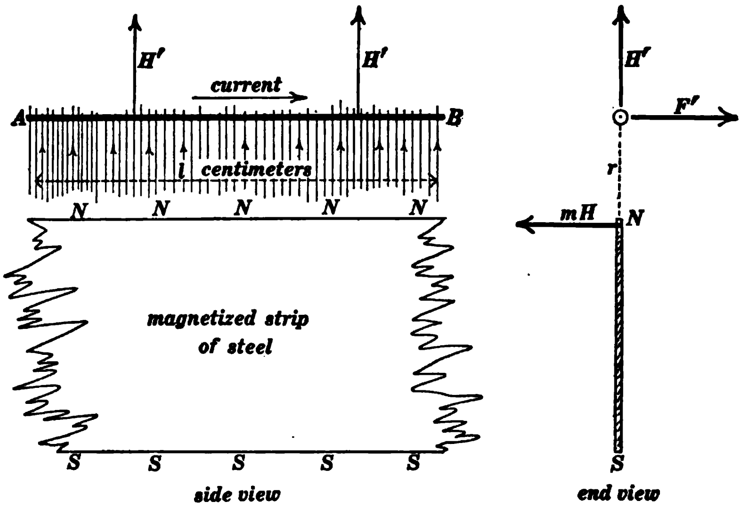

Fig. 66.

a portion of the polar edge $NNNN$ and a portion of the wire AB each l centimeters in length. The intensity H' of the magnetic field at the wire due to the pole $NNNN$ is equal to $2m/rl$, according to equation (20), and the wire AB is pushed sidewise by a force F' which is equal to $I \times l \times 2m/rl$, according to equation (28), but the force with which the pole $NNNN$ acts on the wire is equal and opposite to the force mH (see Fig. 66) with which the wire acts on the pole. Therefore, ignoring signs, we have $mH = Il \times 2m/rl$, whence

$$H = \frac{2I}{r} \tag{33}$$

56. Magnetic field inside of a long solenoid. — A solenoid is a winding of wire on a long tube as shown in Fig. 67, which is a sectional view. When an electric current flows through the winding of a solenoid the region inside of the solenoid becomes a uniform magnetic field except near the ends of the solenoid as

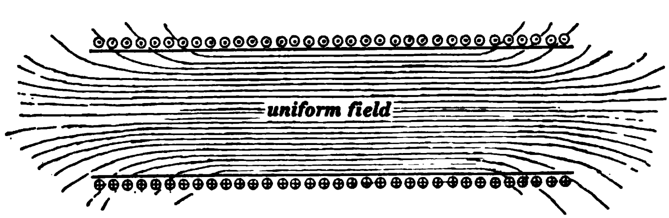

Fig. 67.

shown by the fine lines in Fig. 67, and the intensity of this field is given by the equation

$$H = 4\pi zI \tag{34}$$

in which H is the intensity of the field in gausses, z is the number of turns of wire on each centimeter of length of the solenoid, and I is the strength of the current in abamperes. If the current is expressed in amperes equation (34) becomes

$$H = \frac{4\pi}{10} \cdot zI \tag{35}$$

in which H, as before, is expressed in gausses.

In order to derive equation (34) let us consider the arrangement shown in Fig. 68 consisting of a long coil having z turns

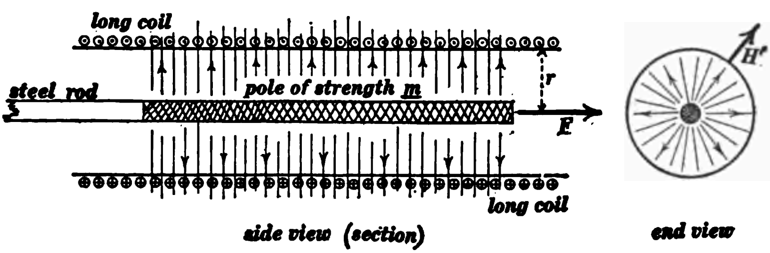

Fig. 68.

of wire on each centimeter of its length, with a steel rod projecting into it. *Let us assume that the total pole strength m on the end of the steel rod is spread uniformly* over a portion of the rod of length l,* as indicated by the shading in Fig. 68. The lines of force emanate from such a long pole in planes at right angles to its length as shown in Fig. 68, and the intensity of this field at the surface of the long coil is $H' = 2m/rl$, according to Art. 40, where r is the radius of the solenoid and l is the length in centi-

* It would be very difficult indeed to magnetize a rod so as to have its pole spread uniformly over a given length of the end of the rod, especially when the rod projects into a solenoid as shown in Fig. 68, because the effect of the solenoid is to tend to concentrate the magnet pole at the end of the rod. The assumed distribution of pole is, however, a possibility if the current in the solenoid is very weak and therefore the assumed distribution is a legitimate basis for the discussion of equation (34).

meters of the portion of the steel rod over which the pole is uniformly distributed. The non-uniformity of the field near the ends of the slim pole is negligible if r is small in comparison with l. The effect of this slim pole is therefore to produce a radial magnetic field over the whole of a portion of the coil of length l. This portion of the coil contains lz turns of wire, and the length of each turn is $2\pi r$ so that the total length of wire in the region where field is produced by the slim pole is $2\pi rlz$. This wire is everywhere at right angles to the field H' (which is due to the slim pole) and it is therefore pushed sidewise by a force $F = I \times 2\pi rlz \times H'$, or, using $2m/rl$ for H', we have

$$F = 4\pi zIm$$

but the force with which the slim pole pushes on the coil is equal and opposite to the force with which the coil pushes on the pole, and the force with which the coil pushes on the pole is equal to the product of the strength of the pole and the field intensity at the pole due to the coil. Therefore the field intensity inside of the coil is equal to $4\pi zI$.

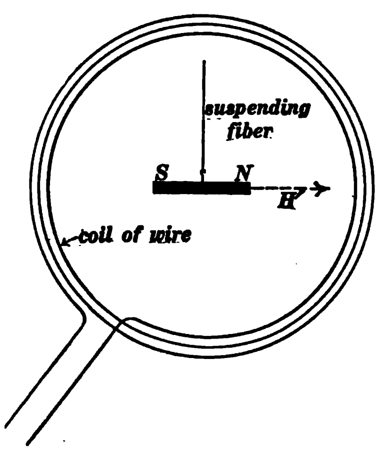

Fig. 69a.

57. The tangent galvanometer. — One of the earliest forms of instrument for measuring the strength of the electric current was the tangent galvanometer. It consists essentially of a circular coil of wire at the center of which a small magnet is suspended, as shown in Fig. 69*a*. This suspended magnet carries a pointer which plays over a divided circle by means of which the angle through which the magnet is turned when a current is sent through the wire may be observed. The coil of wire is mounted with its plane vertical and magnetic north and south.

When no current flows through the coil the suspended magnet points in the direction of the earth's horizontal field H'. A current of I abamperes in the coil produces a magnetic field of which the intensity at the center of the coil is $H = 2\pi ZI/r$ and of which the direction at the center of the coil is at right angles to H'. This field H combines with H' to give a resultant field R, Fig. 69*b*, in the direction of which the suspended magnet now points, ϕ being the angle through which the magnet is turned by the current. From Fig. 69*b* we have

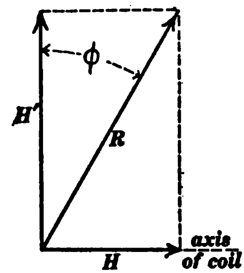

Fig. 69b.

$$\tan\phi = \frac{H}{H'}$$

or, substituting $2\pi ZI/r$ for H and solving for I, we have

$$I = \frac{rH'}{2\pi Z} \cdot \tan\phi \qquad (36a)$$

This equation gives the value of I in abamperes when r is in centimeters and H is in gausses, the values of r, H',* and Z being known and ϕ being observed.

If I be expressed in amperes, then equation (36*a*) becomes

$$I_{\text{amp.}} = \frac{5rH'}{\pi Z} \cdot \tan\phi \qquad (36b)$$

A serious fault in the tangent galvanometer is that the earth's horizontal field H' is never known accurately because it is continually changing in value. When it is desired merely to measure the ratio of two currents, however, the value of H' need not be known (provided it does not change while the following observations are being taken). One current I_1 is sent through the galvanometer, and the corresponding deflection ϕ_1 is observed, giving

*See Art. 42 and Chapter X.

$$I_1 = \frac{rH'}{2\pi Z} \cdot \tan \phi_1 \qquad \text{(i)}$$

Then the other current I_2 is sent through the galvanometer and the corresponding deflection ϕ_2 is observed, giving

$$I_2 = \frac{rH'}{2\pi Z} \cdot \tan \phi_2 \qquad \text{(ii)}$$

Dividing equations (i) and (ii), member by member, we have

$$\frac{I_1}{I_2} = \frac{\tan \phi_1}{\tan \phi_2} \qquad (37)$$

Figure 70 is a general view of a tangent galvanometer. The divisions on the large horizontal circle are not used.

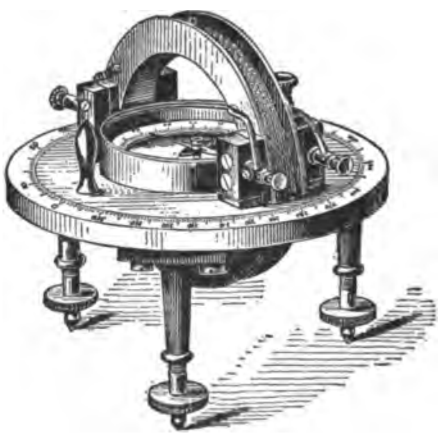

FIG. 70.

58. The action of a uniform magnetic field upon a suspended coil in which an electric current is flowing. (*a*) *Simple case of a rectangular coil with two of its edges parallel to the field.*—Figure 71 represents a rectangular coil of wire suspended between the poles N and S of a large magnet. Let H be the intensity of the magnetic field (assumed to be uniform), let b be the breadth of the coil, let a be the height of the coil, and let Z be the

number of turns of wire in the coil. The field H is parallel to the top and bottom edges, or limbs, of the coil, and at right angles to the two side limbs of the coil. The right-handed limb of the coil in Fig. 71 is pushed forwards (towards the reader) and the left-handed limb of the coil in Fig. 71 is pushed backwards

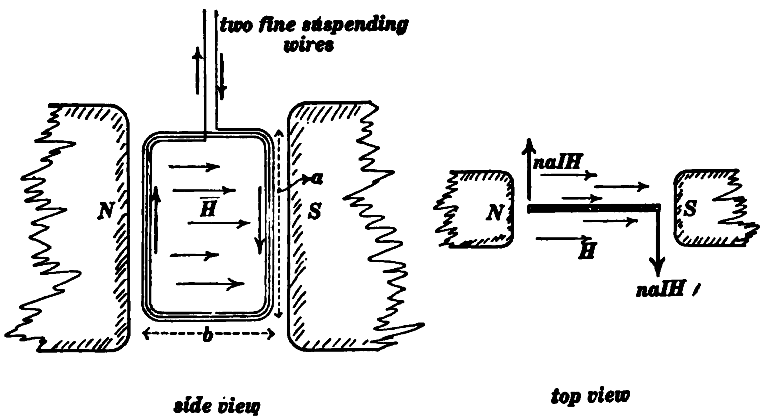

Fig. 71.

(away from the reader), and the force in each case is equal to $Za \times H \times I$, according to equation (28), I being the strength of the current in the coil in abamperes. It is evident that the total force action on the coil is a torque tending to turn the coil about the axis of suspension; the value of the torque may be obtained by multiplying the force acting on each limb of the coil by its lever arm $b/2$ and adding the two results together, which gives

$$T = abZIH \tag{38}$$

in which T is the torque in dyne-centimeters tending to turn the coil and I is the current in the coil in abamperes.

If the rectangular coil is allowed to turn through an angle θ about the axis of suspension in Fig. 71, then only the component, $H \cos \theta$, of the field will be effective in producing torque as shown in Fig. 72, and equation (38) will become

$$T = abZIH \cos \theta \tag{39}$$

When the angle θ is equal to 90° (plane of coil at right angles to the field H), then the torque is equal to zero.

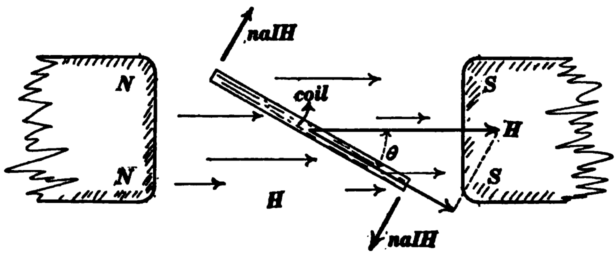

Fig. 72.

(*b*) *Case of a circular coil with its plane parallel to the magnetic field as shown in Fig. 73.* — In this case let us consider a single turn of the coil of which the radius is r. The vertical dotted line in Fig. 74 represents the axis about which the torque is to be determined. Consider an element Δl of the wire. The component of H which is at right angles to Δl is equal to $H \cos \phi$, the product $H \cos \phi \times I \times \Delta l$ gives the force pushing forwards (towards the reader in Fig. 74) on the element Δl,

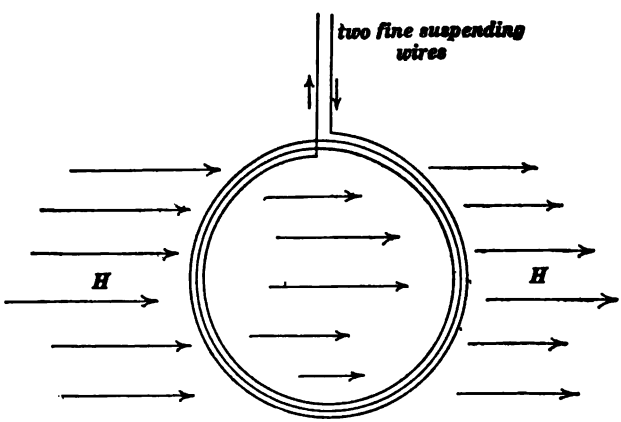

Fig. 73.

and the product of this force times the lever arm $r \sin \phi$ gives the torque action on the element Δl, that is

$$\Delta T = H \cos \phi \times I \times r \sin \phi \times \Delta l$$

but $\cos \phi \times \Delta l$ is equal to the vertical height h shown in Fig. 74, so that $r \sin \phi \times \cos \phi \cdot \Delta l$ is equal to the shaded area shown in the figure. Therefore, representing this shaded area by ΔA, we have

$$\Delta T = IH \cdot \Delta A$$

and, *since this relation is true for every element of the circular coil*, it follows that the total torque is equal to IH times the total area A enclosed by the turn of wire, that is

$$T = AIH \tag{40}$$

in which T is in dyne-centimeters, I is in abamperes, and A is in square centimeters, H being expressed in gausses. If the coil has more than one turn of wire, A is equal to the sum of the areas of all the turns. Thus, if the coil has four turns of wire of which the radii are a, b, c and d respectively then $A = \pi a^2 + \pi b^2 + \pi c^2 + \pi d^2$.

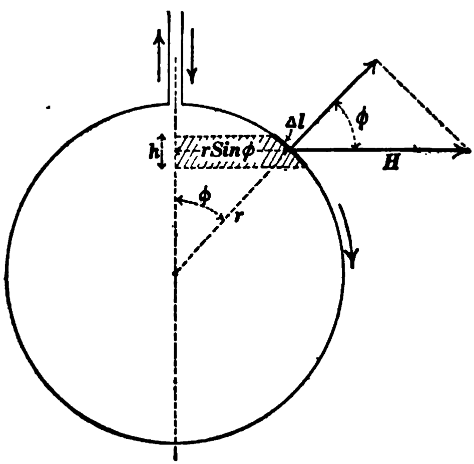

Fig. 74.

59. The electrodynamometer is an instrument for determining strength of an electric current from a measurement of the mutual force action between two coils of wire through both of which the current flows. One of these coils is fixed and the other is suspended. The magnetic field produced by the fixed coil exerts a force upon the suspended coil, and this force, or the movement which it produces, is observed. When the coils are very simple in shape it is possible to calculate (from geometrical and mechanical data alone) the force action between the two coils for a given current, or, conversely, to calculate the value of the current when the force action is observed in mechanical units. Such an electrodynamometer is called an *absolute electrodynamometer*.

The simplest absolute electrodynamometer is that devised by Wilhelm Weber in 1846. It consists of a large stationary coil mounted with its plane vertical, and a small circular coil suspended at its center by two fine wires. The current I to be measured flows through both coils. The magnetic field produced by the outer coil at

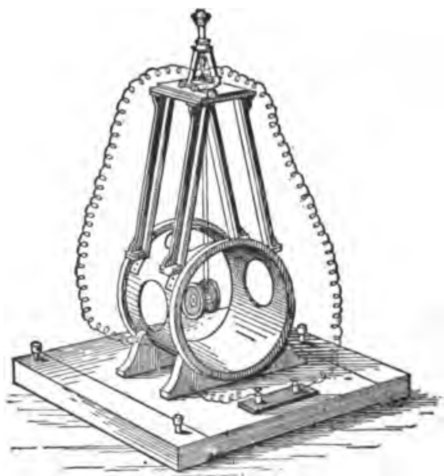

Fig. 75.

its center is $H = 2\pi Z'I/r'$, where Z' is the number of turns of wire in the large coil and r' is its radius. This magnetic field exerts a torque $T = \pi r''^2 Z'' I H \cos\theta$ upon the small coil, where Z'' is the number of turns of wire in the small coil, r'' is its radius and θ is the angle between H and the plane of the small coil or, in other words, θ is the complement of the angle between the planes of the two coils. Substituting the value of $H = 2\pi Z'I/r'$ in the expression for T, we have

$$T = \frac{2\pi^2 Z' Z'' r''^2 I^2}{r'} \cdot \cos\theta \tag{41}$$

This equation permits the calculation of I when Z', Z'', r', and r'' are known, and when T and θ have been observed. When c.g.s. units are used in equation (41), the current is given in abamperes.

Figure 75 shows a slightly modified form of Weber's absolute electrodynamometer in which the small coil is suspended in the approximately uniform field between two large circular coils side by side.

The Siemens electrodynamometer. — The force action between two coils is proportional strictly to the square of the current which flows through the two coils whatever the shape and relative position of the two coils may be, provided only that the relative position of the two coils does not change. Therefore, if

the force action between the coils is measured, first for a current I' and then for a current I'', the ratio I'/I'' is equal to the square root of the ratio of the observed force actions. The electrodynamometer of Siemens is used for measuring current ratios in this way. A general view of this instrument is shown in Fig. 76*a*. The movable coil B (see Fig. 76*b*) is suspended by a fine

Fig. 76a.

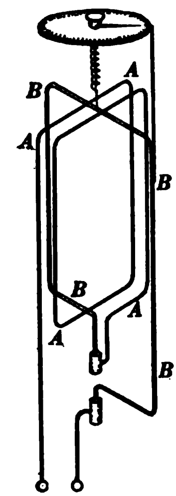

Fig. 76b.

thread and its terminals dip into two mercury cups which permit of its being connected in series with the fixed coil A. When current is allowed to flow through the two coils in series, a torque is exerted upon the movable coil by the fixed coil, and the helical spring is twisted, by turning the milled head, until the movable coil is brought to its standard position, as indicated by the pointer which is attached to B. The angle through which the milled head is turned is indicated by the pointer which is attached to the milled head, and this angle is a measure of the force action between the coils so that this angle is proportional to the square of the current, or the current is proportional to the square root of the angle.

60. The sensitive galvanometer (*Kelvin type*). — From the equation of the tangent galvanometer, equation (36), it is evident that a given current will produce the greatest deflection ϕ of the galvanometer needle when the number of turns of wire in the coil is great, when the radius of the coil is small, and when the directing field H' is weak. A galvanometer constructed so as to meet these conditions and thus give a perceptible deflection for a very small current is called a *sensitive galvanometer*. Such a galvanometer is used chiefly for merely detecting the presence of current in a circuit. The magnet of such a galvanometer is usually suspended by means of a fiber of unspun silk or quartz, and, in order that small deflections may be easily detected, a mirror is usually attached to the suspended magnet so that the angular movement of the suspended magnet may be observed by means of a telescope and scale.

Fig. 77.

Use of a governing magnet. —In order to secure a weak directing field H', the earth's field may be partially neutralized in the neighborhood of the suspended magnetic needle by properly placing a large magnet in the neighborhood of a galvanometer. This large magnet is called a governing magnet.

Use of an astatic system of magnetic needles.—Two similar magnetic needles NS and SN attached to a rod, as shown in Fig. 77, constitute what is called an astatic system. Such a system if suspended in the earth's magnetic field would point indifferently in any direction if the two magnets were exactly alike and exactly opposite in direction. If the two needles NS and SN are nearly alike the earth's field will have but a very slight directing action upon the system. Such a pair of magnetic needles may be suspended with one of its magnets inside of a galvanometer coil as shown in Fig. 78, or with its two magnets surrounded by two properly connected coils as shown in Fig. 79, and the result will be an extremely sensitive galvanometer. The design shown in Fig. 79 is due to Lord Kelvin. A galvanometer constructed after this design with very short magnetic needles,

light connecting rod and mirror, and coils containing many turns of fine wire may be made to indicate distinctly a current as small as one million-millionth of an ampere (10^{-12} ampere).

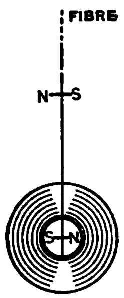

Fig. 78.

The Kelvin galvanometer may be used for the approximate measurement of very weak currents, because the deflection, within a small range, is proportional to the current.

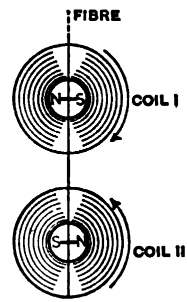

Fig. 79.

61. The sensitive galvanometer (*D'Arsonval type*). — A coil suspended in a magnetic field is acted upon by a torque when a current flows through the coil, and the torque is given by the equation (38), namely:

$$T = abZIH$$

as explained in Art. 58. If the coil is suspended by fine wires this torque will turn the coil more or less, and, in order that the coil may be perceptibly turned by a very weak current, the suspending wires (which serve to lead current to and from the coil) must be very fine, the number of turns of wire in the coil must be great, and the magnetic field H in which the coil is suspended must be intense. In order to obtain a quick movement of the coil it is important to have its breadth b moderately small. Figure 80 shows the essential parts of a sensitive galvanometer constructed according to these principles. It consists of an elongated coil of fine wire suspended in the strong field between the poles of a magnet. This type of galvanometer is due to D'Arsonval. It is not so sensitive as the galvanometer of the Kelvin type, but it is scarcely affected by outside magnetic influences.

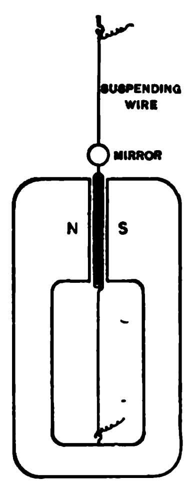

Fig. 80.

The D'Arsonval galvanometer may be used for the approximate measurement of weak currents, because the deflection, within a small range, is proportional to the current; that is

$$i = kd$$

in which i is the current flowing through the galvanometer, d is the deflection in scale divisions, and k is a proportionality factor which is called the *reduction factor* of the galvanometer.

Problems.

80. A horizontal wire 10 meters long, stretched due magnetic east and west, is pushed up by the horizontal component of the earth's field with a force of 2,500 dynes. What is the direction and strength of the current in the wire? The horizontal component of the earth's field is 0.2 gauss. Ans. 125 amperes east.

81. The armature of a dynamo has a length, under the pole-face, of 30 cm. The magnetic field intensity between the pole-face and the armature core is 6,000 gausses. The surface of the armature is covered with straight wires parallel to the axis of the armature. Each of these wires carries a current of 75 amperes. Calculate the force acting on each wire. Ans. 1,350,000 dynes.

82. A horizontal electric light wire stretched due magnetic north and south carries 1,000 amperes of current flowing towards the north. The length of the wire is 250 meters, the intensity of the earth's field is 0.57 gauss and the magnetic dip is 63°. Find the value of the force pushing on the wire and specify its direction. Ans. 1,269,500 dynes west.

83. A circular coil of wire of 20 cm. radius has 15 turns of wire. How much current is required in the coil to produce at the center of the coil a field intensity of 0.57 gauss? Ans. 0.121 abamperes.

84. The two straight parallel wires of an electric light pole-line are 35 inches apart center to center, and a current of 500 amperes is flowing out in one wire and back in the other. Find: (a) The intensity of the magnetic field due to the wires at a point midway between them, and (b) the intensity of the magnetic field due to the

two wires at a point which is 21 inches from the axis of one wire and 28 inches from the axis of the other wire. Ans. (*a*) 4.5 gausses. (*b*) 2.34 gausses.

85. A thin brass tube 2 inches in diameter and 6 feet long is wound with 1,400 turns of wire. (*a*) Calculate the field intensity inside of this coil when a current of 5 amperes flows through the wire. (*b*) Calculate the total energy of the magnetic field inside of the coil. Ans. (*a*) 48.1 gausses. (*b*) 341,220 ergs.

86. A tangent galvanometer gives a deflection of 10° for 1.2 amperes. Calculate the deflection which will be produced by 15 amperes. Ans. 65° 35′.

87. A rectangular frame 25 × 40 cm. has 10 turns of wire wound upon it. The frame is balanced horizontally upon an axis pointing due magnetic east and west. A current of 28 amperes is sent through the wire. Required the distance from the axis at which a 10-gram (9,800-dyne) weight must be hung to balance the torque action due to the earth's magnetic field at a place where its intensity is 0.57 gauss and its dip is 63°. Ans. 0.74 cm.

88. A circular coil has 100 turns of wire. The diameter of the mean turn is 16 centimeters, and a current of 15 amperes flows through the coil. This coil is suspended with its plane lying vertical and magnetic north and south. (*a*) Calculate the torque in dyne-centimeters with which the horizontal component of the earth's field (0.2) acts upon the coil and specify the direction of the axis about which this torque is exerted. (*b*) Calculate the torque in dyne centimeters with which the vertical component of the earth's field (0.68) acts on the coil and specify the direction of the axis about which this torque is exerted. Ans. (*a*) Axis, vertical; torque, 6,032 dyne-centimeters. (*b*) Axis, north and south; torque, 20,508 dyne-centimeters.

89. A circular coil 10 cm. in diameter, having 50 turns of wire, is hung by a phosphor-bronze wire at the center of a large circular coil 120 cm. in diameter, having 500 turns of wire. The suspending wire is free from twist when the planes of the two coils are at right angles, and a torque of 250 dyne-centimeters twists the wire through one radian of angle. How much current must pass through the two coils in series to cause the suspended coil to turn 30° from its position of equilibrium? What

happens if the current is reversed in one coil? What happens if the current is reversed in both coils? Ans. 0.27 ampere.

90. The spiral spring of a Seimens electrodynamometer is twisted through an angle of 225° to balance the force action on the movable coil when a current of 14 amperes flows through the instrument. A twist of 160° is required to balance the force action of a current which is being measured by the instrument. Required the value of this current. Ans. 11.8 amperes.

91. The horizontal component of the earth's magnetic field at the needle of a sensitive galvanometer (Kelvin type) is 0.18 gauss, and its direction is due north. It is desired to produce at the needle a resultant magnetic field of 0.02 gauss intensity in a due easterly direction. Find the distance and direction from the galvanometer needle at which an isolated north magnet pole of strength 600 gausses must be placed to produce the desired result. Ans. 57.6 cm., 6° 20′ west of north.

CHAPTER V.

INDUCED ELECTROMOTIVE FORCE.

THE DYNAMO.

62. Lenz's law. Electromagnetic theory a branch of mechanics.* — The idea of electric current is strictly analogous to the mechanical idea of velocity † and an insight into the nature of induced electromotive force can be obtained only by drawing a parallel between the equations in Mechanics and the equations of Electricity and Magnetism.

The product of the electromotive force E of a generator and the current I delivered by the generator is equal to the power P delivered by the generator to the circuit to which the generator delivers current. That is,

$$P = EI$$

in which P is expressed in ergs per second if E is expressed in abvolts and I in abamperes, or P is expressed in watts if E is expressed in volts and I in amperes.

In order to produce a current I through a circuit of which the resistance is R, an electromotive force equal to RI is required; that is,

$$E = RI$$

Multiplying both members of this equation

The product of the force F exerted on a body which moves at velocity v in the direction of F is equal to the power P developed by the agent which is exerting the force on the body; that is

$$P = Fv$$

in which P is expressed in ergs per second if F is expressed in dynes and v in centimeters per second. There are no names for the units of force and velocity which correspond to the watt as a unit of power.

A force F acts upon a boat and increases the velocity v of the boat until all of the force F is used to overcome the friction of the water. *Let us assume that the friction of the water is proportional to the velocity of the boat*, or equal to rv

* The mechanical analogies which are outlined in this article are exact and complete. Any one who is interested in the full details of this matter should read a remarkable paper on *The Motion of Monocyclic Systems* by H. von Helmholtz, Crelle's Journal, Vol. 97, pp. 111 and 317. A very interesting and instructive book entitled *Applications of Dynamics to Physics and Chemistry*, by J. J. Thomson, touches indirectly upon this matter. See also Art. 125 of this text.

† Electric current *is* velocity and it is entirely meaningless to speak of the velocity with which an electric current flows along a wire. This matter will be made clear when we come to discuss electric waves.

by the current and remembering that EI is equal to the power delivered to the circuit, we have

$$P = RI^2$$

In these equations R may be expressed in ohms, I in amperes, E in volts and P in watts, or R may be expressed in abohms, I in abamperes, E in abvolts and P in ergs per second.

where r is a constant. Then we have,

$$F = rv$$

Multiplying both members of this equation by v and remembering that Fv is the power that is delivered to the boat, we have

$$P = rv^2$$

In these equations c.g.s. units are most conveniently used, that is, F is expressed in dynes, v in centimeters per second, and P in ergs per second. The coefficient r is exactly analogous to the resistance of an electric circuit.

Consider the wires on the surface of the cylinder A in Fig. 63 with electric currents flowing through them as indicated by the dots and crosses. These wires are pushed sidewise by the magnetic field, as explained in Art. 51, and, if the cylinder A is allowed to turn so that the wires A move *with* * this side force, mechanical work is obtained. Where does this work come from? If the cylinder A is forcibly turned so that the wires move *against* the side force with which the magnetic field pushes on them, mechanical work is expended. Where does this work go to? The present chapter is devoted to the consideration of these two questions, and some idea of the conclusions which will be reached may be obtained by a brief discussion of the analogous mechanical problem. A person standing on the swinging span of a drawbridge as shown in Fig. 81 is acted upon by a centrifugal force, as indicated by the arrow, and this centrifugal force depends upon the angular velocity of the moving span. If, while the span is swinging, the person walks towards the center of the span, *he does work in moving himself against the centrifugal force, and* **this work helps to turn the span.** *If the person walks away from the center of the swinging span he is helped by the centrifugal force, or, in other words, he receives energy or work from the swinging span,* **more**

* A body is said to move *with* a force which acts upon it when it moves in the direction of the force. A body is said to move *against* a force which acts upon it when it moves in a direction opposite to the direction of the force.

work is required to keep the span turning than would otherwise be necessary, *and the work received by the moving person is equal to the additional work so expended in turning the span.*

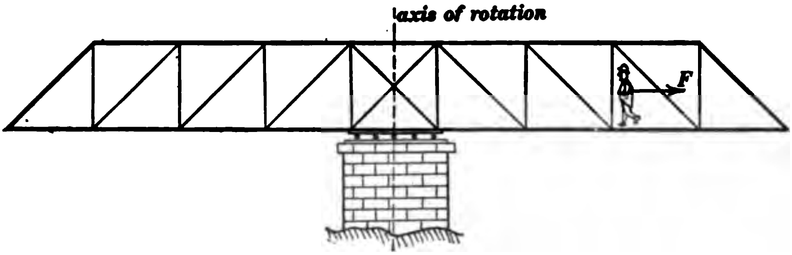

Fig. 81.

Inasmuch as the idea of electric current strength is strictly analogous to the mechanical idea of velocity, the question as to what becomes of the work done in moving a wire *against a force which depends on the current* is strictly analogous to the question as to what becomes of the work done in moving a body *against a force which depends on velocity.* Therefore the above example of a person moving radially on a swinging bridge span is analogous to the following: A wire is connected to a battery so that an electric current flows through it, and the wire is stretched across a magnetic field as shown in Fig. 61. Under these conditions the magnetic field pushes sidewise on the wire, and this side force depends on the current. If the wire be moved sidewise against this force, *work has to be done and* **this work helps to maintain the current.** *If the wire is moved in the direction of the side force, the side force does work in helping to move the wire,* **more work is required to keep the current flowing than would otherwise be necessary,** *and the work received by the moving wire is equal to the additional work thus done in keeping the current flowing.*

In the example of the swinging bridge span, the force exerted by the engine which drives the span must be supposed to be greater or less according as the man is moving outwards or inwards (*with* or *against* the centrifugal force) if the velocity of turning is to be kept constant. In the example of the moving

wire, the battery which supplies the electric current must be supposed to have a greater or less electromotive force according as the wire is moving *with* or *against* the side force due to the magnetic field if the strength of the current is to be kept constant.

The action described above in connection with the motion of a man on a swinging bridge span may be perceived in a very striking way by holding weights in one's hands, swinging round and round on one's heel, and drawing the weights inwards or extending them outwards repeatedly.

The facts outlined above in connection with the moving wire, constitute what is called *Lenz's law*, a more elaborate statement of which will be given later.

63. Induced electromotive force. — Faraday discovered in 1831 that a momentary electric current is produced in a coil of wire when a magnet is *thrust into* a coil or *withdrawn from* the coil, or when an iron rod upon which the coil is wound is *magnetized* or *demagnetized*. The motion of the magnet in the first case or the varying magnetism of the iron rod in the second case, produces a momentary electromotive force in * the coil and this electromotive force in its turn, produces a momentary current if the coil forms a portion of a closed circuit. The electromotive force and electric current produced in this way are called induced electromotive force and induced current.

Examples of Lenz's law. — A current induced in a coil when a magnet is *thrust into* the coil is in such a direction as to tend to *push the magnet out of the coil*, and the work done in moving the magnet against this opposing force is the work which goes to produce the induced current. The current induced in a coil when a magnet is *withdrawn from* the coil is in such direction as to tend to *draw the magnet into the coil* and the work done in moving the magnet against this opposing force is the work which goes to produce the induced current. When an iron rod with a

* One should always speak of the electromotive force between two points, never of the electromotive force in a circuit, except only when one is speaking of an induced electromotive force.

short-circuited winding of wire is magnetized, the current induced in the winding opposes the magnetization and more work is required to magnetize the rod than would be required if the induced current did not exist. This additional work is that which produces the induced current.

64. Electromotive force induced in a straight wire moving sidewise across a uniform magnetic field. — Consider a straight wire BB', Fig. 82, which slides sidewise at a velocity of v centime-

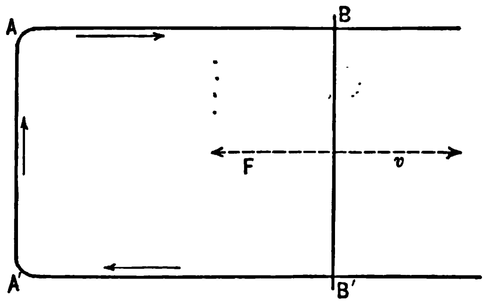

Fig. 82.

ters per second along two straight wires or rails AB and $A'B'$, distant l centimeters from each other. The rails AB and $A'B'$ are connected at AA' so that $ABB'A'$ is a closed circuit. The whole arrangement is placed in a uniform magnetic field of intensity H, the direction of the field being perpendicular to the plane of the figure and towards the reader. The motion of the wire BB' induces in it an electromotive force E which in its turn produces a current I in the circuit $ABB'A'$, and because of this current the magnetic field pushes the wire BB' sidewise with a force F as indicated in the figure. The rate at which work is done in moving the wire BB' against the force F at velocity v is Fv ergs per second, and the rate at which work is done by the electromotive force E in maintaining the current I is EI ergs per second, E being expressed in abvolts, and I being expressed in abamperes. According to Lenz's law, the work done in moving the wire BB' against the force F goes to maintain the current. Therefore we have

$$Fv = EI \tag{i}$$

and from equation (28), in Art. 52,

$$F = IlH$$

whence, substituting this value of F in equation (i), we have

$$E = lHv \tag{42}$$

that is, the electromotive force induced in a wire l centimeters long, moving sidewise at a velocity of v centimeters per second across a uniform magnetic field of intensity H is equal to the product lHv. This product expresses the induced electromotive force in c.g.s. units or abvolts, one abvolt being an electromotive force which will do work at the rate of one erg per second in maintaining a current of one abampere. One volt equals 10^8 abvolts.

65. Expression of induced electromotive force in terms of lines of force cut per second.—During t seconds the sliding piece BB', Fig. 82, moves over a distance vt and sweeps over lvt square centimeters of area. The product of this area by the field intensity H gives the number of lines of force Φ which pass through the area according to equation (18), and this is the number of lines of force cut by the moving wire in t seconds, that is,

$$\Phi = lHvt \tag{i}$$

Dividing both members of this equation by t, we have

$$\frac{\Phi}{t} = lHv$$

but Φ/t is the rate at which the moving wire BB' cuts lines of force, or, in other words, it is the number of lines of force cut per second, and lHv is the electromotive force in abvolts induced in the wire, according to equation (42). Therefore *the electromotive force in abvolts induced in a moving wire is equal to the number of lines of force cut per second by the moving wire.* This result is true for any wire, straight or curved, moving in any

manner in any magnetic field, uniform or non-uniform, although the derivation here given applies to the motion of a straight wire across a uniform field.

66. Expression of induced electromotive force in terms of rate of change of magnetic flux through a circuit.* — The total magnetic flux through the circuit $ABB'A'$, Fig. 82, is given by equation (i), Art. 65, and the rate at which the moving wire BB' cuts lines of force is the rate of increase of Φ. Therefore *the electromotive force induced in a circuit is equal to the rate of change of the magnetic flux through the circuit,* that is,

$$E = -\frac{d\Phi}{dt} \tag{43}$$

Experiment shows this equation to be true when the change of magnetic flux is due to motion and also when the change of magnetic flux is due to varying strength of the magnetic field.

The negative sign in equation (43) has no immediate importance. It is chosen in accordance with the following convention. A *right handed screw* with its axis parallel to the magnetic field H (directed towards the reader in Fig. 82) would have to be turned in a direction *opposite* to the flow of induced current produced by an increasing flux in order to make the screw travel in the direction of H. It is therefore convenient to look upon the induced current or the induced electromotive force as negative when $d\Phi/dt$ is positive.

Equation (43) expresses the electromotive force induced in a single turn of wire. When a region of changing magnetic flux is surrounded by Z turns of wire, then equation (43) expresses the electromotive force induced in each turn of wire, and the total electromotive force is

$$E = -Z\frac{d\Phi}{dt} \tag{44}$$

* Let it be remembered that the fundamental action upon which induced electromotive force depends is the *cutting the lines of force* by a moving conductor or the sweeping of moving lines of force past a stationary conductor.

67. The dynamo.—The dynamo is a machine for the production and maintenance of an electric current when the machine is supplied with mechanical power, or, conversely, for the development of mechanical power when the machine is supplied with electric current. When used for the former purpose the dynamo is called an *electric generator*, and when used for the latter purpose, the dynamo is called an *electric motor*.

The action of the dynamo as a generator is essentially as follows: A wire is forced by an external source of mechanical power to move sidewise across a magnetic field. This motion induces an electromotive force in the wire and this electromotive force produces a current in the circuit which is connected to the ends of the wire. The induced current causes the magnetic field to push on the moving wire in a direction *opposite* to its motion, and the work done in overcoming this opposing force is the work that goes to maintain the induced current.

The action of the dynamo as a motor is essentially as follows: An electric current from an external source is forced through a wire which is allowed to move sidewise in a magnetic field in the direction of the side push upon it, thus developing mechanical power. The motion of the wire induces in it an electromotive force which *opposes* the flow of current through the wire, and the work done by the external source of electric current in forcing the current through the wire in opposition to this induced electromotive force is the work which appears as mechanical energy in the motor.

The above-described action of the dynamo as a generator and as a motor constitutes a complete statement of what is called Lenz's Law, namely, that an induced current leads to the production of a force which opposes the action which produces it and the work done in overcoming this opposing force is the work that goes to produce the induced current.

Types of dynamos.—There are two distinct types of dynamo electric machines, namely, (*a*) alternating-current machines and (*b*) direct-current machines. The alternating-current generator

delivers what is called an alternating current, that is, a current which is subject to rapid periodic reversals of direction. The direct-current generator, on the other hand, delivers a current which is not reversed in direction and which is usually quite steady in value.

68. The alternating-current dynamo. — The simplest form of the alternating-current dynamo is shown in Fig. 83. A wire W,

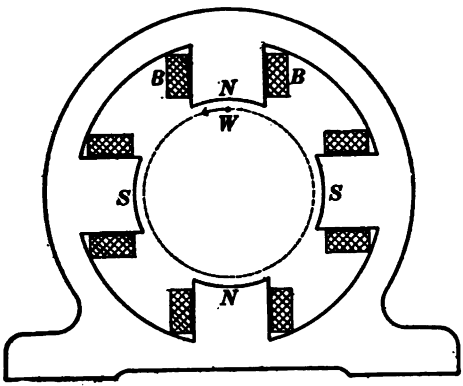

Fig. 83.

perpendicular to the plane of the paper, is moved sidewise along the dotted line so as to cut the magnetic lines of force which emanate from the inwardly projecting poles $NSNS$ of a large electromagnet which is called the *field magnet* of the alternator. While the wire is sweeping across a north pole an electromotive force is induced in it in one direction, and while the wire is sweeping across a south pole an electromotive force is induced in it in the opposite direction. This repeatedly reversed electromotive force is called an *alternating electromotive force* and it produces an *alternating current* in the wire and in an outside circuit to which the ends of a wire may be connected.

In commercial alternators large numbers of wires are used instead of the single wire W shown in Fig. 83, and these wires are placed in slots in the periphery of a rotating cylindrical mass of laminated iron. Thus, Fig. 84 shows 4 wires in 4 slots and

Fig. 85 shows 16 wires in 16 slots. Figures 84*b* and 85*b* are what are called developed diagrams which show how the wires are connected to each other and how they are connected to two

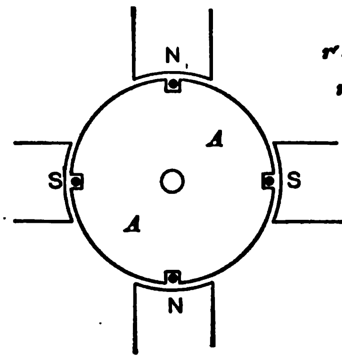

Fig. 84a.

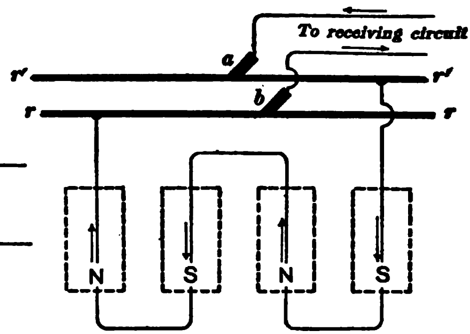

Fig. 84b.

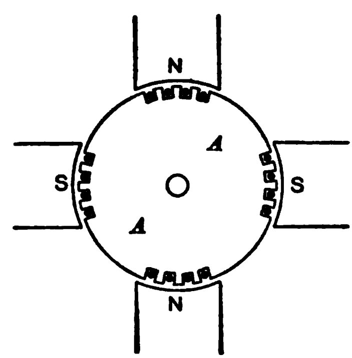

Fig. 85a.

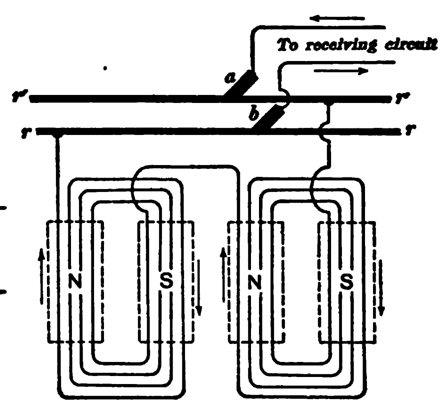

Fig. 85b.

insulated metal rings *r* and *r'* upon which two metal brushes *a* and *b* rub, thus keeping the moving wires connected to an outside receiving circuit.

The laminated iron cylinder *AA* with its winding of wire is called the *armature* of the alternator, the metal rings *r* and *r'* are called *collector rings*. The field magnet of an alternator must

be excited by direct current which is generally supplied by a small auxiliary direct-current generator called the *exciter*.

Definition of the cycle. Frequency. — The electromotive force of an alternator passes through a set of positive values while a group of armature wires is passing a north pole of the field magnet, and through a set of negative values while the given group of armature wires is passing a south pole of the field magnet. The complete set of values, including positive and negative values, is called a *cycle*, the duration of a cycle is called a *period*, and the number of cycles per second is called the *frequency*. If the field magnet of an alternator has p poles ($p/2$ north poles and $p/2$ south poles), then the frequency of its electromotive force is $pn/2$, where n is the speed of the alternator armature in revolutions per second. This is evident when we consider that a complete cycle corresponds to the passage of a given group of armature wires across two field poles, a north pole and a south pole, so that there are $p/2$ cycles in one revolution. The standard frequencies of commercial alternators in practice are 25 cycles per second for large installations for the transmission of power, 60 cycles per second for alternators which supply current for both lamps and motors, and 133 cycles per second for the older styles of alternators which supply current to lamps only.

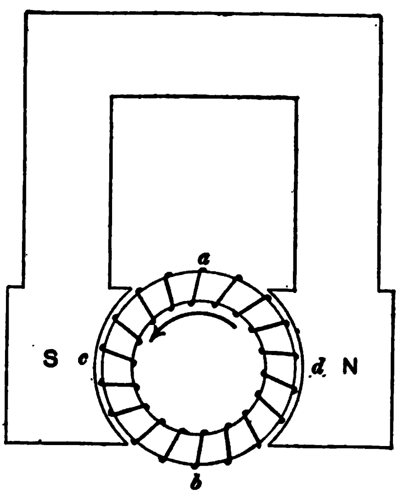

Fig. 86.

69. The direct-current dynamo is somewhat more complicated than the alternator. The following description applies to the direct-current dynamo having an armature of the so-called ring type, and having a bipolar field magnet. An iron ring

ab which is built up of sheet iron stampings, is wound uniformly with insulated wire as indicated in Fig. 86, the ends of the wire being spliced together and soldered so that the winding is endless. This iron ring with its winding of wire is called the *armature* of the machine, and it rotates, as indicated by the curved arrow, between the poles of a strong *field magnet.*

The wires on the outside of the iron ring have electromotive forces induced in them as they move across the pole faces of the field magnet and cut the lines of force. These electromotive forces cannot, however, produce current in the endless wire that is wound on the armature, because exactly equal and opposite electromotive forces are induced on the opposite sides *c* and *d* of the ring, as shown diagrammatically in Fig. 87 in which the

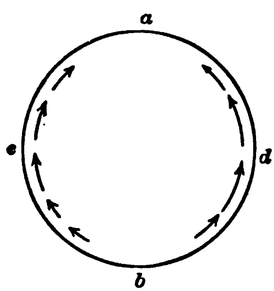

Fig. 87.

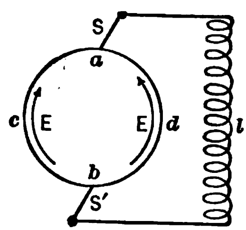

Fig. 88.

circle *adbc* represents the endless wire on the ring. A steady, or very nearly steady, current can, however, be taken from the winding on the ring by keeping the terminals of an external circuit *l*, Fig. 88, in metallic contact with the windings on the ring at *a* and *b*. For this purpose the insulation may be removed from the outer portions of the wire windings on the ring and two stationary metal or carbon brushes *SS*, Fig. 88, may be arranged to rub at *a* and *b* as the ring rotates. In practice wire leads are soldered to the various turns of wire on the ring and connected to insulated copper bars near the axis of rotation as shown in Fig. 89. Sliding contact is then made with these copper bars instead of with the turns of wire at *a* and *b*

directly. This set of copper bars constitutes what is called the *commutator.*

Shunt and series field windings. — The field magnet of a direct-current generator is usually excited by current taken from the machine itself. The winding of wire on the field magnet may consist of many turns of comparatively fine wire having a con-

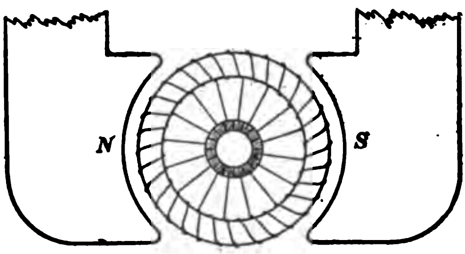

Fig. 89.

siderable resistance. In this case the terminals of the field winding are connected directly to the brushes of the machine, and from two to ten per cent. of the permissible current output of the generator flows through the field windings and excites the field, the remainder of the permissible current output being available for use in the external circuit. In this case the field winding and the outside receiving circuit are in parallel with each other between the brushes, so that the field winding is in the relation of a shunt to the outside receiving circuit. A direct-current dynamo with its field windings arranged in this way is called a *shunt dynamo.*

The winding of wire on the field magnet of a direct-current dynamo may consist of comparatively few turns of heavy wire having a low resistance. In this case the field winding is connected in series with the external receiving circuit, the whole current delivered by the machine flows through the field winding, and from two to ten per cent. of the electromotive force developed by the machine is used to force the current through the field winding, the remainder being available for forcing current through the external receiving circuit. A direct-current dynamo with its field windings arranged in this way is called a *series dynamo.*

The multipolar direct-current dynamo. — Figure 90 shows a ring armature rotating inside of a crown of six inwardly projecting field magnet poles. The electromotive force induced in the windings as they sweep across the pole faces cannot produce current in the endless wire that is wound on the ring, because the electromotive forces induced under the north poles are just balanced by the electromotive forces induced under the south poles, as shown diagrammatically in Fig. 91. To utilize the in-

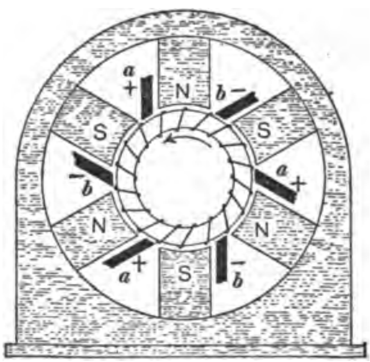

Fig. 90.

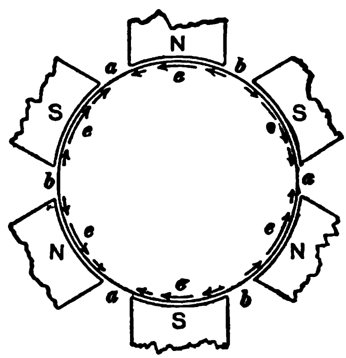

Fig. 91.

duced electromotive forces *eeeeee*, Fig. 91, for the production of direct current, six brushes *aaa* and *bbb*, Fig. 90, should be used. Three of these brushes maintain contact with the windings at *aaa*, and through all three of these brushes current flows out of the armature to one terminal of a receiving circuit. The other three brushes maintain contact with the windings at *bbb*, Fig. 91, all three of these brushes are connected to the other terminal of the receiving circuit and current flows into the armature through all three. The three brushes *aaa* together constitute the positive terminal of the armature, and the three brushes *bbb* together constitute the negative terminal of the armature.

Number of current paths in the armature between positive and negative brushes. — In the bipolar direct-current dynamo two brushes are used as shown in Fig. 88, and the current which enters the armature at the negative brush divides into two parts

and flows through two distinct paths in the armature winding to reach the positive brush. In the particular multipolar dynamo shown in Figs. 90 and 91 the current enters the armature through three brushes, and the current which enters at each of the three brushes divides into two parts and flows through two distinct paths to reach a positive brush. Therefore in this particular machine, having six field poles, there are six current paths through the armature from negative to positive brushes.*

70. Fundamental equation of the direct-current dynamo. — Let Φ be the magnetic flux which enters the armature from the north pole of the field magnet and leaves the armature at the south pole of the field magnet, let Z be the number of conductors on the outside surface of the armature, let n be the speed of the armature in revolutions per second, and let E_a be the electromotive force induced in the armature winding. A voltmeter connected to the brushes of the dynamo would indicate the value of E_a if the current in the armature were negligibly small; when the current in the armature is large, a portion of E_a is used to overcome the armature resistance. The equation which expresses the relation between E_a, Φ, Z, and n is called the *fundamental equation* of the dynamo. This equation is here derived for the simplest case, namely, that of a bipolar dynamo with simple ring-wound armature. In this case

$$E_a = \Phi Z n \text{ abvolts} \tag{45a}$$

or

$$E_a = \frac{\Phi Z n}{10^8} \text{ volts} \tag{45b}$$

Proof of equation (45*a*). — During $1/n$ second the armature makes one complete revolution, so that during $1/2n$ second a given conductor sweeps past a field pole from a to b in Fig. 86 and cuts Φ lines of force. Therefore this conductor cuts lines of force at an average rate which is equal to $\Phi \div 1/2n$, or $2\Phi n$ lines of force per second; which is equal to the *average* electromotive force induced in the given conductor *during the time* that it is moving from a to b in Fig. 86; also this is the *average* electromotive force in all of the conductors between a and b *at any instant.* Therefore, since there are $Z/2$ armature conductors or wires in series between a and b, the electromotive force between a and b is equal to $Z/2 \times 2\Phi n$, or $E_a = \Phi Z n$ abvolts.

71. The induction coil.† — An iron rod wound with insulated wire may be repeatedly magnetized and demagnetized by connecting a battery to the winding and repeatedly making and breaking the circuit. The increasing and decreasing magnetic

* A type of armature winding which is frequently employed provides but two paths through the armature winding irrespective of the number of field magnet poles.

† The induction coil was invented by Ruhmkorff in 1855 and it is frequently called the *Ruhmkorff coil.*

flux thus produced through the rod may be utilized to induce electromotive force in an auxiliary coil of wire wound on the rod. Such an arrangement is called an *induction coil.* The winding through which the magnetizing current from the battery flows is called the *primary coil* and the auxiliary winding in which the desired electromotive force is induced is called the *secondary coil.* The iron rod is always made of a bundle of fine iron wires to prevent the flow of eddy currents as explained in Art. 74.

When the iron rod or core is magnetized a pulse of electromotive force is induced in the secondary coil, and when the iron core is demagnetized, a reversed pulse of electromotive force is induced in the secondary coil. These impulsive electromotive forces may be made very large in value, hundreds of thousands of volts, by making the secondary coil of many turns of wire and by providing for the quickest possible magnetization and demagnetization of the core.

A battery or any ordinary current generator does not magnetize a core very quickly when connected to a winding of wire; in fact, a very considerable fraction of a second is usually required for the core to become magnetized. Therefore, during the magnetization of the iron core of an induction coil the electromotive force induced in the secondary coil is *a comparatively weak pulse of long duration.*

On the other hand, proper arrangements permit of an extremely quick demagnetization of the iron core of an induction coil when the battery is disconnected from the primary winding, and this quick demagnetization induces in the secondary coil *an intense pulse of electromotive force of short duration.*

The quick demagnetization of the iron core of an induction coil is accomplished as follows: Figure 92*a* shows the connections of a battery to the primary coil. The battery is connected and disconnected by making and breaking contact between the metal terminals *tt.* Two large metal plates separated by an insulator (a condenser) are connected to the terminals *tt* as shown. When the points *tt* are connected together the core is slowly

magnetized by the current from the battery. When the points *tt* are separated, the current persists in flowing for a short in-

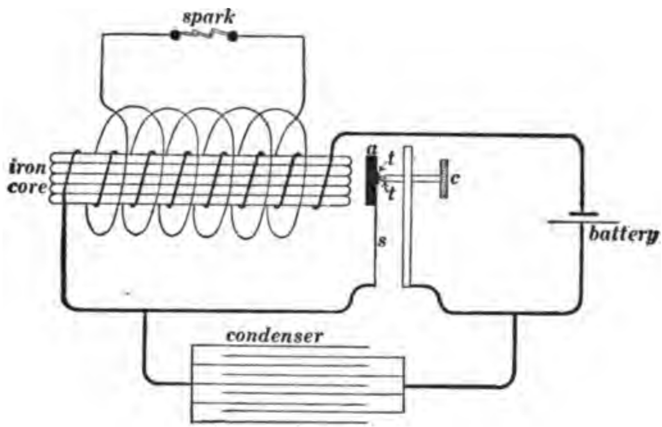

Fig. 92a.

terval of time, this persisting current flows into the condenser plates, and the electric charge which thus accumulates on the plates surges back through the circuit as a reversed current and demagnetizes the iron core.

Figure 92*b* shows a complete induction coil. The condenser is mounted inside of the box-like base.

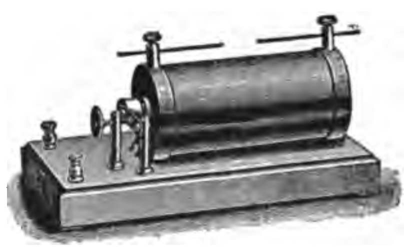

Fig. 92b.

72. The alternating-current transformer consists of two coils of wire, a primary coil and a secondary coil, wound upon an iron core. This iron core is built up of strips of sheet iron, and it usually forms a complete magnetic circuit as shown in Fig. 93. Figure 94 shows a sectional view of Fig. 93; the primary coil is represented by *PP* and a secondary coil by *SS*. Either coil of a transformer may be the primary coil according to the way in which the transformer is used as explained later. The induction coil and the alternating-current transformer are identical, except

that the iron core of the induction coil is *not* a complete magnetic circuit, but has magnet poles at its ends. The effect of these

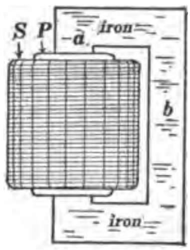

Fig. 93.

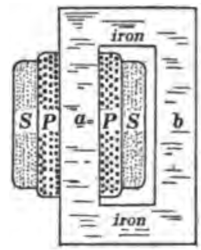

Fig. 94.

magnet poles is to facilitate the quick demagnetization of the core when the primary circuit of the induction coil is broken.

The action of the transformer.—Alternating current is supplied to either coil of the transformer. This alternating current produces rapid reversals of magnetization of the iron core. These magnetic reversals induce an alternating electromotive force in the other coil which delivers alternating current to any circuit to which it may be connected. The coil of a transformer which receives alternating current is called the *primary coil*, and the coil which delivers alternating current is called the *secondary coil*.

Step-up and step-down transformation.—Usually, one coil of a transformer has many more turns of wire than the other. The coil of many turns may act as the primary coil, taking a small current at high electromotive force from an alternator; and in this case the coil of few turns will be the secondary coil, and it will deliver a large current at low electromotive force to a receiving circuit. This action is called *step-down transformation.*

The coil of few turns on the other hand may act as the primary coil, taking a large current at low electromotive force from an alternator; and in this case the coil of many turns will be the secondary coil and it will deliver a small current at high voltage to a receiving circuit. This action is called *step-up transformation.*

The object of step-up and step-down transformation may be explained as follows: The transmission of a given amount of

power electrically may be accomplished by transmitting the large current output of a low voltage generator, or by transmitting the small current output of a high voltage generator. In the former case very large and expensive transmission wires must be used or the loss of power in the transmission wires will be excessive. In the latter case, comparatively small and inexpensive transmission wires may be used without involving an excessive loss of power. Therefore high electromotive force is a practical necessity in the long distance transmission of power. The user of electric power must however be supplied with current at low electromotive force, partly on account of the danger involved in the use of high electromotive forces, and partly on account of the fact that many types of electrical apparatus cannot be operated satisfactorily with high electromotive force; also it is inconvenient and dangerous to generate very high electromotive forces in a complicated machine like an alternator which must be cared for by an attendant. These difficulties are met by employing a transformer for step-up transformation at the generating station and another transformer for step-down transformation at the receiving station.

High efficiency of the transformer.—The transformer is not only cheap to construct and cheap to operate, but it is extremely efficient. The efficiency ranges from 95 or 96 per cent. for small sized transformers to 98 per cent. or more for transformers of large size.

73. Current and electromotive force relations of the transformer.—In the following discussion Z' represents the number of turns of wire in the primary coil, and Z'' the number of turns of wire in the secondary coil. The effect of the electrical resistance of the coils, which is usually quite small in practice, is ignored, and all of the magnetic flux which passes through one coil is assumed to pass through the other coil also.

(*a*) *Electromotive force relations.*—Let E' be the effective value of the alternating electromotive force which acts on the primary coil of a transformer, and let E'' be the effective value of the electromotive force induced in the secondary coil of the transformer. Then

EMF

$$\frac{E'}{E''} = \frac{Z'}{Z''} \tag{46}$$

This relation may be shown to be true as follows: The only thing which opposes the flow of current through the primary coil is the reacting electromotive force induced in

the primary coil by the reversals of magnetization of the core (resistance of primary coil being neglected). Therefore, the electromotive force which acts upon the primary coil is equal and opposite to the electromotive force which is induced in the primary coil by the magnetic reversals of the core. The magnetic reversals of the core, however, induce a certain electromotive force e in each turn of wire surrounding the core. Therefore the total electromotive force induced in the primary coil is $Z'e$ and the total electromotive force induced in the secondary coil is $Z''e$ so that the ratio of the two electromotive forces is equal to Z'/Z''.

(*b*) *Current relations.* — The electromotive force which is induced in the primary coil of a transformer balances the electromotive force which is applied to the primary coil as explained above, and the range of reversals of magnetization of the core must be such as to induce this reacting electromotive force in the primary coil. Therefore the combined magnetizing action of primary and secondary coils is always such as to magnetize the core to that degree which will make the reacting electromotive force in the primary coil equal to the electromotive force of the alternator which is forcing current through the primary coil.

When the secondary coil is on open circuit, just enough current flows through the primary coil to produce the degree of magnetization above specified. Let this value of the primary current, which is called the magnetizing current of the transformer, be represented by i. When a current I'' is taken from the secondary coil a current I' in addition to the magnetizing current i flows through the primary coil. The current i still suffices to magnetize the core, and the magnetizing action of I'' is exactly neutralized by the equal and opposite magnetizing action of I'. The magnetizing action of I'' is measured by the product $Z''I''$ and the magnetizing of I' is measured by the product $Z'I'$, so that, ignoring algebraic signs, we have

L : R

$$Z'I' = Z''I''$$

or

$$\frac{I'}{I''} = \frac{Z''}{Z'} = \frac{E''}{E'} \tag{47}$$

74. Eddy currents. Lamination. — When an iron rod is magnetized or demagnetized, the changing magnetic flux through the central portions of the rod induces electromotive forces around the outer portions of the rod, and these electromotive forces produce what are called eddy currents. Eddy currents are also produced in a mass of metal which is near to a moving magnet or which moves in the neighborhood of a stationary magnet.

Lamination. — Those parts of electrical machinery which are subject to rapid and frequent changes of magnetization are always built up of iron wire or of thin sheets of iron so as to leave the iron continuous in the direction of the magnetization but discontinuous in the direction in which the eddy currents tend to flow.

Such a mass of iron is said to be laminated. The iron parts of dynamo armatures and of transformers are always laminated.

Examples of eddy currents. — A suspended magnet which is set oscillating about its axis of suspension is quickly brought to rest if it is surrounded by a massive ring of copper, because the eddy currents induced in the copper by the moving magnet act upon the magnet with a force which is at each instant opposed to the motion (Lenz's Law).

A sheet of copper which is suddenly thrust between the poles of a strong electromagnet behaves as if it were moving in a viscid liquid. Eddy currents are induced in the copper and, because of these eddy currents, the magnet exerts a force upon the copper which is always opposed to the motion (Lenz's Law).

An interesting effect of eddy currents is their action in preventing the sudden magnetization or demagnetization of a solid iron rod. Thus, a bundle of iron wires surrounded by a winding of wire is magnetized say in one second when the winding is connected to a given battery, and demagnetized in a much shorter time when the battery is disconnected. A solid iron rod of the same size would require perhaps nine or ten seconds to be magnetized by the same coil and battery, and the solid rod would lose its magnetism very slowly when the battery is disconnected. *The eddy currents in the solid rod oppose the magnetization while the rod is being magnetized, and they tend to keep up the magnetization while the rod is being demagnetized* (Lenz's Law). Another interesting effect of eddy currents is that which is exemplified in the ordinary "medical" induction coil, in which the "power" of the coil is adjusted by moving a brass or copper tube which surrounds the iron core of the coil. When the tube surrounds the entire core a sudden break in the primary circuit results in a slow demagnetization of the core because of the eddy currents in the tube which tends to keep up the magnetization, but when the tube is withdrawn the core is demagnetized very quickly when the primary circuit is broken.

PROBLEMS.

92. Let it be assumed that the force required to propel a canal boat is proportional to the velocity of the boat. A force of 50 pounds is required to maintain a velocity of 5 feet per second. (*a*) Find the value of the coefficient by which the velocity of the boat must be multiplied to give the frictional drag and specify the unit in terms of which this coefficient is expressed. (*b*) Find the velocity at which the boat would be propelled by a force of 36 pounds. (*c*) Find the rate at which work is done by a force of 36 pounds in propelling the boat. Ans. (*a*) 10 pounds per (foot per second). (*b*) 3.6 feet/second. (*c*) 129.6 foot-pounds/second.

93. When a force of 50 pounds is applied to the above canal boat the boat starts from rest and after some time reaches its full speed of 5 feet per second. At a given instant the velocity of the boat is 3 feet per second. At this instant: (*a*) Find the rate at which work is done on the boat by the propelling force. (*b*) Find the dragging force which is acting on the boat. (*c*) Find the rate at which work is dissipated in overcoming the friction of the water. (*d*) Explain what is becoming of the difference between (*a*) and (*c*). Ans. (*a*) 150 foot pounds/second. (*b*) 30 pounds. (*c*) 90 foot-pounds/second.

94. An electromotive force of 50 volts acts on a circuit of which the resistance is 10 ohms. At a certain instant during the time that the current is growing from zero to its full value the current has an actual value of 3 amperes. At this instant: (*a*) Find the rate at which the generator delivers work to the circuit. (*b*) Find the dragging force in volts which is opposing the flow of the current through the circuit. (*c*) Find the rate at which work is dissipated in overcoming the resistance of the circuit. (*d*) Explain what is becoming of the difference between (*a*) and (*c*). Ans. (*a*) 150 watts. (*b*) 30 volts. (*c*) 90 watts.

95. A vertical wire 3 meters long is moved sidewise, towards magnetic east or west, at a velocity of 25 meters per second.

Find the electromotive force induced in the wire in volts, the horizontal component of the earth's field being 0.18 gauss. Ans. 0.00135 volt.

96. The pole-face of a dynamo is 30 centimeters long in the direction parallel to the axis of the armature, and the field intensity in the gap space between the pole-face and the armature core is 6,000 gausses. The wires on the armature are 12 centimeters from the axis of the armature, and the speed of the armature is 1,800 revolutions per minute. Find the electromotive force in volts induced in each armature wire (30 centimeters in length) as it passes across the pole-face. Ans. 4.07 volts.

97. A single wire *W*, Fig. 83, is rotated along the dotted line in Fig. 83 at a speed of 25 revolutions per second. The magnetic flux which emanates from each north pole of the field magnet and which enters each south pole is 2,500,000 lines. (*a*) Find the average value of the electromotive force induced in the wire during the time that it sweeps from a point midway between two field poles to the next point midway between two poles. (*b*) Find the number of cycles per second through which this induced electromotive force passes. Ans. (*a*) 2.5 volts. (*b*) 50 cycles per second.

98. The alternator specified in problem 97 has a winding as shown in Fig. 84. Find the average value of the electromotive force induced in the winding during the time that the armature is making one fourth of a revolution (that is, during the time that the slots containing the wires travel from a point midway between the pole pieces to another set of points midway between the pole pieces. Ans. 10 volts.

99. The core of an induction coil carries 100,000 lines of magnetic flux, when current is flowing through the primary coil. When the primary circuit is broken the flux in the core drops to 10,000 lines in 0.003 second. How many turns of wire are required in the secondary coil in order that an average electromotive force of 25,000 volts may be induced in this coil during the 0.003 second? Ans. 83,333 turns.

100. The ring armature of a direct-current bipolar dynamo has 260 turns of wire upon it, the armature is driven at a speed of 1,200 revolutions per minute, and the magnetic flux from a pole-face into the armature core is 2,200,000 lines. Calculate the electromotive force of the dynamo in volts. Ans. 114.4 volts.

101. The armature described in the above problem has upon it 500 feet of pure copper wire 325 mils in diameter. What is the resistance of the armature from brush to brush? Ans. 0.0125 ohm.

Remark. — In a bipolar dynamo the wire on the armature constitutes two paths between the brushes.

102. A transformer takes alternating current from supply mains at 1,100 volts and delivers current to service mains at 110 volts. The primary coil of the transformer has 560 turns of wire. How many turns of wire are there in the secondary coil? The transformer delivers 250 amperes to the service mains. How much current does it take from the supply mains? A usual allowance in transformer coils is 1,000 circular mils sectional area of wire for each ampere. Find size of wire used in primary coil and in secondary coil of the transformer. Ans. 56 turns, 25 amperes, 25,000 cir. mils, 250,000 cir. mils.

CHAPTER VI.

ELECTRIC MOMENTUM. INDUCTANCE.

75. The momentum of the electric current. Spark at break.—The analogy between electric current strength and velocity, as outlined in Art. 62, would lead one to expect an electric current to possess momentum and kinetic energy very much as a moving body possesses momentum and kinetic energy. In fact, this is found to be the case. When an electric circuit is broken, the current continues to flow across the break for a short time, producing an electric arc or spark, and the intensity of this spark is a rough indication of the amount of kinetic energy possessed by the current.

The amount of kinetic energy associated with a given current in a circuit made of a given length and size of wire, *depends upon the shape of the circuit* and *upon the presence of iron near the circuit.* Thus, a current in circuit *a*, Fig. 95, possesses but little kinetic

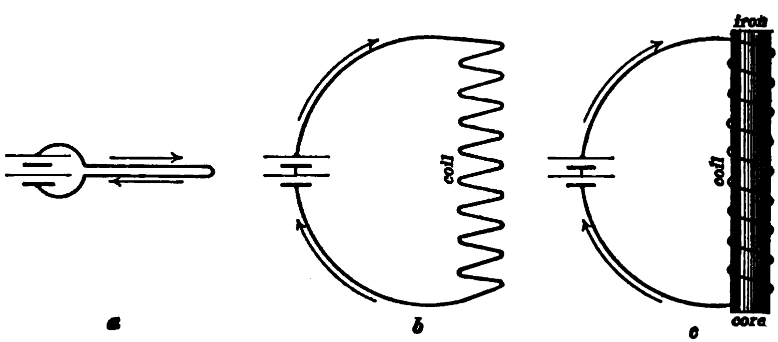

Fig. 95.

energy; the same current in circuit *b* possesses more kinetic energy; and the same current in circuit *c* possesses very much more kinetic energy. When the circuit of an ordinary incandescent lamp is broken a very slight spark only is produced; a

coil of wire having the same resistance as the lamp is connected to the supply mains so as to take the same amount of current as the lamp, and a much more intense spark is produced when this circuit is broken; an iron core consisting of a bundle of iron wires is then placed in the coil and a spark several inches in length may be produced by suddenly breaking the circuit.

The kinetic energy of the electric current resides in the magnetic field which is produced by the current. Thus, a current in the circuit *a*, Fig. 95, produces a very weak magnetic field except in the small region between the wires, and the kinetic energy of the current is small. The same current in the circuit *b* produces an intense magnetic field inside of the coil and the kinetic energy of the current is correspondingly great. The kinetic energy of the current in the coil of wire shown in Fig. 95*c* is much greater than the kinetic energy of the same current in the circuit shown in Fig. 95*b*, but the presence of the iron core in Fig. 95*b* complicates matters greatly, and nearly the whole of this chapter relates to the kinetic energy of currents in the absence of iron.

Practical applications of the spark at break. — In the device which is ordinarily used for lighting gas jets by electricity, an electric circuit is made and broken in the stream of gas which is to be lighted, and the gas is ignited by the spark at break. In order to produce an intense spark, the circuit includes a coil of wire wound on an iron wire core, a so-called "spark coil." This same device is used for igniting the mixture of gas and air in a gas engine.

76. Definition of inductance. — The kinetic energy which is associated with a current in a given circuit is proportional to the square of the current; that is, we may write

$$W = \tfrac{1}{2}LI^2 \tag{48}$$

in which W is the kinetic energy of a current I in a given circuit, and $(\frac{1}{2}L)$ is the proportionality factor. The quantity L is called the *inductance** of the circuit.

* Sometimes called the *coefficient of self-induction* of the circuit.

Discussion of equation (48). — It was pointed out in Art. 53 that to double the strength of the current in a circuit is to double everywhere the intensity of the magnetic field which is due to the current, and it was shown in Art. 44 that the kinetic energy per unit volume of a magnetic field is proportional to the square of the field intensity. Therefore to double the strength of the current in a circuit is to double everywhere the intensity of the magnetic field due to the current, and to quadruple everywhere the energy of the magnetic field, so that to double an electric current is to quadruple the total energy of its magnetic field.

Units of inductance. — If W in equation (48) is expressed in joules, and I in amperes, then L is expressed in terms of a unit of inductance which is called the *henry*, that is to say, a circuit has an inductance of one henry when a current of one ampere in this circuit represents one half of a joule of kinetic energy.

If W in equation (48) is expressed in ergs and I in abamperes, then L is expressed in c.g.s. units of inductance. The c.g.s. unit of inductance is sometimes called the *abhenry.** A circuit has one abhenry of inductance when a current of one abampere in that circuit represents one half of an erg of kinetic energy. There are 10^9 abhenrys in one henry.

Inductance of a coil. — Strictly, one cannot speak of the inductance of anything but an entire circuit, inasmuch as every portion of a circuit contributes its share to the magnetic field at each and every point in the surrounding region; it is, however, allowable to speak of the inductance of a coil when the terminals of the coil are not too far apart, and when the remainder of the electrical circuit does not produce any perceptible magnetic field in the region occupied by the coil.

Non-inductive circuits. — A circuit is said to be non-inductive when the inductance of the circuit is negligibly small, that is, when the electromotive force $L \times di/dt$ † is negligibly small as compared with the electromotive force RI which overcomes the

* The c.g.s. unit of inductance is often called the *centimeter*.

† See next article.

resistance of the circuit. Thus, a given circuit might be considered to be non-inductive under conditions involving *slow changes* of current, whereas the same circuit would not be considered to be non-inductive under conditions involving *rapid changes* of current. When a circuit consists simply of out-going and returning wires, side by side, its inductance is so small that it may be in most cases ignored. The wires used in resistance boxes are usually arranged non-inductively. This may be done by doubling the wire back on itself, and winding this doubled wire on a spool; or the wire may be wound in one layer on a thin paper cylinder, and this cylindrical coil may then be flattened so as to reduce the region (inside) in which the magnetic field is intense.

Measurement of inductance. — The most accurate method for determining the inductance of a coil is by calculation from measured dimensions. This calculation can be carried out only when the coil is very simple in shape, and even then the calculation is in most cases quite complicated.* The simplest case is given in Art. 81. The inductance of an irregularly-shaped coil may be determined by various electrical methods.†

Moment of inertia of a wheel. Analogue of inductance. — The kinetic energy of a rotating wheel resides in the various moving particles of a wheel, in the same way that the kinetic energy of a current resides in the various parts of the magnetic field which is due to the current. If the angular velocity ω of the wheel is doubled the linear velocity of every particle of the wheel is doubled, in the same way that the intensity of the magnetic field at every point in the neighborhod of a coil is doubled when the current in a coil is doubled. Therefore the kinetic energy of every particle of a wheel is quadrupled when its angular velocity is doubled, in the same way that the kinetic energy of every portion of the magnetic field around a coil is quadrupled when the

* See a series of articles in the *Bulletin of the United States Bureau of Standards*, by E. B. Rosa, Vol. 1, pages 125 and 291; Vol. 2, pages 87, 161 and 359; Vol. 3, page 1.

† See *Practical Physics*, by Franklin, Crawford and MacNutt, Vol. 2, page 129; see also *Absolute Measurements*, by Andrew Gray, Vol. 2, Part 2, pages 438–509.

current in the coil is doubled; and consequently the total kinetic energy of a rotating wheel is proportional to the square of its angular velocity, in the same way that the total kinetic energy of a current in a given coil is proportional to the square of the current. That is, we may write

$$W = \tfrac{1}{2}K\omega^2$$

in which W is the kinetic energy of a rotating wheel, ω is the angular velocity of the wheel, and $(\tfrac{1}{2}K)$ is a proportionality factor. The quantity K is called the moment of inertia of the wheel.

77. Electromotive force required to cause a current to increase or decrease. — To maintain a constant current in a circuit an electromotive force equal to Ri must act upon the circuit to overcome the resistance of the circuit. If the electromotive force which acts upon the circuit is greater than Ri, the current increases in value, and if the electromotive force which acts upon the circuit is less than Ri, the current decreases in value. Let the electromotive force which acts upon a circuit exceed Ri by the amount e; then we have

$$e = L\frac{di}{dt} \tag{49}$$

in which L is the inductance of the circuit and di/dt is the rate at which the current increases. When e is negative (electromotive force less than Ri) then di/dt is negative, that is, the current decreases.

Mechanical analogue of equation (49). — To keep a body in uniform motion a force sufficient to overcome the drag of friction must act upon the body. If the force which acts upon the body is greater than the drag of friction, the body gains velocity, and if the force which acts upon the body is less than the drag of friction, the body loses velocity. Let the force which acts upon the body exceed the drag of friction by the amount e, then we have

$$e = L\frac{di}{dt}$$

in which L is the mass of the body and di/dt is the rate at which its velocity changes. Equation (49) is therefore analogous to the fundamental equation in mechanics which expresses the relationship between unbalanced force, mass and acceleration.

Starting from the fact that force equals mass times acceleration, it can be shown that the kinetic energy of a moving body is equal to one-half its mass times its velocity squared, suitable units being employed. The same argument reversed would show that force must be equal to mass times acceleration if kinetic energy is equal to one-half mass times velocity squared; *and an exactly similar argument would establish equation (49) on the basis of equation (48).*

Self-induced electromotive force.—When one pushes on a body causing its velocity to increase the body reacts and pushes back on the hand. This reacting force is equal and opposite to the acting force which is causing the increase of velocity. When the velocity of the body is increasing, its reaction is a force opposed to its motion, and, when the velocity of the body is decreasing, its reaction is a force in the direction of its motion.

Similarly when an electromotive force acts upon a circuit and causes the current to increase or decrease, the changing current reacts, and the reacting electromotive force is equal and opposite to the acting electromotive force which is causing the current to change. Therefore from equation (49) we have

$$e = -L\frac{di}{dt} \tag{50}$$

in which e is the *reaction* of the changing current in a circuit of which L is the inductance, and di/dt is the rate at which the current is changing. This reaction of a changing current is called *self-induced electromotive force.*

78. Growth of current in an inductive circuit.—A steady force E begins to act upon a boat at a given instant, starting it from rest. At the given instant the velocity of the boat is zero, the frictional drag of the water is zero, and all of the force is used to

cause the velocity of the boat to increase. As the boat gains more and more velocity, however, a larger and larger portion of the force E is used to overcome the frictional drag of the water, and a smaller and smaller portion of E is used to cause the velocity of the boat to increase. Finally, after the force has been acting for some time, the boat reaches full speed, and then all of the force E is used to overcome the frictional drag.

An electromotive force E due to a battery or dynamo begins to act on a circuit at a given instant. At this instant the current is zero, and the whole of E acts to cause the current to increase in accordance with equation (49). As the increasing current reaches larger and larger values, however, a larger and larger portion of E is used to overcome the resistance of the circuit, and a smaller and smaller portion of E is used to cause the current to increase. After the electromotive force has been acting for some time the current reaches its full steady value, and then the whole of E is used to overcome resistance. The portion of E which is used at any given instant to overcome resistance is equal to Ri and the portion which is used to cause the current to increase is equal to $L \cdot di/dt$. Therefore we have

$$E = Ri + L\frac{di}{dt} \tag{51}$$

in which i is the value of the growing current at a given instant, and di/dt is its rate of increase at that instant.

Examples.—(*a*) A force of 50 pounds propels a canal boat at a speed of 5 feet per second. Let us assume that the drag of the water is proportional to the velocity of the boat, and let us consider what takes place during the time that the boat is being started from rest by a steady force of 50 pounds, the mass of the boat being 100 tons. At the very start, when the velocity of the boat is zero, the drag of the water is zero, and the propelling force of 50 pounds is used solely to produce acceleration; therefore, from the formula $F = \frac{1}{32} \cdot ma$, we find the acceleration a

to be 0.008 foot per second per second. After the boat has gained a certain amount of velocity, say, 3 feet per second, the drag of the water is $\frac{3}{5}$ of 50 pounds, so that 30 pounds of the propelling force is used to overcome the drag of the water and the remainder, 20 pounds, is used to produce acceleration. Therefore the acceleration is 0.0032 foot per second per second. At the very start when the velocity of the boat is zero no work is being done upon the boat. When the velocity of the boat becomes 3 feet per second, the propelling force does work at the rate of 150 foot-pounds per second; a portion of this power is expended in overcoming the friction of the water, and the remainder goes to increase the kinetic energy of the moving boat. The portion of power which is used to overcome the friction of the water is found by multiplying the velocity of the boat by the portion of the force which is used to overcome the frictional drag. This gives 90 foot-pounds per second. The portion of the power which goes to increase the kinetic energy of the boat is found by multiplying the portion of the propelling force which produces acceleration by the velocity of the boat. This gives 60 foot-pounds per second.

(*b*) An electric generator has an electromotive force of 50 volts and it acts upon a circuit of which the resistance is 10 ohms, so that the steady current that may be produced by the electromotive force is 5 amperes according to Ohm's law. The inductance of the circuit is, say, 2 henrys. At the instant when the electromotive force begins to act on the circuit the current is zero, and all of the electromotive force is used to cause the current to increase so that the rate of increase of the current is 25 amperes per second, according to equation (49). After the current has reached a value of, say, 3 amperes, a portion of the electromotive force of the generator is used to overcome the resistance of the circuit and a portion is used to cause the current to increase. The electromotive force which is used to overcome the resistance is found by multiplying the resistance of the circuit by the current which gives 30 volts, and the remaining 20 volts

cause the current to increase at a rate of 10 amperes per second in accordance with equation (49).*

The ordinates of the curve in Fig. 96 represent the successive

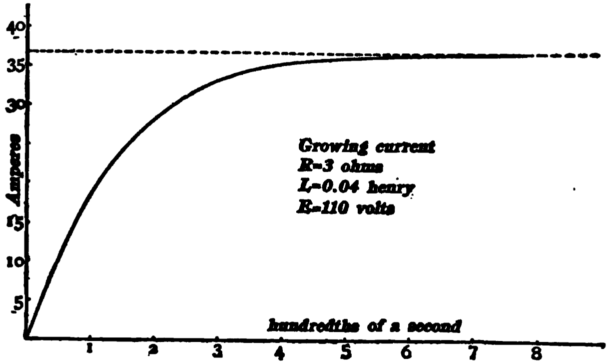

Fig. 96.

values of growing current in a circuit of which the resistance is 3 ohms and the inductance is 0.04 henry, the electromotive force of the generator being 110 volts. The equation to this curve of growing current is

$$i = \frac{E}{R} - \frac{E}{R} \cdot \epsilon^{-\frac{R}{L} \cdot t} \tag{52}$$

in which ϵ is the Naperian base, i is the value of the growing current t seconds after the electromotive force E is connected to the circuit, L is the inductance of the circuit and R is its resistance.

79. Decay of current in an inductive circuit. — A current I is established in an inductive circuit, a piece of metal mm is then laid across the terminals of the circuit and the battery disconnected as indicated by the dotted line in Fig. 97. Under these conditions

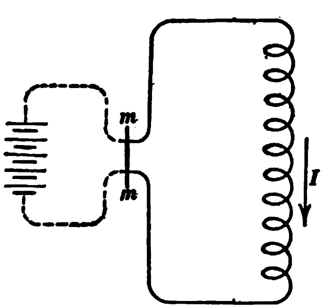

Fig. 97.

*Ohm's law does not apply to a circuit unless EI equals RI^2 as stated in Art. 19.

the current decreases at such a rate that the self-induced electromotive force (reaction of the changing current) $-L \cdot di/dt$ is equal to Ri. This condition is expressed by the equation

$$0 = Ri + L\frac{di}{dt} \tag{53}$$

and this equation may be most easily interpreted as follows: The electromotive force which is acting on the circuit is equal to zero and this electromotive force is divided into the two parts, Ri which is used to overcome the resistance and $L \cdot di/dt$* which is used to cause the current to change.

Examples. — (*a*) The canal boat mentioned in example (*a*) of the preceding article is brought up to a speed of 4 feet per second and then the propelling force ceases to act. The drag of the water is of course equal to 4/5 of 50 pounds when the velocity is 4 feet per second, and this dragging force of 40 pounds produces a negative acceleration or retardation of 0.0064 foot per second per second. The rate at which the kinetic energy of the boat is being dissipated in overcoming the frictional drag of the water may be found by multiplying the frictional drag of 40 pounds by the velocity of 4 feet per second which gives 160 foot-pounds per second.

(*b*) A current of 4 amperes is established in the circuit which is specified in example (*b*) of the preceding article. At a given instant the circuit is closed on itself and the current is left to die away. At this instant the value of Ri is 40 volts, that is, the electromotive force required to overcome the resistance of the circuit when the current is 4 amperes is 40 volts and this electromotive force comes from the reaction of the decreasing current, so that the current must be decreasing at a rate of 20 amperes per second according to equation (49). The rate at which the kinetic energy of the current is being dissipated in overcoming the resistance of the circuit may be found by multiplying the value of Ri

* This part must of course be negative, and therefore di/dt is negative, that is, the current i is decreasing.

in volts by the value i in amperes (equals Ri^2) which gives 160 watts.

The ordinates of the curve in Fig. 98 represent the successive values of a decaying current in a circuit of which the resistance

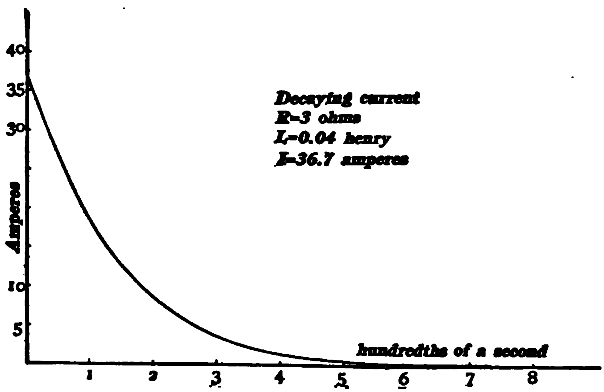

Fig. 98

is 3 ohms and the inductance is 0.04 henry, the initial value of the current being 36.7 amperes.

The equation of the curve of decaying current in Fig. 98 is

$$i = I \cdot \epsilon^{-\frac{R}{L} \cdot t} \tag{54}^*$$

in which ϵ is the Naperian base, I is the value of the decaying current at the instant from which time is reckoned, and i is the value of the decaying current t seconds later.

80. The choke coil. — A coil having considerable inductance is frequently used for the choking of rapid fluctuations of current. Such a coil is called a *choke coil*. When a choke coil is connected to the terminals of an alternator the rapidly alternating electromotive force of the alternator produces but little current through the coil. This is analogous to the fact that a rapidly alternating

* Equations (52) and (54) are obtained by integrating the differential equations (51) and (53) with due reference to the known value of the current at the instant $t = 0$.

force (a force which is repeatedly reversed in direction) produces but little to and fro motion of a heavy body even though the frictional opposition to motion be negligibly small.

One of the most important uses of the choke coil is in connection with the lightning arrester. Figure 99 represents a dynamo G supplying current to a trolley wire. When this wire is struck by lightning a sudden rush of current takes place through G to earth, and this rush of current may prove disastrous to the dynamo by breaking through the insulation instead of following the windings of wire in the machine. By placing a choke coil C in the position shown in the figure, the lightning discharge is

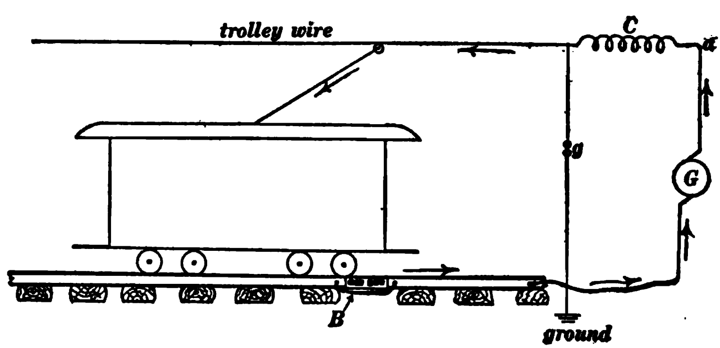

Fig. 99.

made to break through a short air gap g and flow to earth harmlessly. When the air gap g has been broken down in this way, that is, when a spark or arc has been established across the gap, it is a good conductor, and the dynamo G is short-circuited. Therefore a lightning arrester must be provided with an arrangement for stopping the flow of the dynamo current across the gap g after the rush of current from the lightning stroke has ceased. This is sometimes done, as in the Thomson arrester, by means of a strong magnet which produces an intense magnetic field in the region of the gap and pushes the arc sidewise, and blows it out. This device is called the *magnetic blow-out*. The entire arrangement of a choke coil C, air gap g, and magnetic blow-

out (which is not shown in the figure) constitute what is called a lightning arrester.*

Example. — A coil of heavy copper wire wound in a single layer on a wooden cylinder *AB*, as shown in Fig. 100, is provided with two metal rods *rr* which are separated by a small air gap. One terminal of the coil is connected to the outside coating of a Leyden jar, and a spark is allowed to jump from the jar to the

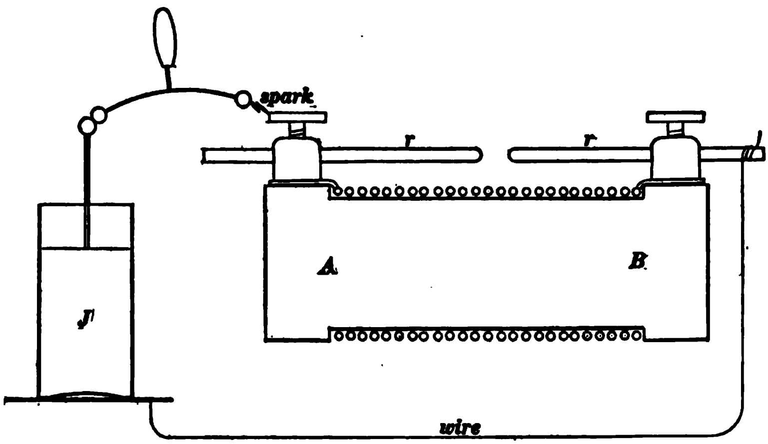

Fig. 100.

other terminal of the coil as shown in the figure. At the instant of formation of the spark the total electromotive force between the coatings of the Leyden jar begins to act on the circuit. If the coil consists of 100 turns of wire wound on a wooden cylinder 4 centimeters in diameter and 30 centimeters long, its approximate inductance is 0.00005 henry, so that, if the electromotive force between the coatings of the Leyden jar is 40,000 volts, the current begins to increase in the coil at the rate of 800,000,000 amperes per second, according to equation (49). The existence of a large electromotive force across the terminals of the coil may be shown by the fact that the discharge of the Leyden jar

* For further information concerning lightning arresters see Franklin and Esty, *Elements of Electrical Engineering*, Vol. I, pages 210–219.

will jump across the air gap instead of going through the coil *AB*. Thus, if the air gap is one centimeter in length it takes 20,000 volts to strike across it, and if a spark does strike across this gap at the instant of the discharge of the Leyden jar, one may be certain that the electromotive force between the terminals of the coil was at least 20,000 volts at the instant of the formation of the spark.

The protective action of the choke coil in Fig. 99 depends upon a rapid increase of current through the coil during an extremely short interval of time just before the gap g breaks down. The dynamo G may be protected from this very brief flow of current by connecting a condenser between the point a and earth, so that this very brief flow of current through the choke coil need not flow through the dynamo, but may go to charge the condenser.

81. Inductance of a long solenoid. — A solenoid is a long coil of wire; two or three layers of wire wound on a long wooden rod, for example. When the depth of the winding of wire is small in comparison with the radius of a solenoid and when the length of the solenoid is great in comparison with the radius, the inductance of the solenoid in abhenrys is given by the following equation

$$L = 4\pi^2 z^2 r^2 l \tag{55a}$$

in which L is the inductance of a solenoid in abhenrys, z is the number of turns of wire on each centimeter of length of the solenoid, r is the radius of the solenoid in centimeters, and l is the length of the solenoid in centimeters. The inductance of the solenoid in henrys is given by the equation

$$L = 4\pi^2 z^2 r^2 l \div 10^9 \tag{55b}$$

in which r and l are expressed in centimeters as in equation (55*a*).

Derivation of equation (55). — The intensity of the magnetic field inside of the solenoid is $H = 4\pi z I$, according to equation

(34), where I is the current in the solenoid in abamperes. Therefore the total energy of the magnetic field inside of the solenoid is equal to $\pi r^2 \times l \times (4\pi z I)^2/8\pi$, according to equation (27), that is, the energy of the magnetic field is given by the equation

$$W = 2\pi^2 z^2 r^2 l I^2$$

but the energy of the magnetic field is equal to $\frac{1}{2}LI^2$, according to equation (48) so that L is equal to $4\pi^2 z^2 r^2 l$.

Equations (55*a*) and (55*b*) are strictly true only for very long coils with thin windings of wire. These equations are frequently useful, however, in determining the approximate inductance of comparatively short solenoids with thick windings of wire.

82. Electric momentum. Flux-turns. — The product of the mass of a moving body and its velocity is called its *momentum*. The product of the inductance of a circuit and the current is called *electrical momentum*. The term electrical momentum is seldom employed, the term *flux-turns* being more usual.

Proposition. — The electrical momentum Li of a coil is equal to the product of the flux through a mean turn of the coil and the number of turns of wire in the coil, that is,

$$Li = Z\Phi \tag{56}$$

in which L is the inductance of the coil in abhenrys, i is the current in the coil in abamperes, Φ is the number of lines of magnetic flux passing through a mean turn of the coil, and Z is the number of turns of wire in the coil. The truth of this equation may be made evident as follows: The self-induced electromotive force in a coil is due to the increasing flux produced by the increasing current, so that the self-induced electromotive force is equal to $-Z \cdot d\Phi/dt$, according to equation (44), where Φ is the magnetic flux through a mean turn of the coil due to the current in the coil. The self-induced electromotive force is also equal to $-L \cdot di/dt$, according to equation (50). Therefore we have

$$L\frac{di}{dt} = Z\frac{d\Phi}{dt} \tag{i}$$

whence by integrating * we have $Li = Z\Phi$.

83. The dependence of the inductance of a coil on the number of turns of wire in the coil and upon the size of the coil. — The de-

* This simple integration occurs so frequently in arguments of this kind that it is worth while to consider its meaning as follows: In order to permit of a verbal expression of equation (i), divide both members by Z, giving $d\phi/dt = L/Z \times di/dt$, which means that the flux Φ increases always L/Z times as fast as i, so that, if Φ and i start from zero together, then Φ must always be L/Z times as large as i.

pendence of inductance upon the shape of a coil is much too complicated to permit of its general discussion in this text. The ratio of the inductances of two coils *of exactly the same shape* depends, however, in a very simple way upon the sizes of the coils and upon the relative number of turns of wire in them, as follows:

The inductance of a coil of wire wound on a given spool is proportional to the square of the number of turns of wire. — Thus, a given spool wound full of number 16 wire has 500 turns, and an inductance of 0.025 henry; the same spool wound full of number 28 wire has ten times as many turns, and its inductance is one hundred times as great, or 2.5 henrys.

The inductance of a coil of given shape, the number of turns of wire being unchanged, is proportional to its linear dimensions. — Thus, if a given spool of wire be imagined to be increased in dimensions in every detail in the ratio of 1:10, size of wire being increased in the same ratio so that the number of turns will be unchanged, then the inductance of the spool would be increased ten times.

84. Kinetic energy associated with independent currents in two circuits. Definition of mutual inductance. — Consider two adjacent circuits one of which may be called the *primary circuit* and the other the *secondary circuit* to distinguish them. Let I_1 be the current in the primary circuit and I_2 in the secondary circuit. The total kinetic energy associated with these two currents consists of three parts: (*a*) A part which is proportional to I_1 squared, (*b*) a part which is proportional to I_2 squared, and (*c*) a part which is proportional to $I_1 I_2$. Therefore we may write

$$W = \tfrac{1}{2}L_1 I_1^2 + \tfrac{1}{2}L_2 I_2^2 + M I_1 I_2 \qquad \text{(i)}$$

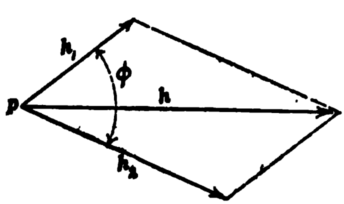

Fig. 101.

in which W is the total kinetic energy of the two currents, and $(\frac{1}{2}L_1)$, $(\frac{1}{2}L_2)$, and M are the proportionality factors. The quantities L_1 and L_2 are the inductances of the respective circuits inasmuch as equation (i) reduces to equation (48) when either current is zero. The quantity M is called the *mutual inductance* of the two circuits. It may be either positive or negative. Mutual inductance is expressed in terms of the same units as inductance.

Proof of equation (*i*). — Consider a point p in the neighborhood of the two circuits. Let h_1, Fig. 101, be the intensity at p of the magnetic field due to I_1 alone, and let h_2 be the intensity at p of the magnetic field due to I_2 alone. The resultant magnetic field at p is h, as shown in Fig. 101, and we have

$$h^2 = h_1^2 + h_2^2 + 2h_1h_2 \cos \phi$$

Consider the energy ΔW in a small element of volume at the point p. This energy is proportional to h^2 so that it may be considered in three parts which are proportional to h_1^2, to h_2^2, and to h_1h_2, respectively. But h_1 is proportional to I_1, and h_2 is proportional to I_2, so that the kinetic energy in an element of volume at the point p may be considered in three parts which are proportional respectively to I_1^2, to I_2^2, and to I_1I_2. What is true of the energy in an element of volume at the point p is true of the energy in every other element of volume, that is, the energy in every element of volume consists of three parts which are proportional to I_1^2, to I_2^2, and to I_1I_2, respectively, so that the total energy consists of three such parts.

Problems.

103. The current in a circuit has a value of 26 amperes at a given instant. Three hundredths of a second later the current is 10.3 amperes. What is the average rate of change of the current during the interval? Is this rate positive or negative? Ans. — 523.3 amperes per second.

104. Calculate the kinetic energy in joules of a current of 160 amperes in a circuit having an inductance of 0.05 henry. Ans. 640 joules.

105. An electromotive force of 25 volts is connected to a circuit of which the resistance is 0.6 ohm and the inductance is 0.05 henry. At what rate is the current increasing: (*a*) At the instant the electromotive force is connected to the circuit; (*b*) at the instant that the current reaches a value of 10 amperes, and (*c*) at the instant that the current reaches a value of 35 amperes? Ans. (*a*) 500 amperes per second, (*b*) 380 amperes per second, (*c*) 80 amperes per second.

106. The field winding of a dynamo has 50 ohms resistance and, approximately, 7.5 henrys of inductance. Assuming that the current grows in the coil in accordance with equation (52), calculate the time required for the current in the winding to reach 2 amperes when the winding is connected to a generator of which the electromotive force is 110 volts. Ans. 0.359 second.

107. A current has been left to die away in a circuit of 0.6 ohm resistance and 0.05 henry inductance. Find the rate of

change of the current as it passes the values 100 amperes, 10 amperes, and one ampere. Ans. —1,200 amperes per second, —120 amperes per second, —12 amperes per second.

108. Find the approximate inductance in henrys of a cylindrical coil 25 centimeters long, 5 centimeters mean diameter, wound with one layer of wire containing 150 turns. Ans. 0.000222 henry.

109. The choke coil of a lightning arrester consists of 50 turns of wire wound in one layer on a cylinder of which the diameter is 15 centimeters and the length is 50 centimeters. (*a*) Calculate the approximate inductance of this coil. (*b*) Calculate the approximate rate of increase of current in the coil at the instant that a lightning discharge jumps across two centimeters of air in preference to going through the coil. Ans. (*a*) 0.000111 henry. (*b*) 360,000,000 amperes per second.

Note. — The electromotive force required to strike across 2 centimeters of air is approximately 40,000 volts.

110. (*a*) Calculate the magnetic flux through the solenoid which is specified in problem 111 when a current of 30 amperes flows through it. (*b*) Calculate the value of the electrical momentum of this current in flux-turns. (*a*) 4,440 maxwells. (*b*) 666,000 flux-turns.

111. The field coil of a dynamo has 5,000 turns of wire and, when a current of one ampere flows through the field winding, 1,500,000 lines of force are produced through the field core. Assuming that the flux is proportional to the exciting current, find the inductance of the field coil in henrys. Ans. 75 henrys.

112. A battery having an electromotive force of 10 volts and a resistance of one ohm is connected to a coil of wire wound on an iron core. The coil has 1,000 turns of wire and its resistance is 4 ohms. What is the current in the coil when the magnetic flux in the core is increasing at a rate of 500,000 lines per second? Ans. 1 ampere.

113. A certain spool wound full of wire 0.1 centimeter in diameter has an inductance of 0.08 henry. The same spool is wound full of wire 0.32 centimeter in diameter. What is its inductance? Ans. 0.000762 henry.

114. A spool 5 times as large as the spool mentioned in problem 116 but similar in shape, is wound with wire 6 millimeters in diameter. What is its inductance? Ans. 0.193 henry.

CHAPTER VII.

ELECTRIC CHARGE. THE CONDENSER.

85. Electric charge. — A current of water through a pipe is a transfer of water along the pipe. Let q be the amount of water which, during t seconds, flows past a given point in the pipe, then the quotient q/t is the rate of flow of water through the pipe, and this rate of flow may be spoken of as the strength I of the water current. Suppose the strength I of the water current to be given (rate of flow of water in units of volume per second) then the amount of water flowing past a given point of the pipe in t seconds is given by the equation:

$$q = It$$

Similarly, an electric current in a wire may be looked upon as a transfer of electricity along the wire, and the quantity q of electricity which flows past a point on the wire during t seconds may be defined as the product of the strength of the current and the time, that is,

$$q = It \tag{57}$$

If the strength of the current is variable, then equation (57) must be written in the form

$$\Delta q = I \cdot \Delta t \tag{58}$$

in which Δq is the small quantity of electricity which flows past a given point on the wire during the short intervals of time Δt.

Quantity of electricity is usually spoken of as *electric charge* or simply as *charge*.

Quantity of water is the fundamental and easily measured thing in hydraulics and water current is most conveniently defined as quantity of water passing per second. In the case of electricity, the fundamental and easily measured thing is electric current, and

quantity of electricity is most conveniently defined as the product of electric current and time.

Units of electric charge. — The ampere-second is the amount of electricity which flows in one second through a wire which carries a current of one ampere. The ampere-second is usually called the *coulomb*. One ampere-hour is the quantity of electricity flowing in one hour through a wire carrying one ampere. The ampere-hour is extensively used among electrical engineers in specifying the discharge capacity of storage batteries. The *abcoulomb* is the quantity of electricity which flows in one second through a wire carrying a current of one abampere. One abcoulomb is equal to ten coulombs.

86. Measurement of electric charge. The ballistic galvanometer.* — A very large electric charge may be determined by observing the time during which the charge will maintain a sensibly constant measured current. Thus a given storage battery can maintain a current, say, of 10 amperes for 8 hours, so that the discharge capacity of the storage battery is equal to 80 ampere-hours. The charges most frequently encountered in practice, however, are too small to be measured in this way, and for such charges the ballistic galvanometer is used as follows:

The charge to be measured is sent through a galvanometer in the form of a pulse of electric current of very short duration. This pulse of current sets the needle of the galvanometer swinging. The maximum deflection d of the needle at the first swing is called the *throw* of the galvanometer, and this throw, if it is not too large, is proportional to the amount of charge q which is carried through the galvanometer by the pulse of current. That is,

$$q = kd \tag{59}$$

the quantity k is called the *reduction factor* of the galvanometer, and it is usually determined in practice by observing the throw produced by a known charge. Equation (59) is true for a galva-

* See Chapter X for a more complete discussion of the ballistic galvanometer and of the measurement of electric charge.

nometer with a heavy needle (or with a heavy moving coil in the case of the D'Arsonval type of instrument) which is not subject to any perceptible air friction as it vibrates. Such a galvanometer is called a *ballistic galvanometer.*

The reduction factor of a ballistic galvanometer may be calculated from the equation

$$k = \frac{It\sqrt{\lambda}}{\pi b} \tag{60}$$

in which I is a known steady current which produces a steady deflection b of the galvanometer, t is the period of one complete oscillation of the galvanometer and λ is the ratio of two successive throws of the freely swinging needle (or coil) of the galvanometer, swings, the so-called damping ratio of the galvanometer.*

87. The flow of current in unclosed circuits. Electrically charged bodies. — Consider two insulated metal bodies A and B, Fig. 102*a*, which at a given instant are connected, as shown, to the terminals of a battery or to any source of electromotive force. When the wire is connected a momentary pulse of current flows through it out of one body and into the other, and the bodies A and B are said to become *charged with electricity.*

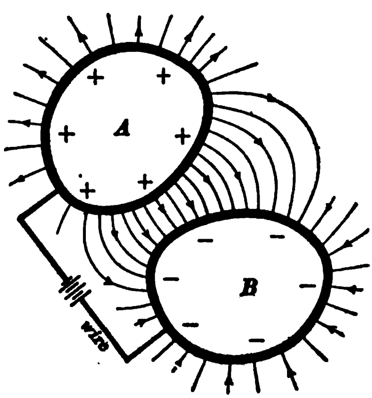

Fig. 102a.

The body into which the momentary current flows is said to become *positively charged* and the body out of which the momentary current flows is said to become *negatively charged*, that is, the charge on one body is $+q$ and the charge on the other body

* See *Electrical Measurements* by Carhart and Patterson, pages 207–213. See *Absolute Measurements in Electricity and Magnetism*, by Andrew Gray, Vol. II. pages 390–396. See Maxwell's *Electricity and Magnetism*, Vol. II, pages 374–391,

is $-q$. Electrically charged bodies always occur thus in pairs, the positive charge on one body being always associated with an equal negative charge on some other body or bodies.

Example. — Two large sheets of tin foil separated from each other by waxed paper are connected through an incandescent lamp to supply mains. If this arrangement is connected to direct-current supply mains a single pulse of current flows through the lamp at the moment of connection, and the lamp filament is not perceptibly heated. If the arrangement is connected to alternating-current supply mains a pulse of current flows through the wire at every reversal of the alternating electromotive force and the lamp filament may be heated to incandescence.

The electric field. The dielectric. — The region between the two bodies A and B, Fig. 102*a*, is understood to be filled with some electrical insulator such as air, oil or glass. An insulator between two charged bodies is called a *dielectric*. This dielectric is the seat of a peculiar stress which is called the *electric field*. The lines of force * of this electric field trend somewhat as shown in the figure, touching the surfaces of the metal bodies A and B at right angles. These lines of force are thought of as going out from the positively charged body and coming in towards the negatively charged body.

Electrostatic attraction. — The charged bodies A and B, Fig. 102*a*, attract each other. This attraction, which is called electrostatic attraction, shows that the lines of force of an electric field are in a state of tension and have a tendency to shorten very much as the lines of force in a magnetic field. This tension of the lines of force pulls outwards on the surface of A and on the surface of B at each point.

The electrostatic attraction of two metal bodies which are connected to a battery or dynamo may be shown as follows: A gold leaf is hung alongside of a vertical brass strip. When the gold

* The electric field is similar in many respects to the magnetic field, having a definite intensity and a definite direction at each point.

leaf is connected to one terminal of a dynamo and the brass strip to the other terminal, the gold leaf is attracted by the brass. It is necessary in this arrangement to cover the face of the brass strip with a layer of paper to avoid short-circuiting the dynamo through the gold leaf.

The outward pull of the electric field on the surface of a charged body is very strikingly shown by pouring a viscous liquid over the sharp lip of a charged metal ladle. The liquid is pulled into fine jets by the lines of force which emanate from the surface of the liquid as it passes over the lip. When melted rosin is used in this way the jets congeal into very fine fibers which float about in the air.

Need of high electromotive force and of good insulation. — The phenomena described above, in fact most of the phenomena of electrostatics, are easily perceptible only when the bodies are charged by electromotive forces of many thousands of volts. The most convenient method of producing these large electromotive forces is by means of the Holtz or Wimshurst electrical machine, and, when such a machine is used, the bodies A and B must be well insulated, because such electrical machines cannot supply charge at a rapid rate, that is, such machines can deliver only very small currents. (See Arts. 106 and 108.)

The electric spark. — When the electromotive force acting to charge two bodies A and B, Fig. 102*a*, is increased more and more, a value is eventually reached which *breaks down* or *ruptures* the dielectric and allows the charge on the bodies to pass in the form of an electric spark.

Mechanical analogue of electrically-charged bodies and of the electric field. — Imagine two cavities A and B, Fig. 102*b*, in an extended elastic solid such as rubber or jelly. Imagine these cavities to be filled with water and to be connected to a pump by means of a pipe so that the pump may draw a certain amount of water out of one cavity and force it into the other, thus causing one cavity to contract and the other cavity to expand, and causing the surrounding mass of rubber or jelly to be strained, the lines

of stress or strain being somewhat as shown in the figure. The expanded cavity is analogous to a positively charged body, the contracted cavity is analogous to a negatively charged body, the stressed condition of the rubber or jelly is analogous to the electric field between two charged bodies and the pressure-difference of the pump is analogous to the electromotive force of the battery in Fig. 102*a*.

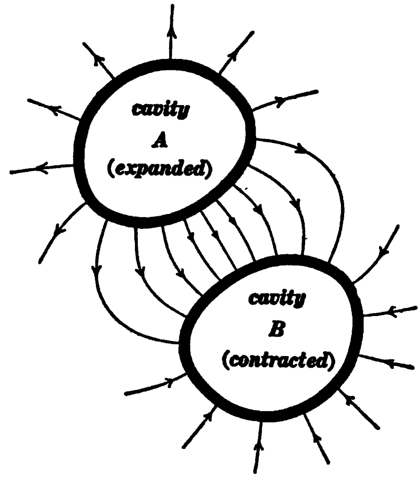

Fig. 102b.

88. Electrostatic capacity. The condenser.—The amount of charge q which flows out of B and into A, Fig. 102*a*, when the battery is connected is proportional to the electromotive force of the battery. Therefore we may write

$$q = CE \qquad (61)$$

in which q is the charge that is drawn out of B and forced into A, in Fig. 102*a*, by a battery of which the electromotive force is E, and C is a constant depending upon the size and shape of A and B and upon the nature of the intervening dielectric. This quantity C is called the *electrostatic capacity* or simply the *capacity* of the pair of bodies A and B. If the bodies A and B are in the form of flat plates of metal separated by a thin layer of dielectric their electrostatic capacity is large. Such an arrangement is called a *condenser*. Condensers are usually made of sheets of tin foil separated by sheets of waxed paper or mica. The *Leyden jar* is a condenser made by coating the inside and outside of a glass jar with tin foil.

Measurement of capacity.—The simplest method of measuring the capacity of a condenser is to charge the condenser by a battery of known electromotive force E and then measure the

charge $q(=CE)$ by discharging the condenser through a ballistic galvanometer. See Chapter X.

Units of capacity.—A condenser is said to have a capacity of one *farad* when one coulomb of charge is drawn out of one plate and forced into the other plate by an electromotive force of one volt; C in equation (61) is expressed in farads when q is expressed in coulombs and E in volts. The farad is an extremely large capacity as compared with capacities ordinarily met with in practice, and the *microfarad* (one millionth of a farad) is frequently used as a unit. The term *abfarad* is occasionally used to designate the c.g.s. unit of capacity. A condenser would have one abfarad of capacity if one abcoulomb of charge would be drawn out of one plate and forced into the other plate by an electromotive force of one abvolt. One abfarad is equal to 10^9 farads.

Electric absorption.—When a condenser, which has been charged for some time, is discharged and then left standing, a small amount of additional charge collects on the condenser plates so that a second or third discharge may be taken from the condenser. It seems as if a portion of the initial charge on the condenser were absorbed by the dielectric, this absorbed charge being slowly given back to the condenser plates when these have been discharged. This phenomenon of electric absorption is strictly analogous to the following: A rubber tube which is stretched for some time and then released, comes nearly back to its initial length at once, and then continues to shorten for a long time. If the end of the tube is fixed immediately after the release, the tendency of the tube to continue to shorten will develop a stretched condition in the tube which will show itself by a sudden slight shortening when the tube is released a second time.

89. Mechanical analogue of the condenser.—Figure 103 shows two metal plates AA and BB separated by a dielectric DD and connected to a battery. Figure 104 shows a box separated into two compartments AA and BB by means of a rubber

diaphragm DD and the two compartments are connected to a pump P. The electromotive force of the battery in Fig. 103 forces a certain amount of charge q into the plate AA, draws the same amount of charge out of the plate BB, and subjects the dielectric DD to an electrical stress. The pressure-difference developed by the pump P in Fig. 104 forces a certain

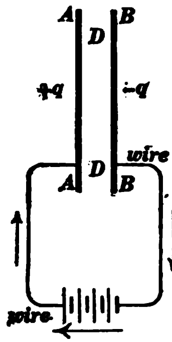

Fig. 103.

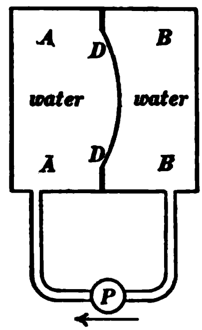

Fig. 104.

amount of water q into the compartment AA, draws the same amount of water out of the compartment BB, and subjects the rubber diaphragm DD to a mechanical stress. If the pump P in Fig. 104 is removed and the two compartments connected by a pipe, the mechanical stress of the diaphragm DD will be relieved by a momentary flow of water from A to B through the pipe. If the battery in Fig. 103 is removed and the two plates connected by a wire, the electrical stress of the dielectric DD will be relieved by a momentary flow of electric current through the wire. When the pump P in Fig. 104 is connected, as shown, it causes water to flow out of B into A until the pressure-difference developed by the pump is balanced by the *elastic reaction* of the diaphragm DD, and the amount of water drawn out of B and forced into A is proportional to the pressure-difference developed by the pump. When the battery B is connected to the plates in Fig. 103, it causes an electric current to flow out of B and into A until the electromotive

force of the battery is balanced by what one may perhaps call the *electro-elastic reaction* of the dielectric DD, and the amount of charge drawn out of B and forced into A is proportional to the electromotive force of the battery.

The extent to which the diaphragm DD, Fig. 104, yields is measured by the amount of water q which is drawn out of B and forced into A, and *the yield per unit of pressure-difference* is a sort of coefficient of elasticity of the diaphragm. The extent to which the dielectric DD, Fig. 103, yields is measured by the amount of charge q which is drawn out of the plate B and forced into plate A, and *the amount of yield per unit of electromotive force of the battery* $(q/E = C)$ is a sort of coefficient of electro-elasticity of the layer of dielectric and it is called the *capacity* of the condenser. It is important to remember that the capacity of a condenser is not analogous to the cubic capacity of a vessel but that it is analogous to the cubic capacity of a rubber bag, the amount of water that a rubber bag will hold depends upon the pressure.

90. Inductivity* of a dielectric.—If the diaphragm DD in Fig. 104 were made of a stiff material like steel, the amount of yield per unit of pressure-difference would be very much less than if the diaphragm were made of a substance like rubber. This is analogous to the fact that the capacity of the condenser AB, Fig. 103, with plates of a given size at a given distance apart depends upon the nature of the dielectric between the plates. The quotient: *Capacity of a condenser with given dielectric, divided by the capacity of the same condenser with air between its plates* is called the *inductivity* of the dielectric. For example, the inductivity of petroleum is about 2.04, that is, the capacity of a given condenser is about 2.04 times as great when the dielectric is petroleum as it is when the dielectric is air. A condenser is called an *air condenser*, a *mica condenser*, a *paraffin condenser*, etc., according to the dielectric between its plates. The accompanying table gives the inductivities of a few dielectrics.

* Sometimes called specific inductive capacity.

TABLE.

Inductivities of various substances.

Glass	3–10	Shellac	2.95–3.60
Sulphur	2.24–3.84	Mica	4–8
Vulcanite	2.50	Quartz	4.5
Paraffin	1.68–2.30	Turpentine	2.15–2.43
Rosin	1.77	Petroleum	2.04–2.42
Wax	1.86	Water	73–90

The inductivity of a dielectric is determined by measuring the capacity of a condenser first with air between its plates and then with the given dielectric between its plates. See Chapter X.

91. Dependence of the capacity of a condenser upon size and distance apart of its plates. — Using a ballistic galvanometer as explained in Art. 88, it may be shown experimentally * that the capacity C of a condenser, an air condenser, for example, is proportional † to the area a of one of its plates, and inversely proportional to the distance x between its plates; that is, C is proportional to a/x, so that we may write

$$C = \frac{1}{B} \cdot \frac{a}{x} \tag{62}$$

in which C is the capacity of the air condenser, a is the area of one plate (sectional area of the dielectric), x is the distance between the plates, and $1/B$ is a constant.

When C is expressed in farads, a in square centimeters, and x in centimeters, then the value of $1/B$, as determined by experiment is 884×10^{-16}, so that equation (62) becomes

$$C_{\text{farads}} = 884 \times 10^{-16} \times \frac{ka}{x} \tag{63}$$

* Indeed it may be shown from geometrical considerations that C must be proportional to a/x; the value of the proportionality factor must however be determined by observation. It is possible to calculate the value of $1/B$ from the observed velocity of light as explained in Art. 146.

† When a is large compared with x, the non-uniformity of the electric field near the edges of two parallel oppositely charged plates is negligible, and it is ignored throughout this discussion.

in which k is the inductivity of the dielectric, x is the thickness of the dielectric in centimeters, and a is the area in square centimeters of one plate of the condenser (sectional area of the dielectric).

92. Work done by an electromotive force in pushing a given amount of charge through a circuit. — Consider an electromotive force E maintaining a current I in a circuit. The rate at which this electromotive force does work is equal to EI, which, multiplied by a time t, gives the work done during that time, so that $W = EIt$. But the product It is equal to the charge q which is transferred during the time t, therefore we have

$$W = Eq \tag{64}$$

in which W is the work done by an electromotive force E during the time that charge q is pushed through the circuit. The work W is expressed in joules when E is expressed in volts and q in coulombs.

93. The potential energy of a charged condenser. — A charged condenser represents a store of potential energy in much the same way that the distorted diaphragm DD in Fig. 104 represents a store of potential energy, or in the same way that a bent spring represents a store of potential energy. When a spring is bent, the bending force is at first equal to zero, it increases in proportion to the amount of bending, and the average value of the bending force is equal to one half its ultimate value (that is, the value which corresponds to a given amount of bend). Let E be the ultimate value of the bending force and q the distance through which the end of the spring is moved, then $\frac{1}{2}E$ is the average value of the bending force, which, multiplied by q, gives the work done in bending the spring or the potential energy of the bent spring. Therefore, the potential energy of the bent spring is given by the equation

$$W = \tfrac{1}{2}Eq$$

in which E is the ultimate value of the bending force, and q is

the distance through which the end of the spring is moved during the bending.

In a similar manner, it may be shown that the potential energy of a charged condenser is

$$W = \tfrac{1}{2}Eq \qquad (65a)$$

in which E is the electromotive force between the plates of the charged condenser, and q is the amount of charge which has been drawn out of one plate and pushed into the other. The potential energy W is expressed in joules when E is expressed in volts and q in coulombs.

A weight is hooked to the lower end of a vertical spring, as shown in Fig. 105, and then the weight is released. In this case the full value of the weight acts upon the spring from the start, and the weight oscillates up and down for some time before coming to rest in its equilibrium position. Let E be the pull of the earth upon the weight and let q be the distance from the initial to the equilibrium position, then the total work done by gravity after the weight has come to rest is equal to Eq (pull of earth multiplied by distance through which the weight has moved), but the potential energy which is stored in the spring after the weight comes to rest in its equilibrium position is equal to $\tfrac{1}{2}Eq$ that is, one half of the work which has been done on the weight by gravity is stored in the spring as potential energy and the remainder of the work has been dissipated by the oscillations of the weight.

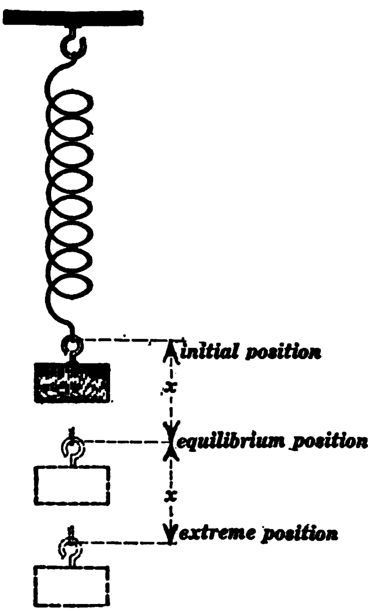

Fig. 105.

When a battery is connected to the terminals of a condenser the full electromotive force of the battery begins to act at once, and the current surges back and forth through the circuit until the system finally settles to equilibrium. When this final state of equilibrium is reached, a definite amount of charge q will have been pushed into the condenser by the battery, and the total amount of work done by the battery will be Eq; but the amount of potential energy stored in the condenser is $\frac{1}{2}Eq$, and therefore an amount of work $\frac{1}{2}Eq$ has been dissipated by the electrical oscillations of the system, exactly as in the case of the spring and weight above described.

In order that all the work done in stretching a spring may be stored in the spring as potential energy, the stretching force must begin at zero and increase gradually as the spring is bent more and more; in order that all the work done in charging a condenser may be stored in the condenser as potential energy, the charging electromotive force must begin at zero and increase gradually as the condenser becomes charged. If the final value of the charging electromotive force is E its average value is $\frac{1}{2}E$, which, multiplied by the amount of charge q that has been pushed into the condenser, gives the potential energy of the condenser.

The potential energy of a charged condenser may be expressed in terms of E and q, or in terms of C and E, or in terms of C and q by using equation (61). Thus, by substituting CE for q, equation (65*a*) becomes

$$W = \tfrac{1}{2}CE^2 \tag{65b}$$

and by substituting q/C for E, equation (65*a*) becomes

$$W = \frac{1}{2} \cdot \frac{q^2}{C} \tag{65c}$$

94. Transference of charge by a moving ball. Intensity of electric field.—Two metal plates A and B, Fig. 106, are connected to a battery of which the electromotive force is E, and a

very small metal ball b is suspended between A and B by a silk thread. If this ball is started it continues to vibrrate back and forth from plate to plate, and at each movement it carries across a definite amount of charge q. Every time the ball carries charge q across from plate to plate an amount of charge q flows through the battery, the battery does an amount of work equal to Eq and this work reappears as mechanical work done on the ball as it is pushed across from plate to plate by the electric attraction or repulsion. Let F be the force which pushes on the ball, then Fx is the work done by this force in pushing the ball across from plate to plate,* so that $Fx = Eq$, or

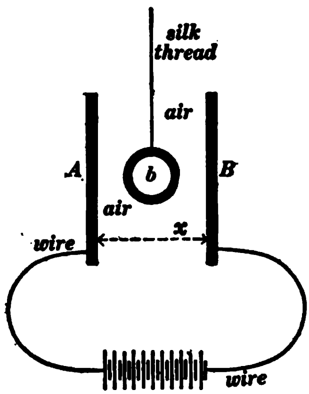

Fig. 106.

$$F = \frac{E}{x} \cdot q \qquad \text{(i)}$$

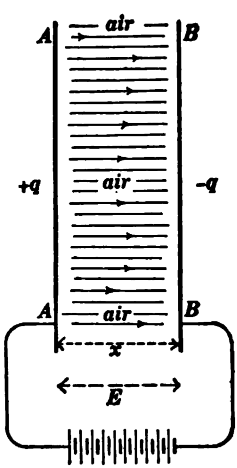

Fig. 107.

Any region in which a charged body is acted upon by a force † is called an *electric field;* thus the region between the plates A and B, Fig. 106, is an electric field, as indicated by the fine lines of force in Fig. 107.

The force F with which an electric field pulls on a charged body (of small size) placed at a given point in the field is proportional to the charge q on the body so that

* The ball is supposed to be quite small so that the distance moved by it may be taken to be equal to the distance x between the plates. Under these conditions the force which acts upon the ball is constant throughout its movement from plate to plate.

† That is, a force which depends upon the charge on the body and which does not exist when the body has no charge.

$$F = fq \tag{66}$$

in which f is the proportionality factor. This quantity f is called the *intensity* of the electric field at the point.

Comparing equations (i) and (66), it is evident that the intensity of the electric field between the parallel plates AB in Fig. 106 is

$$f = \frac{E}{x} \tag{67}$$

That is, the intensity of the electric field between the plates is equal to the electromotive force between the plates divided by the distance between the plates. When E is expressed in volts and x in centimeters, the field intensity is expressed in volts per centimeter. The intensity of an electric field may also be expressed in abvolts per centimeter.

Direction of electric field at a point.—The direction of an electric field at a point is the direction in which the field pulls on a positively charged body placed at that point. A line of force in an electric field is a line drawn so as to be in the direction of the field at each point.

95. Dielectric strength.—The ability of a dielectric to withstand electrical stress or electric field is called the *strength* of the dielectric. The strength of a dielectric is measured by the intensity of the electric field in volts per centimeter which is just suffi-

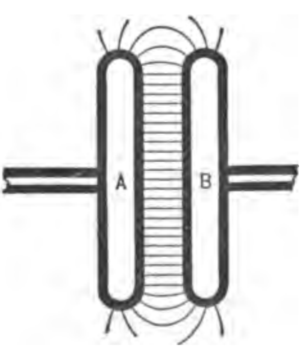

Fig. 108.

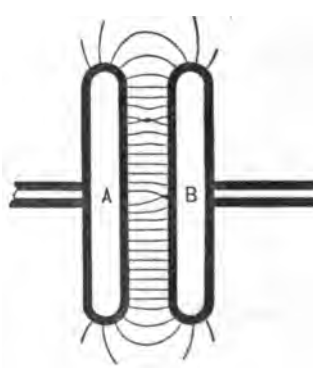

Fig. 109.

cient to rupture it. Thus, air at ordinary atmospheric pressure is ruptured by an electric field of which the intensity is about 24,000 volts per centimeter, and kerosene is ruptured by an electric field of which the intensity is about 50,000 volts per centimeter. The dielectric strength of a substance varies greatly with its degree of purity.

When an insulating substance is placed between two flat metal plates *A* and *B*, as shown in Fig. 108, the substance is subjected to a uniform electrical stress (uniform electric field) when the plates are connected to an electrical machine, and the electromotive force required to rupture the substance is quite accurately* proportional to the thickness of the insulating layer, provided the insulating substance is homogeneous like air or oil; or, in other words, a fairly definite intensity of electric field (volts per centimeter) is required to rupture a homogeneous substance like air or oil, and such a substance has therefore a fairly definite dielectric strength. Most solid substances, however, are non-homogeneous. Thus, the rubber gum which is extensively used for insulating wires is "filled" with finely divided clay and is therefore non-homogeneous. Sheets of window glass are usually filled with fine bubbles and are therefore non-homogeneous. Thick sheets of vulcanized fiber are usually charged with moisture in the interior and dry near the surface, and they are, therefore, non-homogeneous. The electromotive force required to rupture a non-homogeneous substance is not even approximately proportional to the thickness of the layer, and it is therefore customary to specify the dielectric strength of solid insulating substances by giving the electromotive force required to rupture a specified thickness.

The least roughness of the surface of the metal plates *A* and *B*, in Fig. 108, or particles of dust floating in the dielectric, produce great variations in the value of the electromotive force required to rupture a dielectric. The action of these irregularities of surface and of floating particles is shown somewhat exag-

* See Art. 127.

gerated in Fig. 109. In this figure a floating particle and a minute projecting point on the plate B are represented. The intensity of the electric field near the point and near the ends of the particle is much greater than the average intensity E/x between the plates, and the dielectric begins to give way at these places when the field intensity *there* reaches the breaking value.

TABLE.*

Dielectric strengths.

Substance.	Strength in Volts per Centimeter.	Substance.	Strength in Volts per Centimeter.
Oil of turpentine	94,000	Beeswaxed paper	540,000
Paraffine oil	87,000	Air (thickness 5 cm.)	23,800
Olive oil	82,000	CO_2 "	22,700
Paraffine (melted)	56,000	O "	22,200
Kerosene oil	50,000	H "	15,100
Paraffine (solid)	130,000	Coal gas "	22,300
Paraffined paper	360,000		

Lines of force of the electric field between two oppositely charged metal spheres are shown in Fig. 110. In this case the electric field is not uniform, and the intensity of the electric field in volts per centimeter near the surface of one of the spheres may be sufficient to start a rupture, although the intensity of the field at a distance from the surface of one of the spheres may be much less than that which corresponds to the rupture of the dielectric. Furthermore, the electric field intensity in Fig. 110 is not, of course, equal to the electromotive force between the spheres divided by their distance apart, because the field is non-uniform. Therefore the electromotive force required to rupture a dielectric between two metal spheres is not proportional to their distance apart.

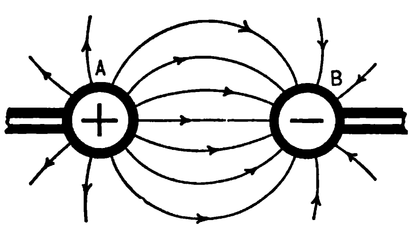

Fig. 110.

*From the measurements of Macfarlane and Pierce, *Physical Review*, Vol. I, page 165.

96. The spark gauge. — The electromotive force required to produce a spark between two polished metal spheres of given size in air varies in a definite manner with the length of the air gap. If the electromotive forces required for different distances be once determined by observation, then any electromotive force may be determined by measuring its sparking distance between the pair of spheres. An arrangement for measuring electromotive force in this way is called a *spark gauge* or a *spark micrometer*. The spark gauge is adapted only to high electromotive forces, and the results obtained by it are subject to large errors.

TABLE.

Sparking distances in air at 18° C. and 745 mm. pressure.*

s = length of air gap in centimeters.
r = radius of spheres in centimeters.

s	$r = 0.25$	$r = 0.5$	$r = 1.0$	$r = 2.5$ cm.
cm.	Volts.	Volts.	Volts.	Volts.
0.1	4,830	4,800	4,710	
0.2	8,370	8,370	8,100	
0.3	11,370	11,370	11,370	
0.4	13,800	14,400	14,400	
0.5	15,600	17,400	17,400	18,300
0.6	17,100	19,800	20,400	21,600
0.7	18,300	21,900	23,100	24,600
0.8	18,900	24,000	26,100	27,300
0.9	19,500	25,500	28,800	30,000
1.0	20,100	27,000	31,200	32,700
1.1	20,700		33,300	35,700
1.2	21,000		35,400	38,400
1.3	21,600		37,200	41,100
1.4	21,900		38,700	43,800
1.5	22,200		40,200	46,200
1.6			41,400	48,600

97. Electric flux. — The product of the intensity of an electric field and an area at right angles to the direction of the field is called the *electric flux* across the area, that is, we may write

$$\Phi = fa \tag{68}$$

in which Φ is the electric flux across a square centimeters of

* Heydweiller, *Wied. Ann.* 48, p. 235, 1893.

area at right angles to an electric field of intensity f. When f is expressed in volts per centimeter and a in square centimeters, the flux Φ is expressed in terms of a unit which may be called the volt-centimeter. Electric flux is in many respects similar to magnetic flux, but the two must not be confused, although the same letter Φ is here used for both.

98 Amount of electric flux which emanates from an electric charge. — It was shown in Art. 39 that $4\pi m$ units of magnetic flux emanate from a magnet pole of which the strength is m, that is, the strength of a magnet pole may be expressed in terms of the magnetic flux which emanates from it. There is also a simple proportional relationship between the amount of charge on a body and the amount of electric flux which emanates from the body, and the amount of charge on a body may be expressed in terms of the electric flux which emanates from it. The relationship between electric charge and flux will be established for the simplest case, namely, the case in which a charge is spread uniformly over a flat surface, as on the flat metal plate of a condenser.

Consider the two parallel metal plates AA and BB, Fig. 107, the area of each plate being a square centimeters, their distance apart being x centimeters, and the electromotive force between them being E volts. The lines of force of the electric field in the region between the plates are indicated by the fine arrows, and the intensity of this field is equal to E/x, according to equation (67), so that the total electric flux Φ emanating from the plate AA is $a \times E/x$. The capacity of the condenser AA BB in farads is $884 \times 10^{-16} \times a/x$, according to equation (63), so that the charge on one plate is equal to $884 \times 10^{-16} \times aE/x$ ($q = CE$) which is equal to $884 \times 10^{-16} \times \Phi$, because aE/x is equal to Φ; therefore

$$q = 884 \times 10^{-16} \times \Phi$$

or

$$\Phi = Bq \tag{69}$$

in which

$$B = 1\ 131 \times 10^{13} \tag{70}$$

(see Art. 91), that is, B lines of electric flux emanate from one coulomb of positive charge or converge towards one coulomb of negative charge. *It is a great help towards a clear understanding of electrostatics to think of electric charge, positive or negative, as the beginning or ending of lines of electric force.*

99. Electric field due to a concentrated charge. — Consider a concentrated positive charge q from which lines of electric force emanate in all directions, as shown in Fig. 111, and let it be re-

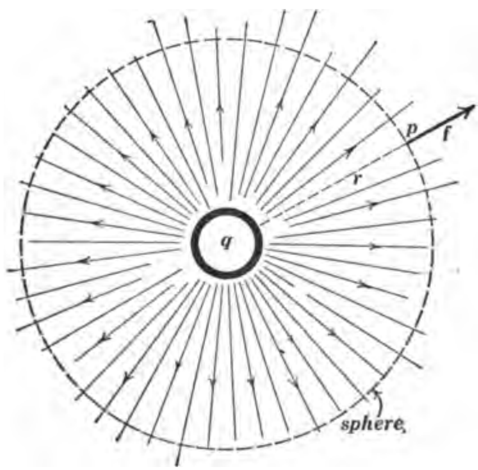

Fig. 111.

quired to find the intensity f of the electric field at a point p distant r centimeters from the center of q. Describe a sphere of radius r with its center at the center of q. The area of the surface of this sphere is $4\pi r^2$, and the electric field f is everywhere at right angles to this surface and everywhere the same in value at the surface. Therefore the electric flux across the surface of the sphere is equal to $4\pi r^2 \times f$, and this must be equal to Bq according to equation (69), where B has the value given in equation (70). Therefore we have

$$Bq = 4\pi r^2 f$$

or

$$f = \frac{B}{4\pi} \cdot \frac{q}{r^2} \tag{71}$$

in which f is the electric field intensity in volts per centimeter at a point r centimeters from a concentrated charge of q coulombs.

Equation (71) applies not only to the ideal case of a concentrated charge, but also to the case in which the charge is uniformly distributed over a sphere.

100. Electrostatic attraction and repulsion of concentrated charges. — Consider a concentrated charge q' at a point distant r centimeters from another concentrated charge q''. The electric field intensity at q' due to q'' is given by equation (71), namely,

$$f = \frac{B}{4\pi} \cdot \frac{q''}{r^2}$$

and according to equation (66), the force with which this field acts upon the charge q' is equal to the product $q'f$, that is,

$$F = \frac{B}{4\pi} \cdot \frac{q'q''}{r^2} \tag{72}$$

in which F is the form in *joule-units* * with which the two concentrated charges q' and q'' repel each other when they are at a distance of r centimeters apart, q' and q'' being expressed in coulombs. When q' and q'' are both positive or both negative, the force F is a repulsion; when one is positive and the other is negative, the force F is an attraction.

The unit of charge of the "electrostatic system." — The c.g.s. units of magnetic pole strength, magnetic field intensity, electric current strength, electromotive force, electric charge, electrostatic capacity, etc., which have been used in the foregoing chapters constitute what is called the *electromagnetic system of units.* This "electromagnetic system" starts out with the definition of a unit magnetic pole as a pole of such strength that it will exert a force of one dyne upon an equal pole at a distance of one centimeter. The so-called *electrostatic system of units* starts out with the definition of the unit charge as a charge which will exert a force of one dyne upon an equal

* A *joule-unit* of force is a force which will do one joule of work in pulling a body through a distance of one centimeter; it is equal to 10^7 dynes.

charge at a distance of one centimeter, so that, if the two charges q' and q'' are expressed in "electrostatic" units, and F in dynes, equation (72) becomes

$$F = \frac{q'q''}{r^2} \tag{i}$$

It is convenient to call the electrostatic system of units the *Faraday units* in order to distinguish them from the c.g.s. units which are employed throughout this text. Thus, the Faraday unit of electric charge is a charge which will exert a force of one dyne upon an equal charge at a distance of one centimeter in air; the Faraday unit of electric field intensity is an electric field of such strength that it will exert a force of one dyne upon a body which carries one Faraday unit of charge; the Faraday unit of electric current is the flow of one Faraday unit of charge per second through a wire; the Faraday unit of magnetic field intensity is a field which will push sidewise with a force of one dyne upon each centimeter of a wire carrying one Faraday unit of electric current; the Faraday unit of magnetic pole is a pole of such strength that it will be acted upon by a force of one dyne in a magnetic field of one Faraday unit intensity; and so on. The electrostatic system of units (Faraday units) are extensively used in advanced treatises on Electricity and Magnetism. In this text, however, the electromagnetic system of units will be used, that is to say, either the c.g.s. units, such as the abampere, the abohm, the abvolt, the abcoulomb, the abfarad, etc., or the so-called practical units such as the ampere, the ohm, the volt, the coulomb, the farad, etc.

Number of Faraday units of charge in one abcoulomb. — A given pair of charges attract (or repel) each other with a definite force at a given distance apart according to equation (72), in which q and q' are expressed in coulombs and F is expressed in joule-units of force. If F is expressed in dynes, the number which expresses it must be ten million times as large, so that the right-hand member of equation (72) must be multiplied by 10^7 to give F in dynes. The force with which two concentrated charges, each equal to one abcoulomb (10 coulombs), would repel each other at a distance of one centimeter apart is $100B/4\pi$ joule-units of force, or $10^9B/4\pi$ dynes, according to equation (72); by substituting this value of force in equation (i), above, placing $q' = q''$, and solving for q' we find the number of Faraday units of charge in one abcoulomb, namely, 3×10^{10}. This result is equal to the velocity of light in air in centimeters per second. See Art. 146.

101. Electrostatic attraction of parallel plates. — Consider two parallel metal plates connected to a battery as shown in Fig. 107. Let a be the area of each plate, x their distance apart, and E the electromotive force between them. The two plates constitute a condenser of which the capacity is

$$C = \frac{1}{B} \cdot \frac{ka}{x} \tag{i}$$

according to equation (63), where k is the inductivity of the

dielectric between the plates. The energy of the charged condenser is

$$W = \frac{B}{2} \cdot \frac{q^2}{ka} \cdot x \tag{ii}$$

according to equation (65*c*).

Let the battery be disconnected and the plates A and B insulated so that q cannot change, and imagine the plates to be pulled apart so as to increase their distance by the amount Δx (the dielectric being a fluid like air or oil). Then the energy of the condenser will be increased by the amount

$$\Delta W = \frac{B}{2} \cdot \frac{q^2}{ka} \cdot \Delta x \tag{iii}$$

This expression for the increase of energy of the charged condenser is easily derived from equation (ii) by assuming x to increase slightly. *This increase of energy of the charged condenser comes from the work done in separating the plates against their mutual attraction.* Let F be the force of attraction of the plates, then $F \cdot \Delta x$ is the work done against their mutual attraction, and this is equal to ΔW in equation (iii) so that

$$F \cdot \Delta x = \frac{B}{2} \cdot \frac{q^2}{ka} \cdot \Delta x$$

or

$$F = \frac{B}{2} \cdot \frac{q^2}{ka} \tag{73}$$

in which F is the force in joule-units with which two parallel metal plates attract each other, a is the area of each plate in square centimeters, q is the charge on each plate (positive on one, negative on the other) in coulombs, and B is equal to 1.131×10^{13}.

It is noteworthy that the force of attraction is independent of the distance between the plates and inversely proportional to the inductivity of the dielectric, *the charge being given.* The plates are supposed to be very large in comparison with the distance between them, according to Art. 90.

Attraction for a given electromotive force.—The charge q in the above discussion is equal to the capacity of the condenser times the electromotive force between the plates, according to equation (61), that is,

$$q = \frac{1}{B} \cdot \frac{kaE}{x}$$

according to equations (61) and (62). Substituting this value of q in equation (73), we have

$$F = \frac{1}{2B} \cdot \frac{kaE^2}{x^2} \tag{74}$$

in which F is the force in joule-units with which two metal plates attract each other in air or oil, E is the electromotive force between the plates, a is the area of each plate, x is the distance between the plates in centimeters, and B is equal to 1.131×10^{13}.

It is worthy of note that the force of attraction of parallel plates is inversely proportional to the square of the distance between them and directly proportional to the inductivity of the dielectric *for given electromotive force.*

102. The absolute electrometer is an arrangement for determining the value of an electromotive force by measuring the force of attraction of parallel metal plates. The value of the electromotive force is calculated from equation (74) when k ($k = 1$, for air), a, x, and B are known, and F is observed in joule-units. Figure 112 shows the essential features of the absolute electrometer.

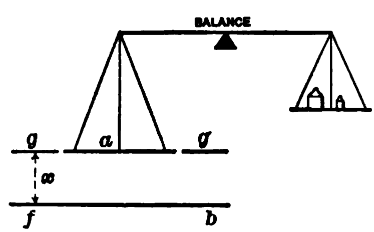

Fig. 112.

A portion of area a of the upper plate is hung from one end of a balance beam so that the force with which this portion is attracted by the lower plate may be counterpoised by weights placed upon the scale pan and thus determined. The stationary portion gg of the upper plate completely surrounds the portion a and is called the *guard ring.* Equation (74) is

true only for plates which are very large in comparison with their distance apart, and this guard ring makes it possible to realize this condition approximately without making the moving part a of the upper plate inconveniently large.

The value of the constant B, which appears in equation (62), Art. 90, which also appears in the equations for electrostatic attraction, and which is equal to the amount of electric flux which emanates from one coulomb according to equation (69), may be most easily determined by means of the absolute electrometer. A known electromotive force E is connected to the plates of an

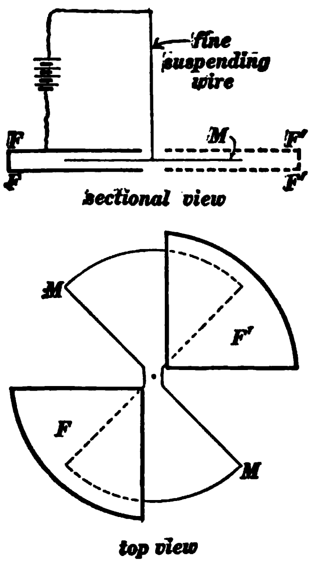

Fig. 113a.

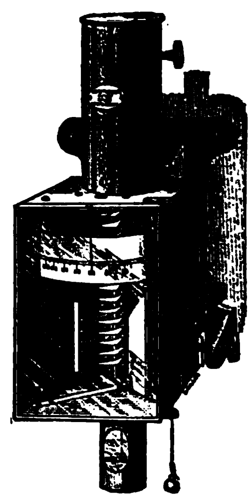

Fig. 113b.

absolute electrometer, and the force F is measured, the area a of the movable plate and the distance x between the plates being known, then the value of B may be calculated from equation (74), k being equal to unity for air.

103. The electrostatic voltmeter consists essentially of a fixed metal plate and a delicately poised or suspended metal plate which carries a pointer which plays over a divided scale. The

electromotive force which is to be measured is connected to the two plates and the scale is numbered so as to indicate the value of the electromotive force directly.

The fixed and movable plates of the electrostatic voltmeter are usually arranged as shown in Fig. 113*a*, in which *FF* and *F'F'* are the fixed plates, and *MM* is the movable plate. Figure 113*b* shows an electrostatic voltmeter designed by Lord Kelvin for measuring electromotive forces ranging from 80 to 140 volts. The movable plate in this instrument consists of a large number of vanes which are drawn into the spaces between a large number of stationary plates essentially as in Fig. 113*a*.

104. Energy and tension of the electric field in air. — Consider the charged metal plates *AA* and *BB* in Fig. 107; the capacity of the plates considered as a condenser is

$$C = \frac{1}{B} \cdot \frac{a}{x}$$

according to equation (62), where $B = 1.131 \times 10^{13}$. The energy of the charged condenser *AABB*, Fig. 107, is

$$W = \tfrac{1}{2} CE^2$$

according to equation (65*b*). Therefore, using $1/B \cdot a/x$ for C, we have

$$W = \frac{1}{2B} \cdot \frac{aE^2}{x} \tag{i}$$

This energy of the charged condenser resides in the region between the plates, that is, in the electric field. The volume of this region is ax. Therefore, dividing both members of equation (i) by ax, we find

$$\left.\begin{array}{r}\text{Energy of an electric field in}\\ \text{joules per cubic centimeter}\end{array}\right\} = \frac{1}{2B} \cdot f^2 \tag{75}$$

in which f, which is written for E/x, is the intensity of the electric field between the plates in volts per centimeter.

The force of attraction of the two metal plates *AA* and *BB*, Fig. 107, is given by equation (74), and this force is transmitted

across from plate to plate in the form of a tension of the electric field. Therefore dividing both members of equation (74) by a, we have the tension of the electric field in joule-units of force per square centimeter of section, and in the case of air (k equals unity), we have

$$\left.\begin{array}{r}\text{Tension of an electric field in joule-units}\\ \text{of force per square centimeter of section}\end{array}\right\} = \frac{1}{2B}\cdot f^2 \qquad (76)$$

in which f, which is written for E/x, is the intensity of the electric field in volts per centimeter.

105. Electric potential. — The two heavy black circles in Fig. 114 represent two long parallel metal cylinders one of which is

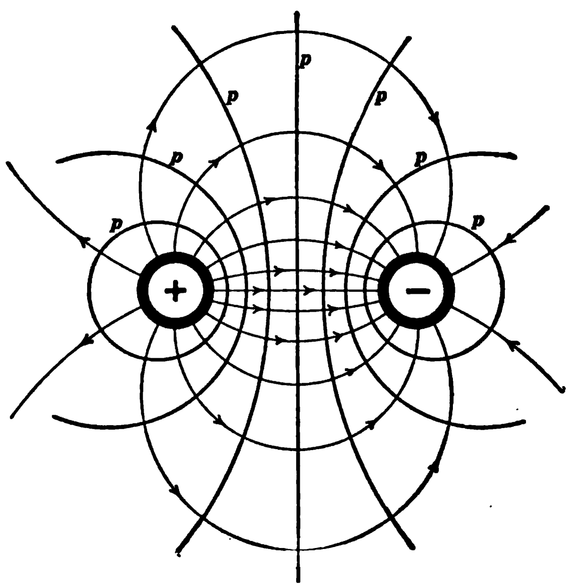

Fig. 114.

positively charged and the other of which is negatively charged, and the fine curved lines (with arrowheads) represent the lines of force of the electric field in the region between the charged cylinders. The intensity of the electric field at a given point is so many *volts per centimeter* parallel to the lines of force at that

point. Let the plane of the paper in Fig. 114 be a horizontal plane, and imagine a hill built upon this plane in such a way that its *slope lines* as seen projected upon the base plane coincide with the lines of force in Fig. 114. If the height of this hill is measured in volts then its slope may be expressed in *volts per centimeter* at each point, in fact its slope will be a complete representation of the electric field in the plane of Fig. 114. *The height, at a point, of an imagined hill whose slope is everywhere equal to the electric field is called the electric potential at that point.* The heavy curved lines *ppp* in Fig. 114 are the contour lines, or lines of equal level, on the potential hill which is imagined to be built as described above. The potential is therefore the same at every point along each of the heavy curved lines and these lines are therefore called *lines of equipotential.*

The above example refers to the distribution of electric field in two dimensions, and in this case the potential hill may be actually constructed as a geometrical hill. In general, however, this is not possible, that is to say, it is not possible to construct a geometrical representation of the potential hill. A clear idea of potential in this general case may be obtained as follows: Imagine any given distribution of electric field, the electric field surrounding a charged sphere, for example, and imagine the region surrounding the sphere to vary in *temperature* from point to point in such a way that the temperature gradient (degrees per centimeter) at each point may be equal to the electric field (volts per centimeter) at that point. Then the temperature at each point represents what is called the electric potential at that point. In this example of the field surrounding a charged sphere, the lines of force are radial straight lines and any surface drawn so as to be at each point at right angles to the lines of force is a surface of equipotential.

In order to completely establish the value of the electric potential at different points in space, a region of *zero potential* must be arbitrarily chosen. Then the potential at any other point is equal to the electromotive force E between the arbitrarily chosen

region of zero potential and the given point. The product Eq is equal to the work required to carry charge q from the region of zero potential to the given point, and it is therefore equal to the *potential energy* of the charge q when it is placed at the given point.

The idea of potential is important in the mathematical theory of electricity and magnetism, but its use by students who are beginning the study of the subject of electricity and magnetism tends to turn the attention away from physical realities.

PROBLEMS.

115. During 0.03 second a charge of 15 coulombs passes through a circuit. What is the average value of the current during this time? Ans. 500 amperes.

116. Suppose the strength of a current in a circuit to increase at a uniform rate from zero to 50 amperes in 3 seconds. Find the number of coulombs of charge carried through the circuit by the current during the 3 seconds. Ans. 75 coulombs.

117. A condenser of which the capacity is known to be 5 microfarads, is charged by a Clark standard cell of which the electromotive force is 1.434 volts and then discharged through a ballistic galvanometer. The throw of the ballistic galvanometer is observed to be 15.3 scale divisions. What is the reduction factor of the galvanometer? Ans. 469×10^{-9} coulombs per division.

118. A condenser of unknown capacity is charged by 10 Clark cells in series, giving an electromotive force of 14.34 volts, and then discharged through the ballistic galvanometer specified in problem 117; the throw of the ballistic galvanometer is observed to be 18.6 divisions. What is the capacity of the condenser? Ans. 0.608 microfarad.

119. An electromotive force acting on a condenser increases at a uniform rate from zero to 100 volts during an interval of 1/200 of a second. The capacity of the condenser is 20 microfarads. Find the value of the current during the 1/200 of a second. Ans. 0.2 ampere.

120. An alternating electromotive force which is represented by the ordinates of the zigzag line in Fig. 115 acts on a circuit which contains a condenser. The resistance of the circuit is negligible, and the capacity of the condenser is 20 microfarads.

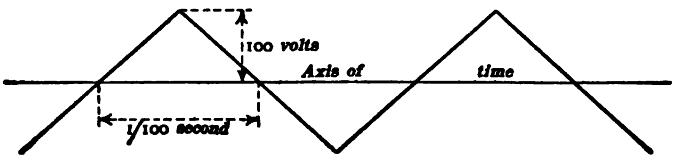

Fig. 115.

Plot the curve of which the ordinates represent the successive instantaneous values of the current.

121. Two parallel metal plates at a fixed distance apart with air between are charged as a condenser and discharged through a ballistic galvanometer. The plates are then submerged in turpentine and again charged and discharged through a ballistic galvanometer. The charging electromotive force is the same in each case and the throw of the ballistic galvanometer is observed to be 7.6 divisions in the first instance and 16.7 in the second instance. Find the inductivity of the turpentine. Ans. 2.2.

122. The metal core and metal sheath of a submarine cable are separated by an insulating layer of gutta-percha and they constitute the two plates of a condenser. One mile of a submarine cable has a capacity of 0.06 microfarad. What is the capacity of 100 miles of the cable? Ans. 6 microfarads.

123. A condenser is to be built up of sheets of tin foil 12 centimeters × 15 centimeters. The overlapping portions of the sheets are 12 centimeters × 12 centimeters. The sheets are separated by leaves of mica 0.05 centimeter thick. How many mica leaves and how many tin foil sheets are required for a one-microfarad condenser? Assume the inductivity of the mica to be equal to 6. Ans. Mica, 655; tin foil, 656.

124. A condenser is made of two flat metal plates separated by air. Its capacity is 0.003 microfarad. Another condenser has

plates twice as wide and twice as long. These plates are separated by a plate of glass (inductivity 5) which is four times as thick as the air space in the first condenser. What is the capacity of the second condenser? Ans. 0.015 microfarad.

125. Two metal plates, 100 centimeters × 100 centimeters, are separated by 2 centimeters of air. This condenser is charged by a battery having an electromotive force of 2,000 volts. What is its energy in joules? Ans. 0.000884 joule.

126. A flat glass plate, inductivity 5, size 100 centimeters × 100 centimeters × 2 centimeters, is slid between the metal plates specified in problem 125, the battery being left connected to the metal plates. What is the energy of the condenser after the glass is in place? Ans. 0.00442 joule.

127. The 2,000-volt battery is disconnected from the metal plates specified in problem 126 after the glass is in place and the metal plates are thoroughly insulated. The glass plate is then withdrawn, the whole charge being left on the metal plates. What is the electromotive force between the metal plates after the glass plate is withdrawn? What is the energy of the condenser after the glass plate is withdrawn? How much has the energy been increased by withdrawing the glass? How much force was necessary to withdraw the glass, ignoring friction, weight, etc.? (Assume that the glass is withdrawn sidewise, not cornerwise.) Ans. 10,000 volts, 0.0221 joule, 0.01768 joule, 1,768 dynes.

128. The air condenser specified in problem 125 is charged with 2,000 volts, the battery is disconnected and the plates are then moved to a distance 3 centimeters apart, charge on the plates remaining unchanged. What is the electromotive force between the plates after the movement? What is the increase of energy due to the movement? How much force was necessary to produce the movement, ignoring friction, weight, etc.? Ans. 3,000 volts, 0.000442 joule, 4,420 dynes.

129. What is the intensity of the electric field between two

parallel metal plates, 15 centimeters apart, the electromotive force between the plates being 25,000 volts? Ans. 1,667 volts per centimeter.

130. A large metal ball is placed in the uniform electric field between the plates specified in problem 129 and the ball is acted upon by a force of 2.5 dynes. What is the charge on the ball in coulombs? Ans. 15×10^{-11} coulomb.

131. A spark gauge *s* is connected with two metal plates *AB* as shown in Fig. 116. A sheet of paraffined paper 0.002 of an inch in thickness is placed between *A* and *B* and the spark gap *s* is slowly increased until the paraffined paper is punctured. The spark gap is then measured and found to be equal to 0.16 centimeter. The radius of the spheres of the spark gauge is 0.25 centimeter. Find the value of the electromotive force required to puncture the piece of paper. Ans. 6,950 volts.

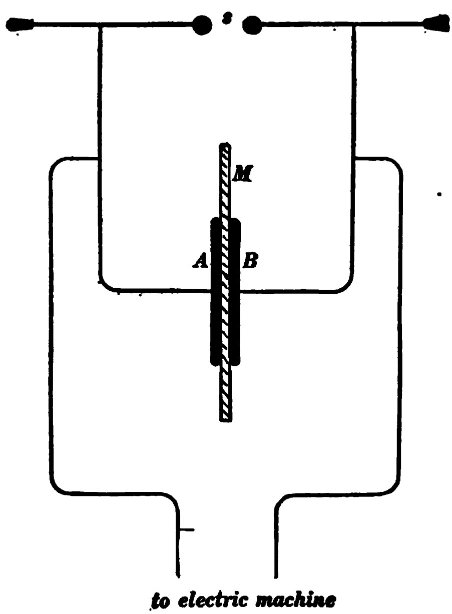

Fig. 116.

133. What is the intensity of the electric field in volts per centimeter at a distance of 200 centimeters from the center of a sphere upon which 0.0002 coulomb of charge is uniformly distributed? Ans. 4,500 volts per centimeter.

134. Find the maximum charge that can be held on a sphere 200 centimeters in diameter in air on the assumption that the electric field intensity at the surface of the sphere cannot exceed 22,000 volts per centimeter without breaking the air down electrically. Ans. 0.000979 coulomb.

135. A very long metal cylinder, 10 centimeters in diameter, is charged and the amount of charge on each centimeter of length of the cylinder is 3×10^{-12} coulomb. Find the intensity of the electric field at a point distant 50 centimeters from the axis of the cylinder. Ans. 0.108 volt per centimeter.

136. Find the maximum charge that can be held on each unit length of the cylinder specified in problem 135 on the assumption that the electric field intensity at the surface of the cylinder cannot exceed 22,000 volts per centimeter without breaking down the air. Ans. 62×10^{-9} coulomb.

137. A liquid covers a horizontal plane at a uniform depth of 10 centimeters. At a point in this plane there is a hole through which the liquid flows at the rate of 15 cubic centimeters per second. Find the direction and magnitude of the velocity of the liquid on the plane at a point distant r centimeters from the center of the hole. Ans. $0.238/r$ centimeters per second.

138. Two parallel metal plates each one centimeter in diameter are placed in pure distilled water at a distance of 10 centimeters apart and an electromotive force of 100 volts is connected to the two plates. Find the force in dynes with which the plates attract each other, the inductivity of the water being equal to 90. Ans. 32×10^{-4} dynes.

139. The entire region throughout a room is a uniform electric field directed vertically upwards and its intensity is 2,000 volts per centimeter. (*a*) Choosing the floor as the region of zero potential, what is the potential at a point 150 centimeters above the floor? (*b*) What kind of lines are the lines of force, straight or curved, and in what direction? (*c*) What kind of surfaces are the surfaces of equipotential, plane or curved, and in what direction do these surfaces lie? Ans. (*a*) 300,000 volts. (*b*) Straight lines, perpendicular to floor. (*c*) Plane surfaces, parallel to floor.

140. Given two parallel metal plates 15 centimeters apart to which a 10,000-volt battery is connected. Imagine a line X drawn straight from plate to plate. Choose the negatively

charged plate as the region of zero potential and plot a curve of which the abscissas are distances measured along the line X and of which the ordinates represent the values of the potential at points along this line.

141. An inclined plane is viewed from above. A series of contour lines and of slope lines are drawn upon the plane. Make a diagram showing the appearance of these lines as projected upon the base plane.

142. A circular cone is viewed from above and a series of contour lines and slope lines are seen projected upon the base plane of the cone. Draw a diagram showing the appearance of these lines on the base plane.

CHAPTER VIII.

THE PHENOMENA OF ELECTROSTATICS.

106. The voltaic cell (or dynamo) versus multiplying devices for the production of large electromotive forces. — A locomotive engineer, knowing that an ordinary locomotive can exert about 15,000 pounds of draw-bar pull, and, wishing to observe the behavior of a bar of steel when subjected to a stretching force of 150,000 pounds, might arrange to use ten locomotives hitched together to exert the desired force; but if the bar of steel should break, then the dormant energy of the locomotives would come into action, about 10,000 actual horse-power would have to be taken care of, and a terrible wreck would probably be the result. *The value of a locomotive lies in the fact that it can continue to pull even when the thing it pulls on yields, as it were, at a speed of 60 miles per hour;* but for exerting a large force upon a thing which does not yield rapidly, some sort of a force-multiplying device, such as a screw or a lever, is more convenient and incomparably cheaper and safer than a battery of locomotives.

An electrician, knowing that an ordinary dry cell has an electromotive force of about 1.5 volts, and, wishing to observe the effects when the air between two metal plates is subjected to a high electromotive force, might think of connecting 100,000 dry cells in series to exert 150,000 volts; but if the air should break down, then the dormant energy of the battery would come into action, about 1,000 actual horse-power would have to be taken care of, the apparatus would in all likelihood be destroyed, and if the body of the electrician should by accident become a portion of the battery circuit, he would be instantly killed. *The value of the battery or dynamo lies in the fact that it can continue to push even when the circuit upon which it pushes yields at a "speed" of many amperes;* but for exerting a large electromotive force across

a fairly good insulator which does not yield * to any great extent, some sort of an electromotive-force-multiplying device is more convenient and incomparably cheaper and safer than a battery of dynamos or voltaic cells.

107. The alternating-current transformer or induction coil.— One method for multiplying electromotive force is by means of the alternating-current transformer, which is strictly analogous to a mechanical lever pinned to a very massive movable body, instead of to a rigid fulcrum, as shown in Fig. 117. Such a fulcrum gives way under a steady force, but it is sufficiently immovable under the action of an alternating force, that is, a force which is repeatedly reversed in direction. The alternating-current transformer or induction coil cannot be used to produce a large steady electromotive force.†

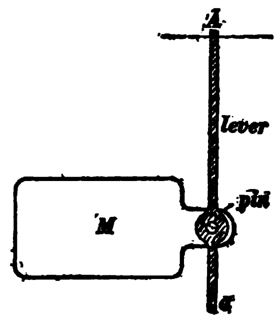

Fig. 117.

The mechanical analogue of the alternating-current transformer. —A lever Aa, Fig. 117, of negligible mass, is attached to a very massive body M by a pin connection. An alternating force acts on the end A causing it to oscillate back and forth along the dotted line with great alternating velocity I, and the end a of

* An electromotive force of 100,000 volts connected to two metal plates each one meter square with a plate of flint glass between them two centimeters thick would produce about one ten-million-millionth of an ampere through the plate of glass. See table of specific resistances in Art. 14.

† Accessory devices may, however, be used in conjunction with an alternating-current transformer to produce an approximately steady uni-directional electromotive force. One of these devices, the mercury-arc rectifier, is strictly analogous to the mechanical device called a *ratchet* which permits a body to move in one direction only; and another device is the commutator which is analogous to the *crank* which converts the alternating force of a locomotive engine into a pulsating, uni-directional, draw-bar pull, which may become fairly steady in value if the moving locomotive is very massive. It is not an exaggeration to say that no one can understand the mercury-arc rectifier or the commutator or any other electrical or magnetic device or phenomenon unless he can reduce it in his mind to its mechanical equivalent. See the Preface to this volume.

the lever oscillates back and forth with a small alternating velocity i. If the motion of the end a of the lever is opposed by a considerable frictional resistance, requiring a large alternating force E to overcome it, then a certain small alternating force e must act on the end A of the lever to produce the required alternating force E.

One who is familiar with the action of the alternating current transformer may follow out this mechanical analogue in all of its details. The alternating velocity of the end A corresponds to the primary current I', and the alternating velocity of the end a corresponds to the secondary current I''. Immovability of the end a corresponds to open secondary circuit, and entire freedom of motion of the end a corresponds to short-circuited secondary. If the mass M were infinite, the end A could not move at all when the end a is fixed (open secondary), but if the mass M is finite, then a given alternating force E' acting on the end A would cause some motion of the end A, even if the end a were rigidly fixed, and this motion of the end A corresponds to the magnetizing current of the transformer.

108. The electrical doubler. — The device which is most frequently used for building up intense electrical fields (high electromotive forces) is shown in its simplest form in Figs. 118 to 121.

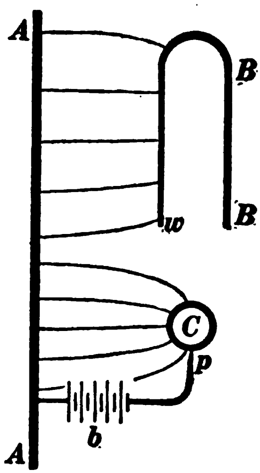

Fig. 118.

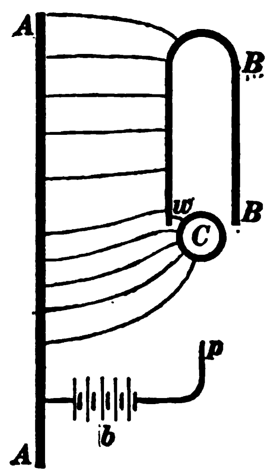

Fig. 119.

It is desired to build up a very intense electrical field between a metal plate AA and one side of a hollow metal vessel BB. A metal ball C, called a *carrier*, is attached to an insulating handle

by means of which it can be brought into contact with the point p and then pushed into the interior of BB and brought into contact with BB, repeatedly. Each time the carrier touches the point p it receives a certain amount of charge from the battery b, or in other words, a bundle of lines of force comes into existence between C and AA as shown in Fig. 118. As the car-

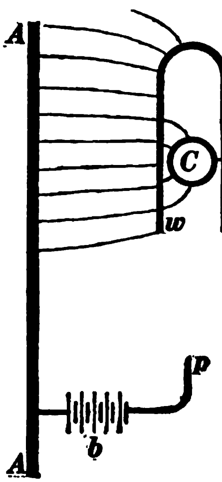

Fig. 120.

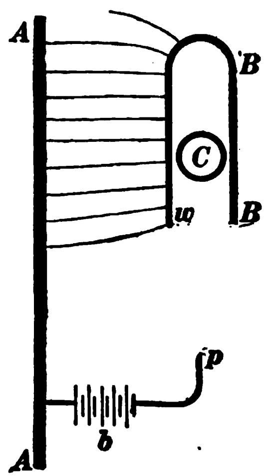

Fig. 121.

rier is moved into the hollow vessel BB, the bundle of lines of force trends as shown in Fig 119, and work has to be done to move the carrier against the pull of these lines of force. As the carrier is moved into BB the lines of force from C to AA are cut in two, as it were, one after another, by the metal wall at w, the portions of the lines of force which pass from C to AA, as shown in Fig. 120, are then obliterated by bringing C into contact with B, and the carrier C is left entirely neutral as shown in Fig. 121. Each repetition of the above movements of the carrier C "strings" an additional bundle of lines of force from A to B and thus increases the intensity of the electrical field between A and B.

The production of a very large electromotive force between A and B in Figs. 118 to 121 by the to and fro motion of the

carrier C is somewhat analogous to the production of an intense stress in a steel block $AABB$, Fig. 122, by stretching small rubber bands, like R, and placing them over the block, one after an other. The simplest mechanical analogue of the electric doubler, however, is the building up of intense stresses in a drum by winding upon it a string or wire under considerable tension.*

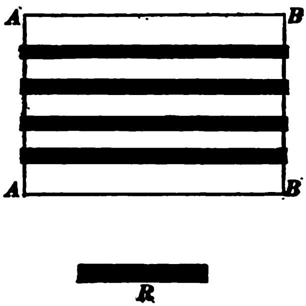

Fig. 122.

In the electrical doubler which is represented in Figs 118 to 121, the carrier C receives charge repeatedly from a battery b. The frictional electrical machine and the influence electrical machine employ the principle of the electrical doubler. In these machines, however, the carrier is charged not by a battery of voltaic cells, but by either of two peculiar electrical processes, namely, (*a*) *charging by contact and separation*, or (*b*) *charging by influence*.† These two processes are described in

* A very interesting example of this action is described on page 339 of the third volume of Lord Kelvin's *Popular Lectures and Addresses*. When Lord Kelvin was carrying out his first experiments on deep sea sounding, the long piano-steel wire which was used was wound upon a heavy metal drum, and the stress in the drum became great enough to bend it out of shape.

† The simplest device combining *charging by influence* and the *doubling action* which is described in connection with Figs. 118 to 121 is shown in Fig. 123. The hollow metal vessels A and B have a certain amount of charge to begin with. Two flat metal carriers C and D each having an insulating handle, are placed between A and B and brought into contact with each other as shown. The result is that the lines of force from A to B arrange themselves as shown in the figure, and then the carriers C and D may be separated from each other, carrier C being moved

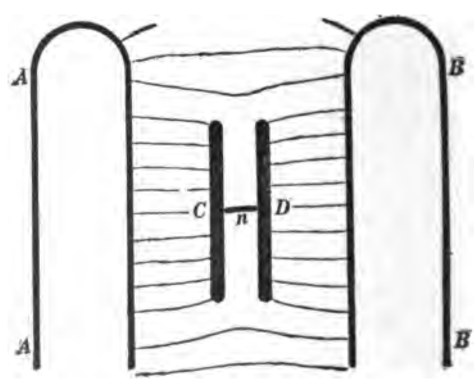

Fig. 123.

the following articles, and in order to obtain a clear understanding of the frictional electric machine, of the Toepler-Holtz electrical machine, and of the Wimshurst electrical machine, it is important to keep in mind the fact that every one of these machines involves the principle of the electrical doubler, inasmuch as the carrier or carriers pass between two conducting bodies to both of which they give up their charges, so that these two conducting bodies take the place of the hollow vessel *BB* in Figs. 118 to 121.

109. Charging by contact and separation. — The production of electric charge by the rubbing together of certain substances is one of the most familiar of the phenomena of electricity. When a cat is stroked with the hand in a dry room, the cat's fur and the hand become oppositely charged, and the crackling sound which is produced is due to the production of minute electrical sparks which may be seen if the room is dark. A hard rubber comb becomes strongly charged when it is passed through very dry hair, and the comb will attract small bits of paper or pith. When pencil marks are erased from a very dry piece of paper by means of a rubber eraser, the paper becomes charged and it clings to the drawing board or table.

Two substances when brought into contact always tend to settle to a state of equilibrium in which electric lines of force pass from one substance to the other across the very thin air space between them. Thus, two flat plates of zinc and copper settle to equilibrium with an electromotive force of about 0.9 volt between them, so that the intensity of the electric field in the very narrow space between the plates may be several thousands volts per centimeter. If the plates are thoroughly insulated and moved apart, the electric field intensity (volts per centimeter) re-

into the interior of *B* and brought into contact with *B*, and carrier *D* being moved into the interior of *A* and brought into contact with *A*. The result is that the bundle of lines of force from *A* to *C* in Fig. 123 is stretched across from *A* to *B*, and the bundle of lines of force from *D* to *B* is stretched across from *A* to *B* thus increasing the total number of lines of force from *A* to *B*. The *revolving doubler* of Lord Kelvin is a mechanical device for performing the operations here described, the carriers *C* and *D* being mounted upon a rotating, insulated arm.

mains unaltered, so that the electromotive force between the plates may be increased to several thousands of volts. Thus, the very fine lines in Figs. 124 and 125 represent the electric field

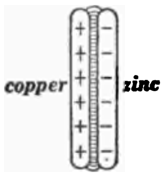

Fig. 124.

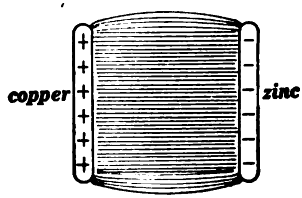

Fig. 125.

between the copper and zinc plates when they are close together and after they have been separated to a considerable distance.

In order to produce an intense electric field by separating two metal plates, the plates must be very flat, and they must be separated in such a way as to avoid a lingering contact between them. When both of the substances are good insulators, however, they always retain their charges (one positive and the other negative) when they are moved apart, and the intervening region becomes an intense electric field. This phenomenon is called *charging by contact and separation.* In order to bring sealing wax and fur, or glass and silk into intimate contact, vigorous rubbing is necessary, and therefore charging by contact and separation is frequently spoken of as *charging by friction.*

To understand the phenomenon of charging by contact and separation it is important to keep in mind that the charging is done by contact (no one knows exactly how), and that the creation of an intense electrical field throughout a large region is accomplished by separation. In this case the electrical field is wound up,* as it were, by pulling the charged surfaces apart, and the work done in pulling the charged surfaces apart against their force of attraction (tension of the lines of force) is the work that goes to establish the field in the larger and larger region between the receding surfaces.

* In the sense of winding up a spring so as to put it under stress.

110. The frictional electric machine. — This machine consists of a rotating glass disk *DD*, Fig. 126, the various parts of which come in succession into intimate contact with two leather cushions *AA* which are impregnated with an amalgam of tin, zinc and mercury. The surface of the glass plate as it leaves these cushions is left highly charged with positive electricity, and

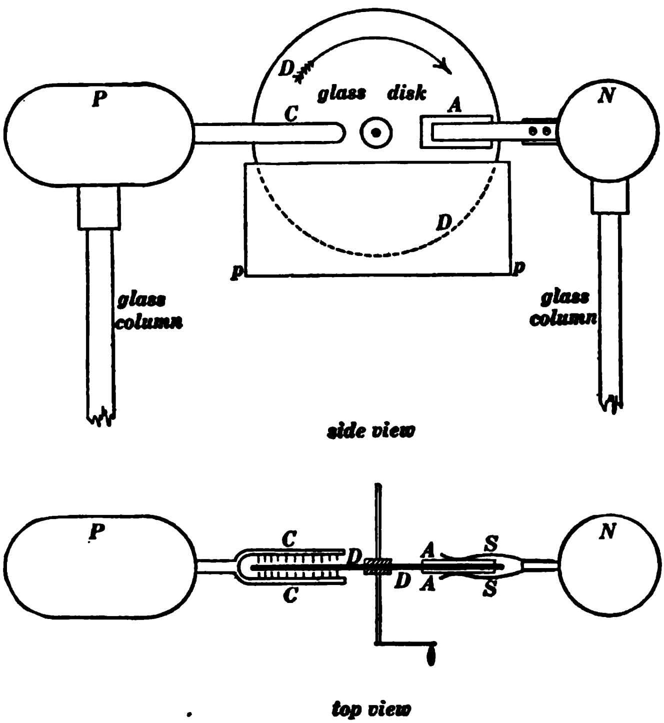

Fig. 126.

the cushions are left negatively charged. The negative charge flows into the insulated conductor *N* which is connected to the cushions by means of the metal springs *SS*, and the positive charge is carried on the surface of the glass disk to the collecting combs *CC* whence it flows into the insulated conductor *P*. Two silk aprons *pp*, one on each side of the rotating disk, tend

to prevent the escape of the positive charge from the surface of the disk.*

111. Charging by influence. — The simplest example of charging by influence is that which is described in connection with Fig. 123. Charging by influence is essentially the cutting of lines of force in two by two sheets of metal so that the ending of the lines of force on one sheet constitute a new negative charge and the beginning of the lines of force on the other sheet constitute a new positive charge. Let *A*, Fig. 127, be a charged body from which lines of force emanate. When a metal ball *B* is brought near to *A*, the lines of force converge upon one side of *B* and diverge from the other side as shown in the figure; if a second metal ball *C* is brought into contact with *B*, as shown in Fig. 128, then the lines of force converge upon *B* and diverge from *C*, and the two balls *B* and *C* retain their charges when they are separated and removed to a distance from *A*. This operation is called *charging by influence*, and it results in the production of equal amounts of positive and negative electricity (on *B* and *C* respectively) while the original influencing charge on *A* is undiminished. Charging by influence is exemplified by the operation of the electrophorus.

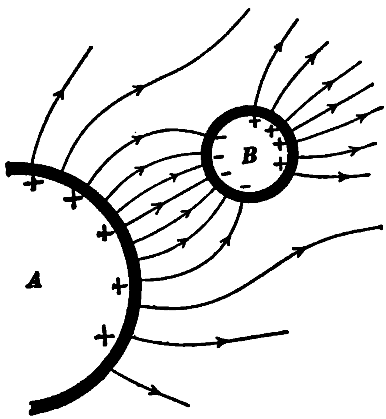

Fig. 127.

112. The electrophorus is a device for the production of charge by influence. It consists of a rosin or hard rubber plate *D*,

* The frictional electric machine involves the principle of the electric doubler, but it is not worth while to examine minutely into the manner in which the lines of force are drawn out as it were and "strung" across from *P* to *N*, as the various parts of the glass plate leave the cushions *AA*. The above account, which is based on the idea that positive and negative electricities are two fluid-like substances, is sufficiently intelligible for present purposes.

Fig. 129, which has been electrified (negatively) by rubbing it with a piece of fur or flannel, and a metal disk *M* with an

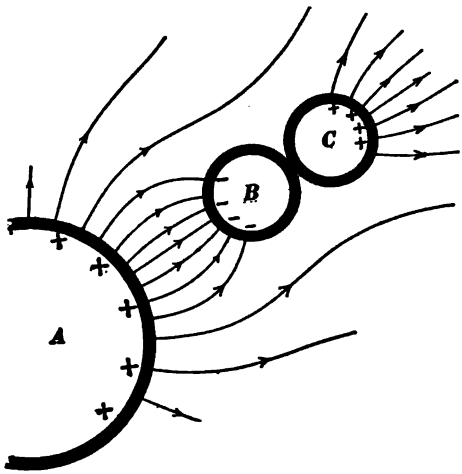

Fig. 128.

insulating handle *H*. When the metal disk is brought near to the negatively charged plate of rosin and touched with the finger it is left with a charge of positive electricity, and this charge remains on *M*, when *M* is removed to a distance from *D*. This operation may be repeated indefinitely.*

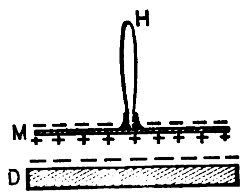

Fig. 129.

113. Influence electrical machines.—The electrophorus is the simplest arrangement for the generation of charge by influence. If the metal carrier *M* of the electrophorus is thrust into a hollow metal vessel and touched to its walls, it gives its entire charge to the hollow vessel, whatever the previous charge on the vessel may be, and thus it is possibe to generate any desired

* The description here given of the operation of the electrophorus is really inadequate. The metal pan which contains the rosin plate plays an important part in the operation of the electrophorus as is evident from the fact that the electrophorus does not operate satisfactorily when the metal pan is insulated from the floor and walls of the room by being placed upon an insulating support.

electromotive force between the hollow metal vessel and the walls of the room. In the Toepler-Holtz machine and in the Wimshurst machine, metal carriers are fixed to a rotating glass disk or disks so that at one part of their path these carriers become charged by influence and at another part of their path they pass between two pieces of metal which act like the hollow metal vessel in Figs. 118 to 121, thus combining the principle of the electrical doubler with the principle of the electrophorus. The inducing charge (which corresponds to the charge on the rosin plate of the electrophorus) in the Toepler-Holtz machine and in the Wimshurst machine is generated by the machine itself.

Reversibility of influence machines.—The Toepler-Holtz machine and the Wimshurst machine may be used as *electric generators* as described below, in which case they must be supplied with mechanical power and they deliver electrical charge at high electromotive force; or they may be used as *electric motors* in which case they must be supplied with electric charge at high electromotive force from some outside source, and they deliver mechanical power. Thus, a very large Toepler-Holtz machine driven at high speed may deliver a steady current of 0.001 of an ampere (one thousandth of a coulomb of charge per second) at an electromotive force of, say, 100,000 volts. This corresponds to an output of 100 watts of power, and if the friction losses in a second similar machine are very small, the second machine may be driven as a motor.

114. The Toepler-Holtz machine.—A general view of the Toepler-Holtz machine is shown in Fig. 130. It is difficult to show in a diagram the essential features of such a machine in which the carriers are arranged on a glass disk. Figure 131 shows a possible form of Toepler-Holtz machine in which the carriers are fixed to a rotating glass cylinder which is surrounded by a stationary glass cylinder upon which the "inductors" *AA* and *BB* (which carry the inducing charges) are supported. The neutralizing rod is a stationary metal rod with metal brushes at its ends, and the figure shows the metal brushes 2 and 4 in

contact with the metal buttons which project from two of the carriers. The result is that these two carriers become charged

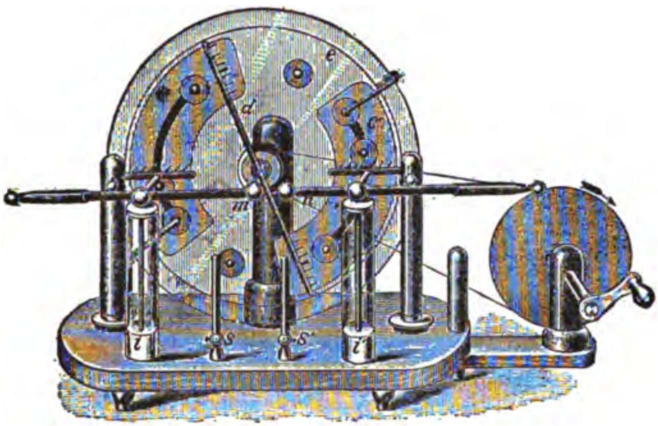

Fig. 130.

under the influence of the positive and negative charges on AA and BB, the upper carrier being negatively charged and the

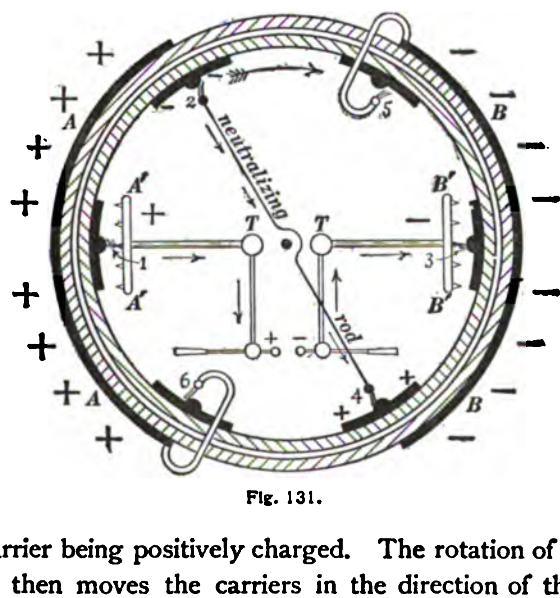

Fig. 131.

lower carrier being positively charged. The rotation of the inner cylinder then moves the carriers in the direction of the curved

arrows until the carriers come under brushes 5 and 6 where they part with a portion of their charges, thus replenishing inducing charges on AA and BB. The carriers are then moved into the space between AA and $A'A'$ on the one hand and into the space between BB and $B'B'$ on the other hand where they come into contact with the brushes 1 and 3, thus giving up the remainder of their charges to the metal terminals TT of the machine. When a spark is formed between the metal terminals TT, the bodies $A'A'$ and $B'B'$ become completely discharged, but the induced charges on AA and BB remain and the machine continues to operate.

The Toepler-Holtz machine is self-exciting, that is to say, the extremely minute electromotive forces due to the contact of the metal brushes with the metal buttons on the carriers are sufficient to start the operation of charging by influence, and the action of

Fig. 132.

the machine is then rapidly intensified by the doubling action which takes place.

115. The Wimshurst machine. — A general view of the Wimshurst machine is shown in Fig. 132. It consists of two oppositely rotating glass disks on each of which a number of metal

carriers are fixed. Stationary neutralizing rods are placed one on each side of the machine, each inclined at an angle of approximately 45° to the horizontal, and the charge on one disk as it travels towards the collectors serves as the inducing charge for the other disk.

The inducing action of the Wimshurst machine may be explained as follows: Figure 133 represents two glass plates *AB*

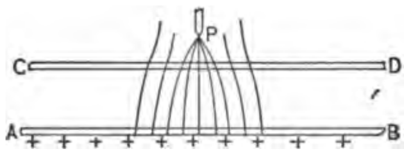

Fig. 133.

and *CD*. One of these plates is charged as indicated by the plus signs, and the lines of force from this charge converge upon the metal point *P* which is at one end of a neutralizing rod.

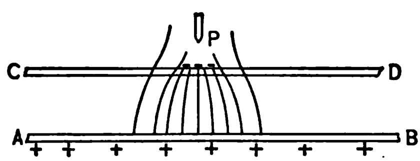

Fig. 134.

The electric field in the neighborhood of *P* is sufficiently intense to break down the air between *P* and the glass plate *CD*, thus leaving negative charge on the glass plate *CD* as shown in Fig. 134.

The small portion of the surface of *CD* which faces the point *P* is thus negatively charged and the amount of charge on this small portion is numerically equal to the amount of positive charge on the larger part of *AB* from which emanate the lines of force that have been broken down between *P* and *CD*. If the plate *CD* is moved to the left in Fig. 134, fresh lines of force crowd into the space between the point *P* and the plate *CD*, they continue to break down as in the first instance, and the entire surface of *CD*, as it moves out from under the point

P, is left much more strongly charged than the plate *AB*. The plate *AB* may itself be charged by moving it in front of a point *P′* under the inducing action of *CD* as shown in Fig.

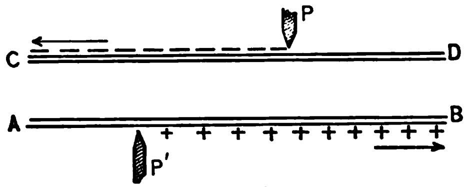

Fig. 135.

135. Under these conditions, the charges on *AB* and *CD* will grow more and more intense until checked by the rapidly increasing leakage from the surfaces of the plates. The negative charge on *CD*, Fig. 135, after it has passed beyond the point *P′*, and

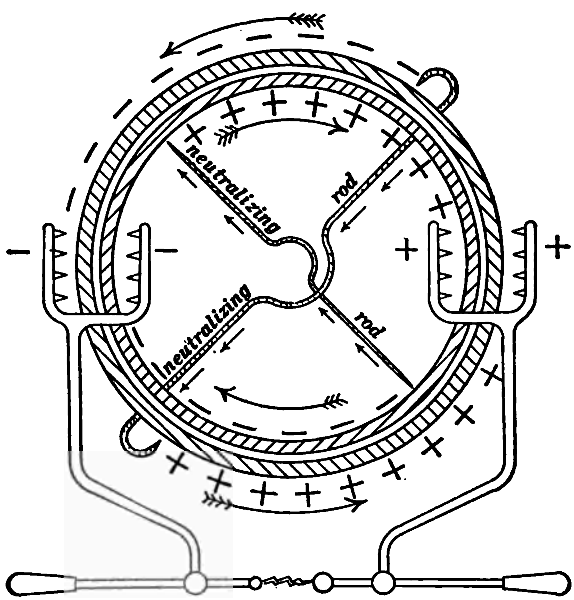

Fig. 136.

the positive charge on *AB*, after it has passed beyond the point *P*, may be collected and used for any purpose.

Figure 136 shows the essential features of a complete Wims-

hurst machine consisting of two coaxial glass cylinders rotating in opposite directions. The negative charges on both cylinders are collected by the double metal comb on the left as the rotating cylinders pass between the prongs of the comb, and the positive charges of both cylinders are collected by the double comb on the right.

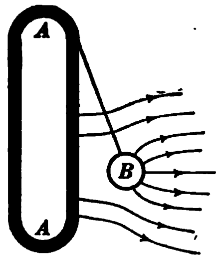

Fig. 137.

116. Electroscopes. — An electroscope is a device for indicating the existence of an electric charge, or for detecting an electric field.

The pith ball electroscope consists of a gilded ball of pith suspended by a silk thread. The presence of an electric field in a given region may be shown by charging the pith ball, and noting the force which acts upon it when it is placed in the given region, the direction of the field being indicated by the direction of the force which acts upon the ball.

A pith ball may be hung alongside of a body of metal, as shown in Fig. 137. If the body of metal is charged, a portion of the charge is given to the ball, and the lines of force which emanate from the ball pull it outwards from the body as shown in the figure.

The essential features of the *gold leaf electroscope* are shown in Fig. 138. A metal rod *R* is supported in the top of a glass case *cc* by means of an insulating plug, a metal disk *D* is fixed to the upper end of the rod, and two gold leaves are hung side by side from the lower end of the rod. The glass case *cc* serves to protect the gold leaves from air currents. The sides of *cc* are lined with strips of metal foil *ff*, and these pieces of metal should be connected to earth. When the disk, rod and leaves are charged, the leaves are pulled apart by the lines of force which emanate from the leaves and terminate on the strips *ff* as shown in Fig. 139. This figure shows the instrument without the enclosing case for the sake of clearness.

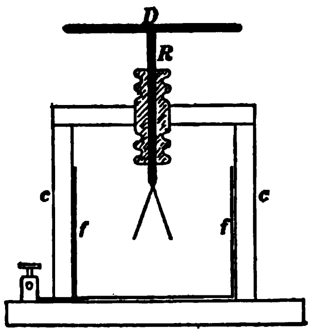

Fig. 138.

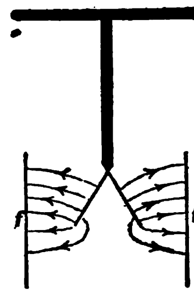

Fig. 139.

The behavior of a gold leaf electroscope when a charged body is brought near to the plate D is as follows: (1) When the electroscope has no initial charge, some of the lines of force pass from the charged body into the disk and then spread out from the leaves to the strips *ff*, causing the leaves to diverge. If the charged body is removed the electroscope becomes neutral and the leaves fall together. If, while the charged body is near D, the disk or rod is touched with the finger, the lines of force passing out from the leaves cease to exist, and the leaves fall together. If now, the charged body is removed, the lines of force passing into the disk from the charged body spread over the disk, rod and leaves, and the electroscope is left charged, as indicated by the divergence of the leaves. This operation, called *charging by influence*, is explained more fully in Art. 111.

(2) When the electroscope has an initial charge, say a positive charge, then a positively charged body brought near to D pushes the initial charge down into the leaves, as it were, and the divergence of the leaves is increased. If a negatively charged body is brought near to D, the positive charge on the leaves is pulled up into the disk, as it were, by the attraction of the negative charge on the body, and the divergence of the leaves is decreased. If the negatively charged body is brought nearer, the leaves will come together; and if the negatively charged body is brought still nearer the leaves will again diverge.

The behavior of a positively charged electroscope when a negatively charged body is brought near to it, is the same as its behavior when it is negatively charged and a positively charged body is brought near to it.

117. Electric charge resides wholly on the surface of a charged conductor. Electrical screening. — The electrostatic phenomena exhibited by charged conductors are precisely the same whether the bodies be solid or hollow; and, if the bodies be hollow, no effect of the charges can be detected inside of them however thin their walls may be. The lines of force of the electric field end at the surface of the charged conductor or, in other words, the electric charge resides wholly on the surface of a charged conductor.

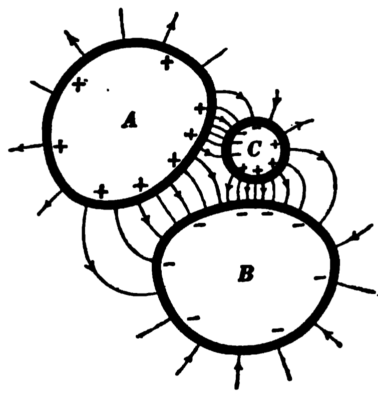

Fig. 140.

A conducting shell, such as a metal box, screens its interior completely, so that no action of any kind reaches the interior from charged bodies outside.* Thus, a hollow metal ball *C*, Fig. 140, screens its interior completely. The lines of force which touch the shell *C* end at its surface. The ending on *C* of the lines of force from *A* is negative charge and the beginning on *C* of the lines of force which reach *B* is positive charge.

The fact that electrical field cannot penetrate into a substance like a metal shows that such substances cannot sustain the peculiar kind of stress which constitutes electrical field any more than a fluid can sustain the kind of stress that exists in a stretched steel wire.

Mechanical analogue of electrical screening. — Consider a solid body *B*, Fig. 141, entirely separated from the surrounding solid by an empty space *eee*. Stress and distortion of the surrounding

* This is not strictly true when the outside conditions are changing rapidly.

solid cannot affect B in any way, and conversely stress and distortion of B cannot affect the surrounding solid because the empty space is incapable of transmitting stress. This empty space in its behavior towards mechanical stress is analogous to a conducting substance in its behavior towards electrical stresses (electrical field).

Fig. 141.

118. A charged conductor shares its charge with another conductor which is brought into contact with it.— Figure 142 shows the lines of force in the neighborhood of a charged conductor A. When another conductor B is brought into contact with A, the lines of force arrange themselves as shown in Fig. 143. The charge which was originally on A spreads over A and B, as indicated by the ending of the lines of force.

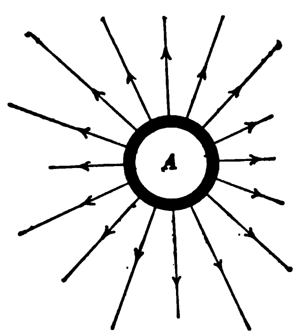

Fig. 142.

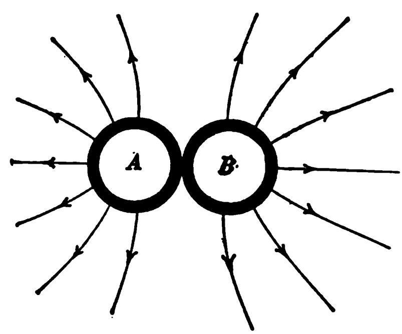

Fig. 143.

119. Faraday's experiment. — A charged body B, Fig. 144, is lowered into a metal vessel and the opening of the vessel is closed with a metal lid. As the body is lowered into the vessel, each line of force that emanates from B is cut in two, as it were, by the wall of the vessel so that, when B is entirely enclosed by the vessel, as many lines of force emanate from the external surface of the vessel as from the body B, and all the lines of

force which emanate from B terminate on the inner surface of the vessel. Therefore, if $+q$ is the amount of charge on B, $-q$ is the amount of charge on the inner surface of the vessel

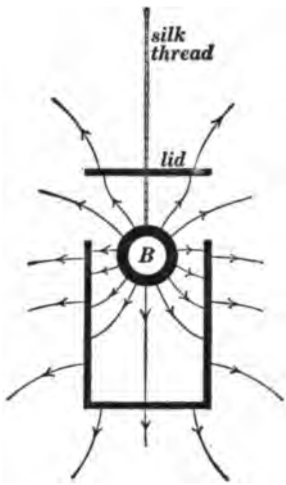

Fig. 144.

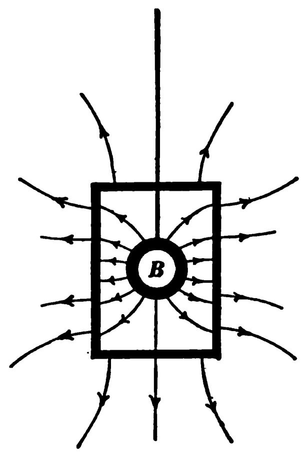

Fig 145.

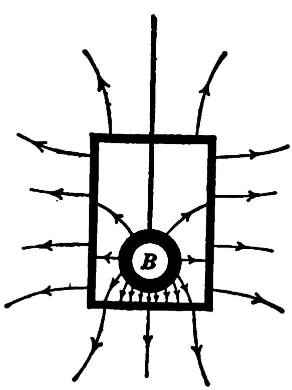

Fig. 146.

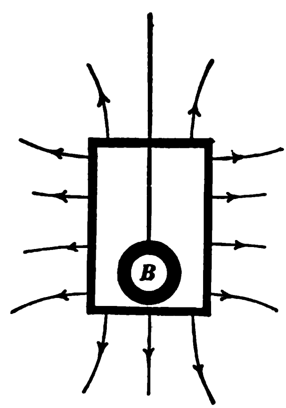

Fig. 147.

and $+q$ is the amount of charge on the external surface of the vessel in Fig. 145.

After the body B has been completely enclosed by the metal vessel as shown in Figs. 145, 146, and 147, the distribution of

the electrical field outside of the vessel does not depend in any way upon the position of the body B inside the vessel, and, if the body B is brought into contact with the wall of the vessel, the lines of force which emanate from B disappear, no charge is left on B and no charge is left on the inner surface of the vessel.

120. Giving up of entire charge by one body to another. — When the body B, Figs. 144, 145, 146, and 147, is lowered into the vessel and allowed to touch the walls of the vessel it loses all of its charge and remains without charge when removed from the vessel, and the charge on the outside of the vessel is equal to and of the same sign as the original charge on B. The body B may thus be said to give up its entire charge to the vessel.

121. Convective discharge and disruptive discharge. — Consider the positive and negative charges at the ends of a bundle of lines of force. In order that these charges may disappear, it is necessary that the lines of force be annihilated. This may be accomplished by the moving of the charged surfaces towards each other until they are coincident, or the dielectric which sustains the electrical stress may break down. In the former case, we have what is known as *convective discharge*, and, in the latter case, we have what is known as *disruptive discharge.*

Convective discharge is to some extent analogous to the relieving of a stretched rubber band by allowing its ends to move towards each other. Disruptive discharge is somewhat analogous to the relief of a stretched rubber band by rupture.

Examples. — (*a*) Two metal plates AA and BB in Fig. 106, being disconnected from the battery, might be discharged (the electric field be made to disappear) by moving the plates together.

(*b*) The transfer of charge by a moving ball, as described in Art. 94, is convective discharge. The ball gathers in the ends of a bundle of lines of force when it touches one plate and it shortens these lines until they disappear as it moves across to the

other plate. Figures 148 to 151 show the successive aspects of the electric field while the ball is moving once across from plate to plate.

(*c*) The electromotive force between the two metal balls *A* and *B*, Fig. 110, may be increased until the intervening dielec-

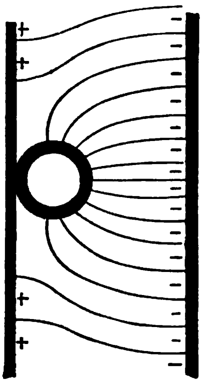

Fig. 148.

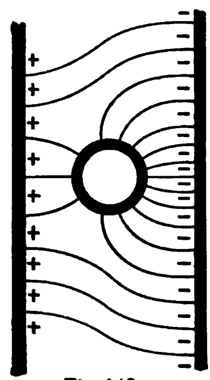

Fig. 149.

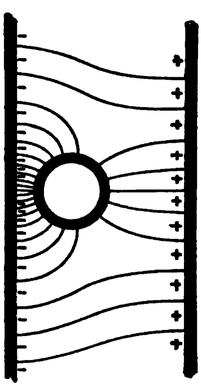

Fig. 150.

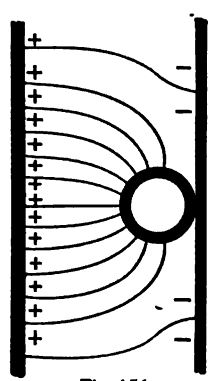

Fig. 151.

tric breaks down, causing the formation of an electric spark. An electric spark is a conducting path, like a wire, and its effect is to completely discharge the two balls *A* and *B*.

122. Progress of the electric spark. — Let *A* and *B*, Fig. 152, be two metal balls upon which electric charge has accumulated until the intensity of the electric field has reached the

breaking point of the intervening dielectric. The rupture of the dielectric starts in the region of greatest electric stress,* as indicated by the short thick line projecting from the surface of *A* in the figure. Along the line of this rupture the dielectric is a

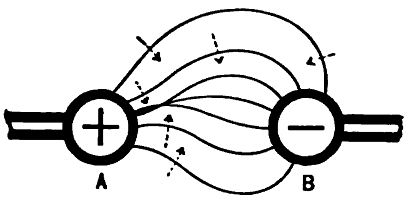

Fig. 152.

good conductor, and the lines of force on all sides move sidewise into the rupture as indicated by the arrows, producing a greatly intensified electric field at the end of the rupture so that the rupture extends further and further until it reaches *B*.

This extension of an electric rupture or spark through a region in which the intensity of the electric field is originally much below the breaking value of the dielectric is analogous to the following: A pane of glass is slightly bent and then scratched near one edge so as to start a crack. The effect of this crack is to greatly intensify the stress in the glass at the end of the crack and the crack therefore quickly runs across the pane.

When the electric rupture has extended itself across from *A* to *B* in Fig. 152, a conducting line or path is established from *A* to *B*, and all of the charge on *A* and *B* disappears, that is to say, the electric field between *A* and *B* disappears.

123. The brush discharge. — The discharge in air from a body of metal which stands at a distance from surrounding bodies is in some respects different in character from the spark discharge between two oppositely charged conductors which are not too far apart. Figure 153 represents the lines of force spreading out

* This rupture always starts in air at the surface of the positively charged ball, unless the surface of the other ball is much more sharply curved.

from a positively charged metal ball. If the ball is sufficiently charged the electric field near its surface reaches the breaking point of the dielectric, and the rupture starts as described in Art.

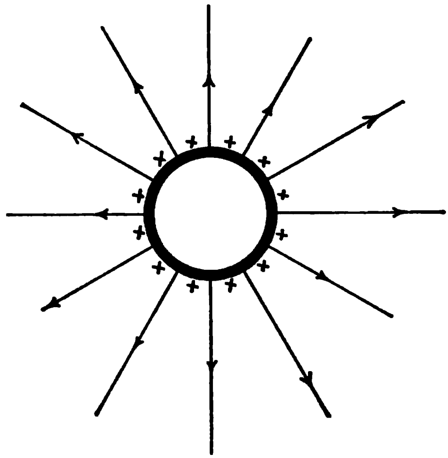

Fig. 153.

122, but in this case the rupture very soon extends into a region where the field was originally very much less intense than at the surface of the ball, and such lines of force as have moved side-

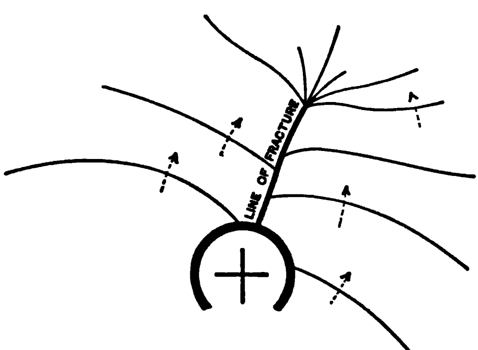

Fig. 154.

wise into the rupture and have partially (that is, through a portion of their length) broken down, now form in a widely divergent bundle from the end of the rupture as shown in Fig. 154 (com-

pare Fig. 154 with Fig. 152). The result is that the rupture divides into many branches which penetrate into the surrounding air in the form of a *tree* or *brush*. This type of discharge is called the *brush discharge*, and it is most readily formed in a region where the lines of electric force are widely divergent, as near a pointed projection on a charged conductor. The brush discharge forms more readily on a positively charged conductor than on a negatively charged conductor, and the positive brush is very different in character from the negative brush.

124. Electric discharge from metallic points. — A body of metal which has a sharp point can scarcely be charged at all, because of the fact that a very slight charge on the body produces a very intense electric field in the neighborhood of the sharp point, the lines of force in this region break down, and the lines of force become detached from the conductor, ending upon charged portions of the surrounding air. Thus, Fig. 155*a* represents a metal ball

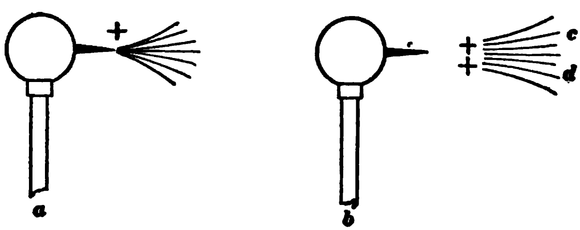

Fig. 155.

with a sharp metal point, and Fig. 155*b* represents the state of affairs after the air has broken down in the neighborhood of the sharp point where the electric field is very intense.

The tension of the lines of force *cd* in Fig. 155*b* pulls the positively charged air away from the point, forming a blast of air. If the ball is connected to an electric machine so as to be continually supplied with charge, new lines of force continually replace those that are broken down and a continuous blast of air is produced which is sometimes strong enough to blow out a candle.

Figure 156 shows the bent end of a metal rod with a sharp point at *P.* When the lines of force emanate from all parts of the rod as shown in the figure, the total force acting on the rod is zero, if it is at some distance from surrounding objects. When, however, the lines of force near the point break down, they no longer pull on the rod, therefore the pull due to the lines of force at *b* is unbalanced, and the rod is acted upon by a force pulling it to the left. The *electric whirl* is an arrangement of pointed rods bent as shown in Fig. 157, and mounted on a pivot on an insulating stand. When this arrangement is connected to an electric machine, it is set into very rapid rotation by the unbalanced pull of the lines of force which emanate from the portions *b* of the rods, as shown in Fig. 156.

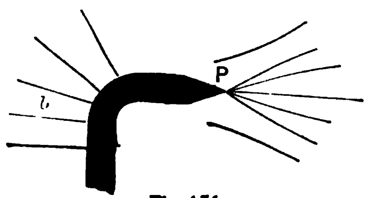

Fig. 156.

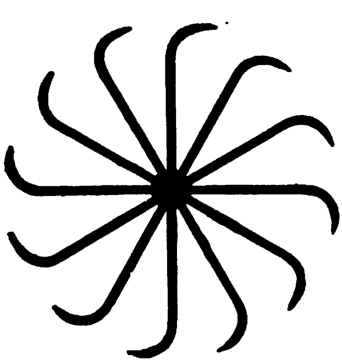

Fig. 157.

125. The mechanical theory of electricity and the atomic theory of electricity.—The study of electricity and magnetism as represented in the foregoing chapters (with the exception of several matters which are discussed in Chapter I) is independent of any consideration of the nature of the physical action which leads to the production of electromotive force by a voltaic cell or dynamo; it is independent of any consideration of the nature of the physical action which constitutes an electric current in a wire; it is independent of any consideration of the nature of the disturbance which constitutes a magnetic field; and it is independent of any consideration of the nature of the disturbance or stress which constitutes an electric field. This kind of study of electricity and magnetism may very properly be called *electro-mechanics.*

The science of mechanics is, in a broad sense, the study of those phenomena which depend upon the mutual actions of bodies in bulk. Thus the study of the behavior of a railway car under the combined action of the pull of the locomotive and the drag of the track belongs to the science of mechanics. The study of the behavior of a magnet in the neighborhood of an electric circuit belongs to the science of mechanics. The study of the behavior of two electrically-charged bodies belongs to the science of mechanics.

Simple mechanics is the study of ordinary bodies at rest or in motion, and one of the most important ideas in the science of simple mechanics is the idea of force; but the science of mechanics is not concerned with, and it sheds no light upon the question as to the exact physical nature of force. Thus, the science of mechanics is not concerned with the question as to the nature of the action which takes place in a gas causing the gas to exert a force on a piston; the science of mechanics is not concerned with the question as to the nature of the action which takes place in the material of a stretched spring causing the spring to exert a force; the science of mechanics is not concerned with the nature of the action between the earth and a heavy weight causing the earth to exert a force on the weight; the science of mechanics is not concerned with the nature of the action which takes place between two bodies which slide over each other and which leads to the production of the force of friction. *It is sufficient for the science of mechanics that these actions are what may be called* **states of permanency** *of the respective systems.* Thus, to say that a gas in a given cylinder pushes with a force of 100 pounds on a piston, is to specify a definite result of a definite condition or state of the gas, and it is this definite result that is important rather than the details of the physical action which is taking place in the gas. In fact, *the science of mechanics owes its existence to the legitimacy and usefulness of the idea of force irrespective of the nature of the physical processes upon which force action depends.**

* A very remarkable discussion "On the Scope of Mechanical Explanation and on the Idea of Force" is given in Appendix B, pages 268–288, of Larmor's *Æther and Matter*, Cambridge, 1900.

Similarly, it is sufficient for the science of electro-mechanics that the physical actions which underlie electromotive force, electric current, magnetic field and electric field are what may be called *states of permanency;* thus, to say that a current of ten amperes flows through a wire is to specify a definite effect of a definite condition or state of the wire, and it is the correlation between the definite condition and the definite effect that is important rather than the details of the physical action which is taking place in the wire. In fact, *the science of electro-mechanics owes its existence to the legitimacy and usefulness of the ideas of electromotive force, electric current, magnetic field and electric field, irrespective of the nature of the physical actions upon which these various things depend.*

The superficial character * of the science of simple mechanics and of the science of electro-mechanics may be further exemplified as follows: Let us consider, on the one hand, the idea of tensile strength. A piece of steel is broken by a tension of 120,000 pounds per square inch, but the exact character of the action which takes place in the steel and which constitutes the tension of the steel, and the exact character of the physical action which takes place in the engine or motor which operates the testing machine and subjects the rod of steel to tension are not matters for consideration. Indeed, nothing at all is known fundamentally as to the physical action which constitutes the tension of a bar of steel. Let us consider, on the other hand, the idea of dielectric strength. A plate of glass is broken down by an electric field of 95,000 volts per centimeter, but the exact nature of the stress which constitutes the electric field and the exact character of the

* What has been said above concerning the scope of mechanics may be exemplified as follows: Simple mechanics is concerned with the correlation of measurable effects, such as the relationship between the size of a beam and the load it can carry, the size of a fly-wheel and the work it can do when it is stopped, the thickness and diameter of a boiler shell and the pressure which it can withstand, the size of a submerged body and the buoyant force which acts upon it, the size and shape of an air column and its number of vibrations per second, and so on. It is evident that such relations as these do not involve any consideration of the intimate nature of the physical actions which are taking place.

physical action which enables a voltaic cell or dynamo to exert the required electromotive force are not matters for consideration, although, as a matter of fact, much more is known concerning the nature of electric field than is known concerning the nature of mechanical stresses in substances like steel.

The science of mechanics, as stated above, deals with those phenomena which depend upon the mutual actions of bodies in bulk. The phenomena of chemical action and those physical phenomena which have to do with the minute details of physical processes, however, have been studied heretofore almost solely on the basis of the atomic theory. Thus, nearly the whole of chemistry is based on the atomic theory; the kinetic theory of gases is a branch of the atomic theory; the theory of crystal formation is a branch of the atomic theory; the study of the phenomena of electrolysis is a branch of the atomic theory; and the study of the phenomena of the discharge of electricity through gases is a branch of the atomic theory.

126. Electrons and ions.—The loss of electricity from a charged body has long been known to be due in part to a leakage of the electricity through the surrounding air and in part to a leakage of the electricity through the insulating supports of the charged body. That is to say, the air conducts electricity to some extent. The electrical conductivity of the air is ordinarily extremely small, but there are a number of influences which cause the air (or any gas) to become a fairly good electrical conductor. Thus, a gas becomes a fairly good conductor when its temperature is raised above a certain point; gas which is drawn from the neighborhood of a flame or electric arc, or from the neighborhood of glowing metal or carbon, is a fairly good conductor; gas which has been drawn from a region through which an electric discharge has recently passed is a fairly good conductor; and the passage, through a gas, of ultra-violet light, of Roentgen rays, or of the radiations from radio-active substances, causes the gas to become a fairly good conductor. The conduc-

tivity which is imparted to a gas by these various agencies may be destroyed by filtering the gas through glass-wool or by placing the gas for a few moments between electrically charged metal plates. This effect of filtration seems to show that the conductivity of a gas is due to something which is mixed with the gas, and the effect of the electric field (between two charged plates) shows that this something is charged with electricity and moves under the action of the field. "We are thus led to the conclusion that the conductivity of a gas is due to electrified particles mixed up with the gas, some positive, some negative. We shall call these electrified particles *ions* and the process by which a gas is made into a conductor we shall call the process of *ionization*."*

The *electron* † is a negatively charged particle of which the mass is about $\frac{1}{1800}$ of the mass of a hydrogen atom. Thus, the cathode rays consist of electrons which are thrown off from the cathode of the Crookes' tube at high velocity, the β-rays from a radio-active substance such as uranium are electrons which are expelled from the atoms of the substance at high velocity.

A *simple ion* is an atom of a gas from which a negatively charged electron has been detached, leaving the remainder of the atom positively charged. Thus, the canal rays in a Crookes' tube consist of simple ions positively charged, and the α-rays which are given off by a radio-active substance such as uranium consist of simple ions positively charged. A *compound ion* consists of a negatively charged electron or a positively charged simple ion to which one or more neutral atoms cling, thus forming a charged atomic aggregate.

Ionization by the electric field.—According to the kinetic theory of gases, a molecule of a gas travels on the average a certain distance between successive collisions with neighboring molecules. This distance is called the mean free path of the molecule. The mean free path of an electron in a gas is about $4\sqrt{2}$‡ times as great as the mean free path of a molecule of the

* See J. J. Thomson, *Conduction of Electricity Through Gases*, page 11.

† Called a *corpuscle* by J. J. Thomson.

‡ According to the kinetic theory.

gas, because of the very small size and great velocity of the electron, whereas the mean free path of a simple or compound ion is equal to or even less than the mean free path of a molecule of the gas. When a gas is subjected to an electric field by being placed between two oppositely charged metal plates, a certain amount of energy is imparted by the electric field to the electrons between successive collisions, and a much smaller amount of energy is imparted to the simple or compound ions between successive collisions (because of their shorter mean free path). If the energy imparted to an electron between successive collisions exceeds a certain value, the electron is able to ionize the atoms of the gas when it collides with them, producing at each collision a new electron and a simple ion. Similarly, if the energy imparted to an ion between successive collisions exceeds a certain value, the ion is able to ionize the atoms of the gas when it collides with them, producing at each collision a new ion and an electron. Thus, the electron must fall freely through a certain difference of potential (about 30 volts) in order to receive enough energy to ionize air molecules, and a positive ion must fall freely through a certain difference of potential (about 440 volts) in order to receive enough energy to ionize air molecules.

127. The electric spark in a gas.—When a gas is subjected to an electric field of which the intensity is sufficient to cause *both* * the electrons and the positive ions to ionize the gas, an extremely rapid increase in the number of electrons and ions takes place, and the result is the production of an electric spark. The mean free path of the positive ions in a gas is inversely proportional to the pressure of the gas so that the electric strength of a gas

* When the intensity of an electric field is sufficient to cause only the electrons to ionize the gas, then all of the electrons which are present in the gas flock towards the positive electrode forming new ions and new electrons on the way, and when they reach the positive electrode the action ceases except for the occasional formation of an electron by outside influences. When the electric field is sufficiently intense to cause electrons and positive ions both to produce ionization, then new ions and electrons are formed everywhere between the electrodes and the number of free ions and electrons increases indefinitely. It is a well-known fact that an electric field must continue to act for an appreciable time before a spark is produced.

should be approximately proportional to the pressure. This is, in fact, the case. Thus, the dielectric strength of air at normal atmospheric pressure is about 20,000 volts per centimeter, at a pressure of 10 atmospheres the strength is about 200,000 volts per centimeter, and at a pressure of 0.1 atmosphere, the dielectric strength is about 2,000 volts per centimeter. The dielectric strength of air reaches a minimum, however, at a pressure of about 2 millimeters of mercury and increases when the pressure falls below this value. An electromotive force, sufficient to produce a spark $\frac{1}{8}$ of an inch long in air at atmospheric pressure, will produce a discharge through 18 or 20 inches of air at 2 millimeters pressure.

The idea of dielectric strength is based on the assumption that the electromotive force required to produce a discharge is proportional to the length of the spark, so that the quotient, volts divided by spark length, may be a constant. This is only approximately true in gases under moderate or high pressure, and when the pressure is very low a greater electromotive force is required to strike across a short gap than is required to strike across a long gap. This curious behavior of gas at low pressure is illustrated by a famous experiment due to Hittorf. Two electrodes were sealed into the walls of two glass bulbs and the tips of the electrodes were one millimeter apart, as shown in Fig. 158. The two bulbs were connected together by a spiral tube 375 centimeters long, and, when the pressure of the gas in the bulbs was reduced to a very low value, the discharge took place through the long tube and not across the one millimeter gap space between the points of the electrodes.*

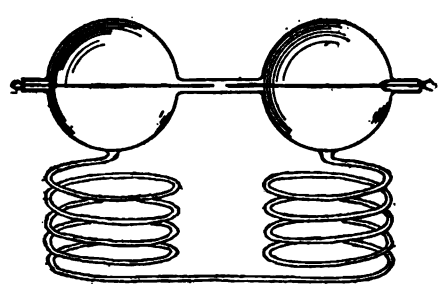

Fig. 158.

* This behavior of a gas at low pressure is fully explained by the atomic theory. See J. J. Thomson's *Conduction of Electricity Through Gases*, pages 430–527.

128. The Geissler tube and the Crookes tube. — The discharge of electricity through gases at low pressures is usually studied by means of a glass bulb through the walls of which are sealed platinum wires which terminate in metal plates called *electrodes*. The current enters at one electrode, the *anode*, and passes out at the other electrode, the *cathode*. This bulb, which is called a *vacuum tube*, is filled with the gas to be studied and the pressure is reduced to any desired value by exhausting the tube by means of an air pump.

Before exhaustion the discharge through the tube is in the form of a sharply-defined spark similar to the spark in the open air.

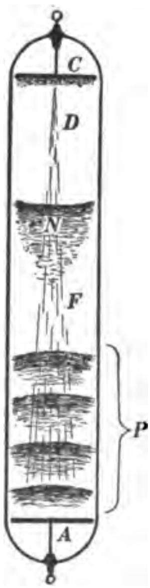

Fig. 159.

When the pressure of the gas in the bulb has been reduced to a few centimeters of mercury, the spark begins to be nebulous and a continued reduction of pressure causes the luminosity ultimately to fill the entire tube. When the pressure has been reduced to a few millimeters of mercury the discharge presents the following features, as shown in Fig. 159. There is a thin layer of luminosity spread over the surface of the cathode C, and beyond this there is a comparatively dark space D called the *Crookes dark space*, the width of which depends upon the pressure of the gas, increasing as the pressure of the gas diminishes. This Crookes dark space extends to a boundary which is approximately a surface traced out by lines of constant length drawn normally to the surface of the cathode. Beyond the Crookes dark space is a luminous region N called the *negative glow*, and beyond the negative glow is another comparatively dark region F which is called the *Faraday dark space*. Beyond the Faraday dark space is a luminous column P extending to the anode A and called the *positive column*. This positive column usually exhibits alternations of bright and dark spaces which are called *striations*. The effects here described are exhibited at their best in a vacuum tube in

which the pressure has been reduced to a few millimeters of mercury. Such a vacuum tube is called a *Geissler tube.* When the exhaustion of the vacuum tube is carried further, the dark space which surrounds the cathode (the Crookes dark space) expands until it fills the entire tube. The glass walls of the tube then show a yellowish-green or blue luminescence (according as the tube is made of soda glass or lead glass) and a slight negative glow may remain in portions of the tube remote from the cathode. These effects, which were first studied by Crookes in England and by Plücker and Hittorf in Germany, are exhibited at their best in a vacuum tube in which the pressure has been reduced to a few thousandths of a millimeter of mercury. Such a vacuum tube is called a *Crookes tube.*

129. Cathode rays and canal rays.—In order that a steady discharge may flow through a vacuum tube, it is necessary that the electric field intensity reach a value sufficient to impart to the positive ions enough energy between collisions to enable them to ionize the gas, because if the electrons (negative ions), only, produce ionization, the discharge through the tube ceases very soon after all of the negative ions have moved across to the neighborhood of the anode. In fact, ionization by positive ions must take place in the neighborhood of the cathode,* and it is this necessity which gives rise to the Crookes dark space. The action which takes place in the Crookes dark space is as follows: Electrons (negative ions) are thrown off from the cathode at very high velocity by the intense electric field in the Crookes dark space, very energetic ionization takes place in the negative glow N, Fig. 159, and the positive ions that are produced in this region attain sufficient velocity in traveling towards the cathode to enable them to ionize the gas in the immediate neighborhood of the cathode. That is, ionization by positive ions takes place in the faint glow which covers the cathode. The mutual dependence of the ionization which takes place in the negative glow and the

* A detailed discussion of this matter may be found in J. J. Thomson's *Conduction of Electricity Through Gases*, pages 529-603.

ionization which takes place in the faint luminosity in the immediate neighborhood of the cathode is shown by placing a small obstacle in the Crookes dark space. This obstacle screens a portion of the cathode surface from bombardment by the positive ions which move from the negative glow towards the cathode so that in the region so shaded ionization does not take place. In the same way the obstacle also screens a certain portion of the negative glow from bombardment by the electrons which are thrown from the cathode and this portion of the negative glow ceases to exist because ionization is no longer produced there. That is to say, the obstacle casts a shadow on the cathode and it also casts a shadow into the negative glow.

The electric field intensity in the Crookes dark space, being necessarily sufficient to enable the positive ions to produce ionization at the surface of the cathode, is able to impart very much greater velocity to the electrons than is necessary to enable them to produce ionization. The result is that the electrons which are thrown off from the cathode travel in straight lines through a long portion of the tube. These high velocity electrons constitute what are called *cathode rays*. The cathode rays are faintly visible throughout the tube because of occasional collisions with the molecules of the gas.

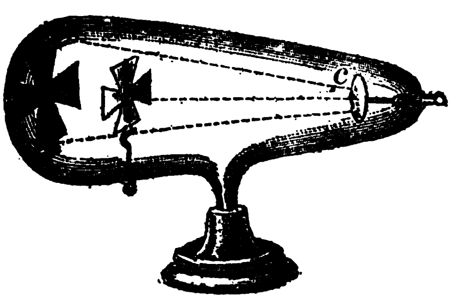

Fig. 160.

When the cathode has a small hole through it, the positive ions which move towards the cathode from the negative glow pass through this hole in the form of a stream of rays which is made visible by the luminosity which accompanies the collisions of the positive ions with the molecules of the gas. Such a stream of positive ions constitutes what has been called the *canal rays*.

An object of any kind placed in the Crookes tube casts a sharp shadow upon the wall of the tube, as shown in Fig. 160.

The wall of the tube shows a brilliant luminescence everywhere except where it is screened by the obstacle from bombardment by the cathode rays.

Magnetic deflection of cathode rays and canal rays. — A moving charged body is equivalent to an electric current, and when a charged body moves across a magnetic field the magnetic field pushes sidewise upon the charged body and causes the charged body to describe a curved path. The magnetic deflection of the cathode rays is easily shown by placing a horse-shoe magnet with its poles placed on opposite sides of the tube shown in Fig. 160. The shadow of the cross is thrown up or down according to the arrangement of the magnet. The magnetic deflection of the canal rays is very slight; a very strong magnetic field is necessary to produce a perceptible deflection. The direction of the magnetic deflection of the cathode rays shows that these rays are negatively charged particles, and the direction of the magnetic deflection of the canal rays shows that these rays are positively charged particles. The magnitude of the deflection of the cathode rays shows that the mass of the cathode particles (electrons) is very small and that their velocity is very great. The magnitude of the deflection of the canal rays shows that the mass of the canal ray particles is relatively great and that their velocity is less than the velocity of the cathode rays. This matter is explained in detail in Art. 135.

An object upon which the cathode rays * impinge is heated, it may be, to a very high temperature. Many substances, however, emit light (without being made perceptibly hot) when subjected to bombardment by the cathode rays. Such substances are said to be luminescent. For example, lead sulphate emits a deep violet light, zinc sulphate emits white light, magnesium sulphate, with a slight admixture of manganese sulphate, emits a deep red light under the action of cathode rays.

* The cathode rays produce effects which are practically important and which can be easily observed. The effects of the canal rays, however, are so slight as to be scarcely perceptible even under the most favorable conditions. Therefore further discussion of the canal rays is not warranted in this brief outline.

The cathode rays pass quite readily through thin metal plates especially through thin plates of aluminum. By using a Crookes tube of which a portion of the wall is made of thin sheet aluminum, the cathode rays may be made to pass through into the outside air. The properties of cathode rays in the air were first studied by Lenard who found that they produce a very high degree of ionization of the air making it a fairly good electrical conductor. Lenard found the cathode rays capable of traversing from 10 to 20 centimeters of air at atmospheric pressure, he found them capable of exciting luminescence, and he found them capable of affecting a sensitive photographic plate.

130. The Roentgen rays. — Objects upon which the cathode rays impinge, not only become heated and luminescent as described above, but they emit a type of radiant energy which was discovered by Roentgen in 1894. Roentgen rays are of the same physical nature as light rays, that is, they consist of waves in the ether, and they are related to light waves very much as a sharp "razor" wave on the surface of water would be related to a long ocean swell, as shown in Fig. 161. Helmholtz pointed out in

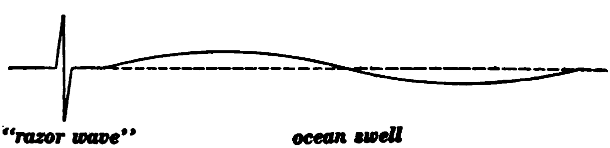

Fig. 161.

1891 that abrupt wave pulses of this kind in the ether would have certain properties, the properties, in fact, which are exhibited by Roentgen rays, as follows: These rays are not reflected in a regular way by the surface of a mirror, and they are not refracted by a lens. They pass through all substances, subject to a certain amount of absorption which is greater the greater the density of the substance, and subject to a certain amount of diffused scattering. The Roentgen rays affect an ordinary photographic plate and they have a powerful ionizing effect on gases.

The fluoroscope. — Many substances such as barium platinocy-

Fig. 162.

anide and calcium tungstate become luminescent under the action of Roentgen rays. This effect is utilized in the fluoroscope which consists of a cardboard screen covered with a layer of barium platinocyanide. When the Roentgen ray shadow of an object, such as the hand, falls on this screen the shadow becomes visible; where the Roentgen rays have been greatly reduced in intensity by the bones of the hand the screen remains dark, where the Roentgen rays have been slightly reduced in intensity by the flesh the screen is moderately luminous, and where the rays have not been reduced at all in intensity the screen is highly luminous. The Roentgen ray shadow of an object may be rendered visible by allowing it to fall upon a photographic plate which is afterwards developed like an ordinary photographic negative. Thus, Fig. 162* is a reproduction of a shadow photograph of a wrist.

The focusing tube. — In order that a shadow may be sharply defined the radiation which produces the shadow must emanate from a very small source. Figure 163 shows a Crookes tube

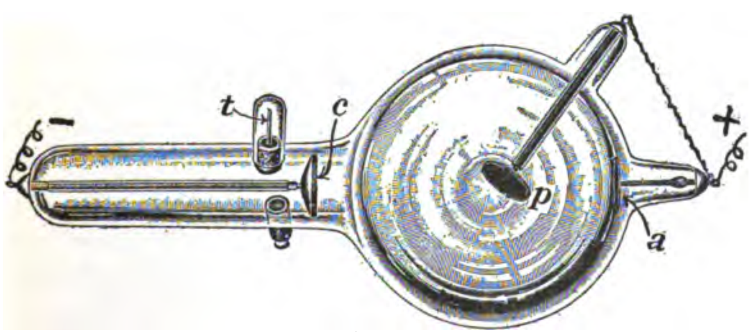

Fig. 163.

with a concave cathode *c* from which the cathode rays converge and strike a small spot on a platinum plate *p*. This small spot is the source of the Roentgen rays. Such a Crookes tube is called a focusing tube, and, by the use of such a tube, very sharply

* From a negative by Dr. E. W. Caldwell, President (1908) of the American Roentgen Ray Society.

defined Roentgen ray shadows may be produced. The platinum plate *p* is usually connected as shown to the aluminum anode *a*. An interesting feature of the Crookes tube, which is shown in Fig. 163, is the small platinum tube *t* which is sealed through the glass wall. When the vacuum in the Crookes tube becomes too high (presumably by the transformation of the residual gases into non-volatile products), the small tube *t* is held for a few seconds in the flame of an alcohol lamp and a sufficient amount of hydrogen passes through the hot platinum to replenish the supply of gas in the Crookes tube.

131. Conductivity of hot gases and flames. — A hot gas is a fairly good electrical conductor and this conductivity has been found to be due to the presence of free ions.* The conductivity of a hot gas or flame is shown by the fact that a charged glass rod may be completely discharged by passing the flame of a Bunsen burner rapidly over its surface.

132. The electric arc. — In order to produce a perceptible discharge of electricity (flow of current) through a gas, a very high electromotive force must be used because of the necessity of producing ionization in the gas by the collision of the moving ions with the gas molecules; and the amount of current which can be made to flow through a gas is usually very small because of the comparatively small number of these ions. When, however, metal or carbon electrodes are heated to a very high temperature they emit electrons (negative ions) in great numbers † and a very considerable current may then be made to flow through the intervening gas. Thus, a current of an ampere or more may be made to flow between a *cold metal anode* and a very *hot metal cathode* in a vacuum tube. When two carbon rods are connected to a battery or dynamo, brought into contact and then separated, the current which begins to flow across the indefinitely small gap between the two carbon rods raises the tips of the carbons to a

* See J. J. Thomson's *Conduction of Electricity Through Gases*, pp. 228–249.
† See J. J. Thomson's *Conduction of Electricity Through Gases*, pp. 188–227.

very high temperature so that electrons (negative ions) are emitted in great numbers. The result is that the current continues to flow between the carbon tips. The column of hot vapor between the carbon tips is called an *electric arc*, and the intense heating of the two carbon tips is due to their bombardment by the ions which move across the arc and carry the electric current. The electric arc may be easily maintained between a hot negative carbon (cathode) and a rapidly rotating disk (a cold anode), but not between a cold cathode and a hot anode. This shows that the emission of negative ions (electrons) by the hot carbon is essential to the formation of the electric arc. The appearance of the arc between carbon electrodes is shown in Fig. 164.*

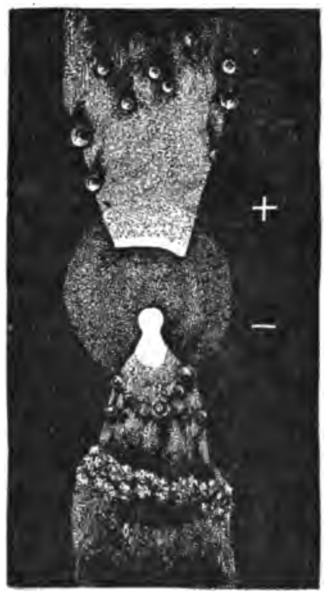

Fig. 164.

133. Chemical effect of the discharge through gases. — The discharge of electricity through gases is accomplished by the ionization of the gas as above explained. This ionization means not only the breaking down of the molecules of a compound gas but also the separation of electrons from the individual atoms of the constituents of the compound gas. The ionization of mixed gases promotes chemical combination. Thus, the nitrogen and oxygen of the air combine slowly under the action of the electric spark.

When oxygen (or air) is ionized, the recombination of the oxygen ions results in the production of ozone. Thus the

* The properties of the electric arc are discussed in great detail in a paper by C. P. Steinmetz, *Trans. International Electrical Congress*, Vol. II, pages 710–730, St. Louis, 1904; in a paper by W. R. Whitney, *Trans. American Electrochemical Society*, Vol. 7, pages 291–299, 1905; and in J. J. Thomson's *Conduction of Electricity Through Gases*, pages 604–620.

peculiar odor which is given off by a Toepler-Holtz machine or a Wimshurst machine is due to the ozone which is formed. The action which takes place in the formation of ozone from oxygen is as follows: Ordinary oxygen is bi-atomic, that is, it contains two atoms of oxygen in the molecule. Ionization causes the disintegration of these bi-atomic molecules forming mono-atomic oxygen, and this mono-atomic oxygen recombines forming a large proportion of bi-atomic oxygen again and a small proportion of tri-atomic oxygen, or ozone. In the production of ozone for commercial purposes a blast of air is driven between two metal plates which are connected to a high voltage alternator. The repeated reversals of the high electromotive force between the plates ionizes the intervening air repeatedly, and the recombination of the ions is accompanied by the formation of a certain percentage of ozone, as above explained. In order to produce a high degree of ionization throughout the entire region between the two metal plates, it is necessary to place a thin plate of glass between the metal plates so as to prevent the formation of a single spark from plate to plate. The effect of this glass plate is to cause a fine brush discharge to take place throughout the entire region. Without the glass plate a single brilliant spark passes through the air. With the glass plate, a diffused violet luminosity is produced throughout the region between the metal plates.

134. Radio-activity.* — The chemical elements uranium, thorium, and radium and their compounds have the property of making a surrounding gas an electrical conductor. Thus, one ten-millionth of a gram of radium bromide which is left as a residue upon a metal plate by evaporating a small quantity of a dilute solution of radium bromide on the plate, causes a gold leaf electroscope to be discharged in a few seconds when the

* The student is referred to the following books for a full discussion of radioactivity: *Radioactivity*, by E. Rutherford, Cambridge, 1905 (second edition); *Radioactivity*, by Frederick Soddy, London, 1904; and *Radioactive Transformations*, by E. Rutherford, New York, 1906.

radium-covered plate is held near to the metal plate of the electroscope. Uranium and thorium have the same effect but the discharge which they produce is not so rapid unless a large quantity of material is employed. This property of these metals and of their compounds is called *radio-activity*, a name which originated because of the peculiar radiations which are given off by radio-active substances and to which the discharging action is due. These radiations are of three distinct kinds, which are called the α-rays, the β-rays, and the γ-rays, respectively. The γ-rays penetrate through a foot or more of solid metal or through many feet of air; the β-rays penetrate through a moderate thickness of a light metal, such as aluminum; whereas the α-rays are stopped by a very thin layer of aluminum or by a layer of air two or three inches in thickness.

The α-rays consist of positive ions each about twice as massive as a hydrogen atom. These ions are projected from the radio-active substance at a velocity of about 20,000 miles per second, and each of them ionizes about 100,000 air molecules before it is brought to rest by repeated collision. After traveling two or three inches through the air, the velocity of these α-particles is reduced to so low a value as to render them no longer perceptible by their ionizing effects.

The β-rays consist of electrons (negative ions) each about $\frac{1}{800}$ as massive as a hydrogen atom. These electrons are projected from the radio-active substance at a velocity which in some cases is nearly as great as the velocity of light (186,000 miles per second). The β-particles also have the property of ionizing the gas through which they pass but not to so great an extent as the α-particles, and they travel several feet through the air before their velocity is reduced to so low a value as to render them no longer perceptible by their ionizing effects.

The γ-rays are extremely abrupt waves in the ether essentially the same in character as Roentgen rays, but much more penetrating than ordinary Roentgen rays. The γ-rays also have the property of ionizing a gas.

The α-rays and the β-rays are deflected by the magnetic field and by the electric field. The direction of the deflection of the α-rays is in each case opposite to the direction of deflection of the β-rays, and therefore it is known that the α-particles are positively charged and that the β-particles are negatively charged. The γ-rays are not deflected by a magnetic field or by an electric field.

The present hypothesis regarding radio-activity is that the atoms of all substances are complex systems of excessively small particles called *electrons*, the atom of each element being a characteristic self-contained group or system of electrons in very violent orbital motion. These systems of electrons (atoms) are supposed to be to some extent unstable, and when instability occurs, the system (atom) collapses into a new configuration and at the same time expels one or more positively or negatively charged electrons or groups of electrons which constitute the α-rays and the β-rays. According to this hypothesis the γ-rays consist of abrupt ether waves which are produced by the sudden collapse of the atomic structure when instability occurs.*

A clear representation of the nature of α-, β-, and γ-rays is shown in Fig. 165. Imagine an atom of the radio-active material to collapse at a given instant sending out a γ-wave, an α-particle, and a β-particle. The relative positions reached by the γ-wave, the α-particle, and the β-particle at a given instant are shown in the figure. The α-particle is a large positively charged particle and the β-particle is a small negatively charged particle.

135. Determination of velocity and mass of the particles which constitute canal rays (or α-rays) and cathode rays (or β-rays). — A narrow stream of rays from a radio-active substance may be

* A very instructive discussion of the electron theory is given by Sir Oliver Lodge in a book entitled *Electrons*, published by Geo. Bell & Sons, London, 1906. The method of measuring the degree of radio-activity of a radio-active substance is explained in Franklin, Crawford and MacNutt's *Practical Physics*, Vol. 2, pages 148–153. An example of the study of a radio-active transformation, that is, of the change which takes place in the radio-active substance, as a result of its radio-activity, is given in Franklin, Crawford and MacNutt's *Practical Physics*, Vol. 2, pages 154 and 155.

obtained by the arrangement shown in Fig. 166 in which AB is a sensitive photographic plate upon which the narrow stream of

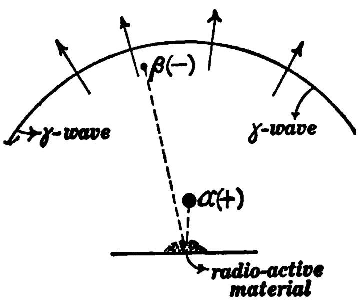

Fig. 165.

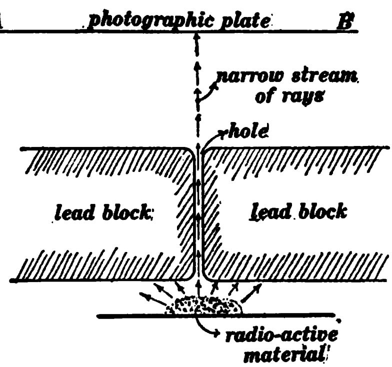

Fig. 166.

rays impinges. Figure 167 shows the effect of an electrical field upon a thin stream of rays from a radio-active substance. The direction of the electric field is shown by the fine horizontal arrows

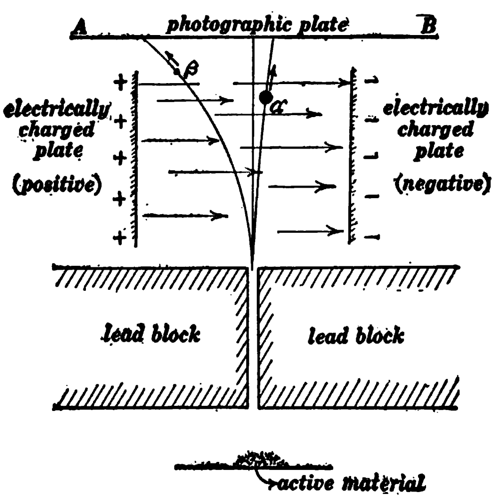

Fig. 167.

(the lines of force of the electric field pass from the positively charged plate to the negatively charged plate). The effect of the

electrical field is to deflect the α-particles in the direction of the field and the β-particles in the opposite direction, while the γ-rays are not affected at all. The amount of deflection in each case may

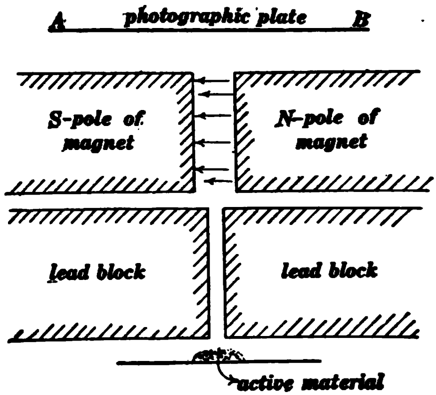

Fig. 168.
α-particles deflected towards the reader.
β-particles deflected away from the reader.
γ-waves not deflected at all.

be determined by developing the photographic plate upon which the rays impinge. The effect of the magnetic field upon the rays from a radio-active substance is shown in Fig. 168 in which the fine horizontal lines represent the lines of force of a magnetic field between the two large magnet poles.

The determination of the velocity of the α- and β-particles is somewhat analogous to the following method for determining

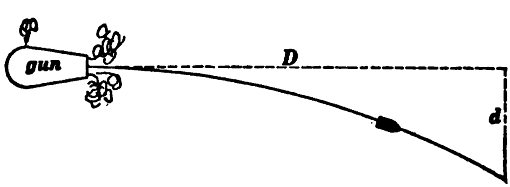

Fig. 169.

the velocity of a cannon ball. The curved line in Fig. 169 represents the orbit of a cannon ball, D being the horizontal distance traveled by the ball in a given time and d being the

vertical distance fallen by the ball under the action of gravity. If D is known and d observed, then the velocity of the cannon ball is given by the equation

$$v^2 = \frac{gD^2}{2d} \tag{i}$$

in which g is the acceleration of gravity.

Action of the electrical field on a moving charged particle.—Consider a charged particle moving upwards through an electrical field as shown in Fig. 170. Let q be the charge on the

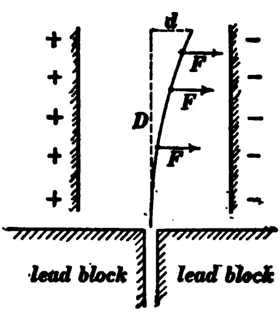

Fig. 170.

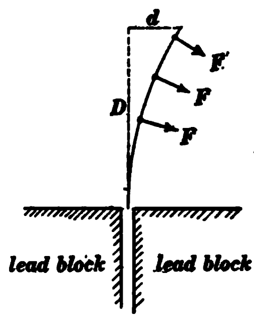

Fig. 171.
Magnetic field perpendicular to plane of paper.

particle in abcoulombs and e the intensity of the electrical field in abvolts per centimeter. Then the force F in dynes pulling on the particle is equal to qe, so that the acceleration of the particle in the direction of F is qe/m. It is evident that the particle moves in the same sort of an orbit as a cannon ball, and that the acceleration qe/m corresponds to the acceleration of gravity g in the case of a cannon ball. Therefore, using qe/m for g in equation (i), we have

$$v^2 = \frac{D^2qe}{2dm} \tag{ii}$$

or

$$\frac{m}{q} = \frac{D^2e}{2dv^2} \tag{iii}$$

Action of the magnetic field on a moving charged particle.—Figure 171 represents a charged particle moving upwards through a magnetic field, the lines of force of which are perpendicular to the plane of the figure. The moving particle is equivalent to an electric current, and the side force F is equal to qvh where q is the charge on the particle in abcoulombs, v is its velocity in centimeters per second, and h is the intensity of the magnetic field in gausses. Therefore the acceleration of the particle in the direction of F is qvh/m. The force F is continuously at right angles to v so that the particle describes a circular orbit. But the acceleration of a particle moving in a circular orbit is v^2/r, and the relation between the radius of the circle r, the semichord D, and the versed sine d is

$$r = \frac{D^2}{2d}$$

Therefore we have

$$\frac{qvh}{m} = \frac{2dv^2}{D^2}$$

whence

$$\frac{m}{q} = \frac{hD^2}{2dv} \tag{iv}$$

Determination of velocity of particles.—Reduced to the simplest terms, the method of determining velocity may be described as follows: An electrical field e in the plane of the paper, Fig. 170, and a magnetic field h at right angles to the plane of the paper in Fig. 170 are adjusted so that together they produce no deflection of the particles which are being studied. When this condition is realized, the force qe with which the electrical field acts on the moving particles is equal and opposite to the force qvh with which the magnetic field acts on the moving particles, so that, disregarding sign, we have

$$qe = qvh$$

or

$$v = \frac{e}{h} \tag{v}$$

that is to say, the velocity of the particles is equal to the ratio of the electric field intensity e in abvolts per centimeter to the magnetic field intensity h in gausses, on the condition that the combined action of the fields produces no deflection of the moving particles.

"*Electrochemical equivalent*" *of* α- *and* β-*particles.* — According to the dissociation theory of electrolysis each atom of hydrogen, for example, in a dilute solution of sulphuric acid is isolated and carries a definite amount of charge, and the ratio (m/q) of the mass m of a hydrogen atom (ion) to the charge q upon it is equal to the electrochemical equivalent of hydrogen, or, in other words, to the number of grams of hydrogen which are liberated during the passage of one coulomb of electric charge through an electrolytic cell containing dilute sulphuric acid. The ratio (m/q) of the mass of a gas ion to the charge upon the ion is called the "electrochemical equivalent" of the gas ion. This ratio is determined by equation (iii) or (iv) when the electric or magnetic deflection of the particle has been observed and when the velocity of the particle is known. The value so determined is given in grams per abcoulomb and it is equal to 5.36×10^{-8} grams per abcoulomb for the β-particles (electrons), from which it follows that the particles have a mass $\frac{1}{800}$ as great as the mass of a hydrogen atom if the charge q is the same in both cases.*

* In regard to the equality of charge on mono-valent ions in electrolytes and on gas ions, see Oliver Lodge's *Electrons*, pages 77–90, where a simple account is given of the work which has been done by J. J. Thomson in the determination of the value of q (or m).

CHAPTER IX.

ELECTRIC OSCILLATIONS AND ELECTRIC WAVES.

136. Mechanical conceptions of the magnetic and electric fields.* —The foregoing chapters are devoted to the discussion of the phenomena of the electric current and the phenomena exhibited by electrically charged bodies. The phenomena of electric oscillations and especially the phenomena of electric waves have not as yet been touched upon. It is usual to treat these phenomena on the basis of the differential equations of the electro-magnetic field, but it is needless to say that this mode of treatment cannot be followed in an elementary text. The most satisfactory elementary treatment of electric oscillations and electric waves is to develop the mechanical conceptions of the magnetic and electric fields and thus arrive at a rational insight into electro-magnetic phenomena. This method is followed in this chapter.

Maxwell was the first to work out mechanical conceptions of magnetic and electric fields, and Maxwell's conceptions are used in the present chapter † although certain inconsistencies arise in the attempt to extend these conceptions to three dimensions.

* Sir Oliver Lodge's *Modern Views of Electricity* is perhaps the best elementary treatise on this subject. This book is now (1908) being rewritten.

† The most complete mechanical conception of the electro-magnetic field is that which is based upon Lord Kelvin's gyrostatic model of the ether. This gyrostatic model of the ether is a mechanical structure which is capable of reproducing most of the known phenomena of electricity and magnetism and of light. See *Æther and Matter*, by Joseph Larmor, Appendix E, Cambridge, 1900. Lord Kelvin's gyrostatic model of the ether has led to a hydrodynamic conception of the ether, due chiefly to Larmor, in which the ether is assumed to be a perfect fluid which is endowed with the necessary elastic properties by an indefinitely fine grained whirling motion. On the basis of Lord Kelvin's gyrostatic conception of the ether and also on the basis of Larmor's turbulent ether, the magnetic field is thought to consist of a simple flow of the ether along the lines of force of the magnetic field. This conception of the magnetic field is very different from the conception which is outlined in this text and which is based upon Maxwell's conception of the ether.

137. Maxwell's mechanical model of the ether. — The ether is to be considered as built up of very small cells of two kinds, positive and negative, in such a way that only unlike cells are in contact. These cells are imagined to be gear wheels provided with rubber-like teeth, as shown in Fig. 172, so that if a cell be turned while the adjacent cells are kept stationary, then a torque due to elastic distortion of the gear teeth is brought to bear upon the turned cell. In subsequent figures, these cells or cog-wheels are represented by plain circles for the sake of simplicity.

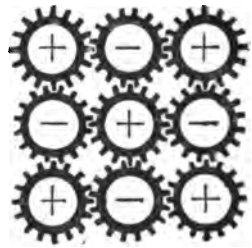

Fig. 172.

Conception of the magnetic field.—The ether cells at a point in the magnetic field are thought of as rotating about axes which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are geared together. This rotatory motion of the ether cells is shown in Fig. 173, which represents a magnetic field perpendicular to the plane of the paper and directed away from the reader; all the positive cells are rotating clockwise and all the negative cells are rotating counter-clockwise. The energy of the magnetic field (see Art. 44) is represented by the kinetic energy of rotation of the ether cells.

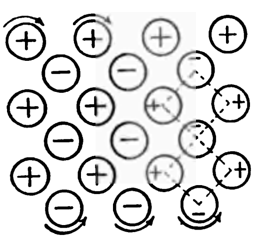

Fig. 173.

Conception of the electric field. — The positive ether cells at a point in an electric field are thought of as being displaced in the direction of the field, while the negative cells are displaced in the opposite direction, and this displacement is assumed to be pro-

portional to the electric field intensity. Thus, Fig. 174 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells as shown by the arrows, that is to say, the distortion of the ether structure in Fig. 174 represents an electric field directed toward the bottom of the page. Figure 175 represents two meshes of the cellular structure of the ether. These two meshes are square in the undistorted ether, as shown in Fig. 173, whereas the downward displacement of the positive cells in Fig. 174 has distorted these meshes, as shown in Figs. 174 and 175. Inasmuch as the cell structure of the ether is assumed to be elastic (the gear teeth in Fig. 172 being made of a substance like rubber), the distortion of the ether structure which is shown in Fig. 174 represents potential energy and this energy is the energy of the electric field (see Art. 104).

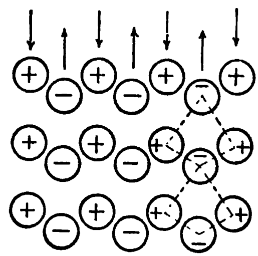

Fig. 174.

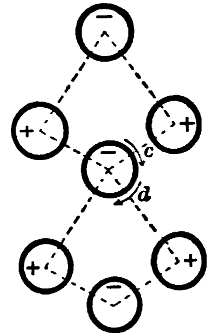

Fig. 175.

Nearly the whole of the following discussion is based upon the torque action which is exerted upon a given cell by the elastic distortion which is represented in Fig. 175. *This torque action is the connecting link between the electric field and the magnetic field and a clear understanding of it is of the utmost importance.* Consider the two positive cells to the right of the middle cell in Fig. 175. Inasmuch as these two positive cells have been displaced downwards with respect to the middle cell, they exert torques upon the middle cell as shown by the arrows *c* and *d*, and these torques are proportional to the intensity of the electric field, that is, to the downward displacements of the cells. The two positive cells to the left of the middle cell in Fig. 175 exert torques which are equal to *c* and *d* respectively, but opposite in direction.

138. The energy stream in the electromagnetic field. — A region in which electric field and magnetic field co-exist may be called an electromagnetic field for the sake of brevity. It has been shown by J. H. Poynting * from theoretical considerations that energy streams through an electromagnetic field in a direction which is at right angles both to the electric field and to the magnetic field at each point, and that the amount of energy per second which streams across one square centimeter of area is proportional to the product of the electric and magnetic field intensities. In case the electric and magnetic fields are not at right angles to each other, the energy stream is proportional to the product of the intensities of the two fields and the sine of the included angle.

Conception of the energy stream. — Consider a row of gear wheels as shown in Fig. 176. Imagine the wheel W to be

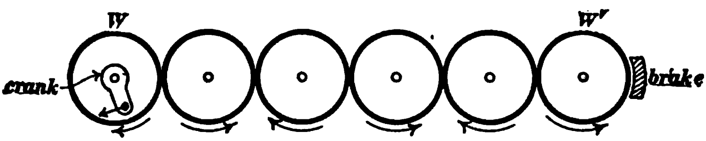

Fig. 176.

turned steadily by a crank, and the wheel W' to be hindered by a brake. The result is that energy is continuously transmitted along the chain of gear wheels from W to W', any given gear of the chain is acted upon by equal and opposite torques by the gear wheels on each side of it, the transmission of energy by the chain depends upon this torque action combined with the motion of the wheels, and the rate at which energy is transmitted along the chain is proportional to the product of the speed of the wheels and the torque action between adjacent wheels.

Imagine the ether cells in Fig. 174 to be rotating, positive cells in one direction, negative cells in the other, about axes perpendicular to the plane of the paper. This rotatory motion constitutes a magnetic field perpendicular to the plane of the paper and perpendicular to the electric field which is towards the bottom of

* See *Philosophical Transactions*, Vol. 175, Part II, page 343, 1884.

the page. On account of the torque action between the cells, as explained in connection with Fig. 175, energy will be transferred to the right (or left) by each horizontal chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field and the intensity of the electric field; and the energy per second flowing across an area (of which the normal is perpendicular to both electric field and magnetic field) is proportional to the product of the respective field intensities and proportional to the area, inasmuch as the area is proportional to the number of rows of cells which are acting as chains of gear wheels. Therefore the energy stream, that is, energy per unit area per second, is proportional to the product of magnetic and electric field intensities and it is at right angles to both.

139. The electric current. — Consider a wire AB, Fig. 177, along which an electric current is flowing from B towards A. The magnetic field on opposite sides of AB is in opposite directions, so that the positive ether cells at p and p' are rotating in opposite directions as shown. An electric current may be maintained for an indefinite length of time, but the opposite rotation of

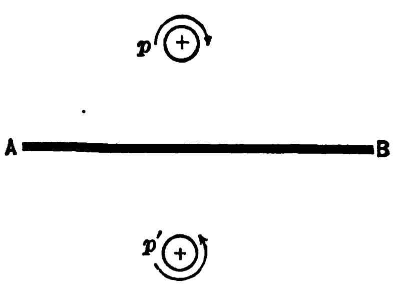

Fig. 177.

positive ether cells on the two sides of AB, Fig. 177, cannot be accommodated by an ever-increasing ether distortion (distortion of the rubber-like teeth of the ether cells as shown in Fig. 172), there must be a *slip* between adjacent cells somewhere between p and p'. This *slip* between adjacent ether cells takes place in the material of the wire and constitutes an electric current.

Steady electric currents flow in closed circuits. — Let AB, Fig. 178, be a wire * in which a steady electric current is flowing from

* In Fig. 178, as in all subsequent figures, a *wire* is to be thought of as an *indefinitely broad metal sheet*, because the cellular conception of the ether is not adapted to three dimensions.

B towards A. Consider the opposite rotation of like ether cells at p and p', and consider a chain of geared cells passing from p to p' around the end of AB. The current through AB may flow for an indefinite time and therefore the opposite rotation of the positive ether cells at p and p' may continue indefinitely, but this continued opposite rotation at p and p' cannot be accommodated by an ever-increasing distortion of the elastic gear teeth of the ether cells along the chain of geared cells which pass around the end of AB. A slip must take place between adjacent cells at some point along this chain. *Therefore the line of flow of the current AB (line of slip of gear cells) must form a closed circuit which cuts across every possible chain of geared cells extending from p to p'.*

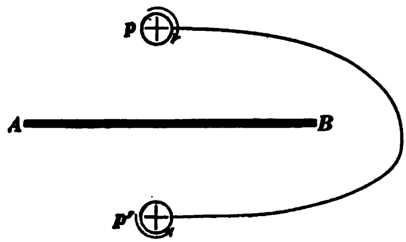

Fig. 178.

When a current flows along a path which *does not form* a closed circuit, then an increasing ether distortion (electric field) is produced around the end portions of the path as explained in Art. 142.

140. Flow of energy in the neighborhood of a wire carrying an electric current. (*a*) *Simplest case, when no electric charge resides on the surface of the wire.*—Let AB, Fig. 179, be a portion of a long wire through which an electric current is flowing. If there

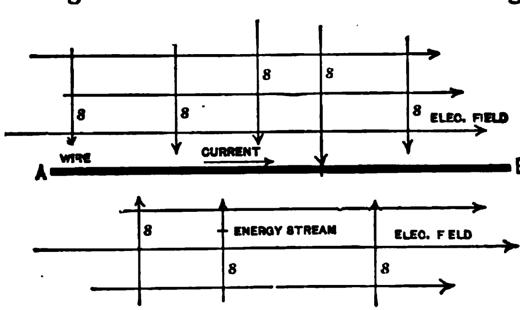

Fig. 179

is no electric charge on the surface of the wire, then the electric field in the neighborhood of the wire is parallel to the wire. The lines of force of the magnetic field, on the other hand, encircle the wire, and therefore the energy streams in towards the wire and on all sides, and is converted into heat in the wire.

Let R be the resistance of the wire in abohms per centimeter of length, and let I be the current in the wire, in abamperes, then RI is the intensity of the surrounding electric field* in abvolts per centimeter. According to Art. 55, the intensity of the magnetic field at a distance of r centimeters from the wire is $2I/r$ gausses. The intensity of the energy stream (units of energy per unit of area per second) at a distance of r centimeters from the wire is proportional to the product of the electric field and magnetic field intensities, and it may therefore be written $k \times RI \times 2I/r$, where k is an unknown proportionality factor. Multiplying this expression for the intensity of the energy stream by the area of a cylindrical surface l centimeters in length and r centimeters in radius (co-axial with the wire), we have the total energy per second streaming in to l centimeters of the wire, and this must be equal to $l \times RI^2$. Therefore, we have

$$2\pi rl \times k \times RI \times \frac{2I}{r} = l \times RI^2$$

whence

$$k = \frac{1}{4\pi}$$

Therefore we have

$$S = \frac{1}{4\pi} \cdot Hf \tag{77}$$

in which S is the energy in ergs per second which streams across one square centimeter of area at right angles to a magnetic field of which the intensity is H gausses and at right angles to an electric field of which the intensity is f abvolts per centimeter, H and f being at right angles to each other.

* The intensity of that component of the electric field which is parallel to the wire.

(*b*) *General case, when electric charge resides on the surface of the wire.* — The component of the electric field which is parallel to the surface of a wire is always equal to the RI drop per centimeter along the wire, but the component of the electric field at right angles to the surface of the wire may have any value whatever, and the electric lines of force which terminate on the surface of the wire on account of the existence of this normal component of electric field involve a stationary* electric charge on the surface of the wire. An example of this general case is shown in Fig. 180. An electric generator G delivers current

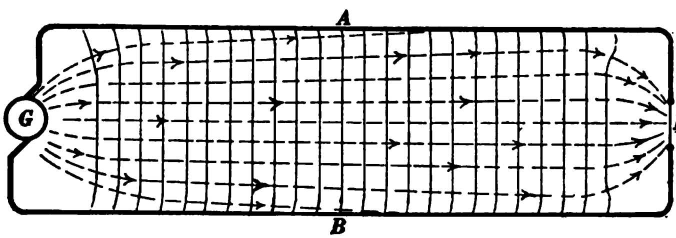

Fig. 180.

over two line wires† A and B to a distant lamp L. The electromotive force across from A to B involves the existence of an electric field the lines of force of which trend somewhat as shown by the full-line curves in the figure. The magnetic field between A and B is everywhere perpendicular to the plane of the paper and everywhere of the same intensity, so that the energy stream lines are a series of lines which are everywhere at right angles to the electric lines of force. The electric lines of force where they touch A and B are slightly inclined to the

* The electric current may be considered to be a transfer of electric charge along the wire but the charge here referred to has nothing directly to do with the current. When a voltaic cell is on open circuit, the electric field in the surrounding region may be such that the volts per centimeter along a given path may vary in the most irregular way; but when this path is occupied by a wire through which the voltaic cell produces a current, then the electric field is modified by the charge on the surface of the wire so as to make the component of the electric field parallel to the wire everywhere equal to the RI drop per centimeter along the wire.

† In order that Fig. 180 may be a complete representation, A and B must be supposed to be broad metal bands.

surfaces of A and B as shown in the figure, the degree of inclination depending upon the RI drop along A and B. Therefore the energy streams out from the generator through the whole of the region between A and B, and, although the energy stream turns in slightly on each line wire, the main portion of the energy converges on the distant lamp L, as shown by the dotted lines in Fig. 180. No attempt is made in Fig. 180 to represent the electric field distribution in the neighborhood of the generator.

141. The charge on a condenser and its disappearance when the condenser plates are connected by a wire.— Consider a closed chain of gear wheels AB, Fig. 181. If the gears are allowed to slip at any point s, the gear f being held stationary and the gear e being turned in the direction of the arrow, then the chain of gears will be distorted as shown in Fig. 182. Conversely, a chain of geared wheels which by elastic action tend to stand in a smooth row,* will be relieved from such a zigzag distortion as is shown in Fig. 182 by permitting the gears to slip at any point, s and the potential energy stored in the distorted chain will be geared towards s from both sides.

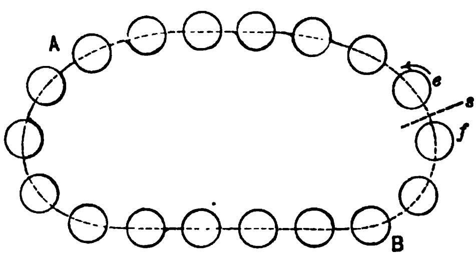

Fig. 181.

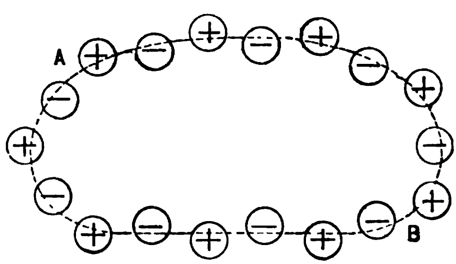

Fig. 182.

* The chains of positive and negative ether cells are thought of as standing in zigzag rows when undistorted, as shown by the horizontal rows in Fig. 173. Hereafter the chains of ether cells are to be thought of as straight (or uniformly curved) when free from distortion, in order that the diagrams may be simpler.

Let *A* and *B*, Fig. 183, be two metal plates, and let the dotted lines represent closed chains of geared ether cells, each chain being like Fig. 181. Imagine the two plates *A* and *B* to be connected by a wire, and an electric current to be forced through this wire by means of a battery, thus causing the plates *A* and *B* to become charged. The forcing of the current through

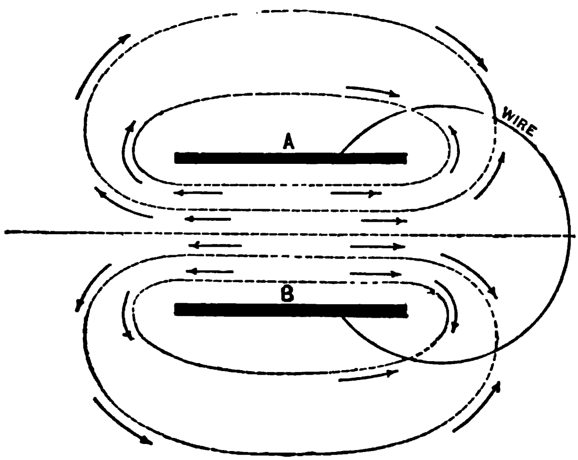

Fig. 183.

the wire means a forced slipping of ether cells at every point of the wire, and each chain of geared cells, initially like Fig. 181, would become distorted like Fig. 182. Throughout the region between *A* and *B* the positive ether cells would be displaced downwards and the negative ether cells would be displaced upwards, that is, the region between *A* and *B* would become an electric field, the direction of which would be outwards from the positively charged plate *A* and inwards towards the negatively charged plate *B*.

Imagine the two metal plates *A* and *B*, Fig. 183, to be charged, that is, imagine the chains of geared ether cells which are represented by the dotted lines in Fig. 183 to be distorted like Fig. 182. Then a wire * connected from *A* to *B* will cut

* Strictly this wire should be thought of as a broad sheet of metal of which the sectional view is shown in Fig. 183. See footnote on page 246.

across every one of the distorted chains of geared ether cells, slipping will begin at every point on the wire, each distorted chain of cells will begin to be relieved from distortion, the energy of each distorted chain will be transmitted along the chain to the wire where it will appear as heat, and the entire region between and surrounding the metal plates *A* and *B* will be relieved from the electrical stress. The explanation here given of the entire relief of the electrical stress between two plates by the establishment of a conducting line (line of slip) between them, applies to two adjacent oppositely charged bodies of any shape. An electric spark is a line of slip (a conducting line) and the energy of the electic field flows in upon the spark as it does upon a wire. The slipping of the ether cells in a conductor is imagined to be opposed by a fractional drag very much as if the gear teeth of the ether cells in a metal were made of a viscous substance like pitch.

142. The electric oscillator.—Let *A* and *B*, Fig. 184, be two metal balls connected to two short rods between which is an air gap. Imagine charge to have been collecting on *A* and *B*

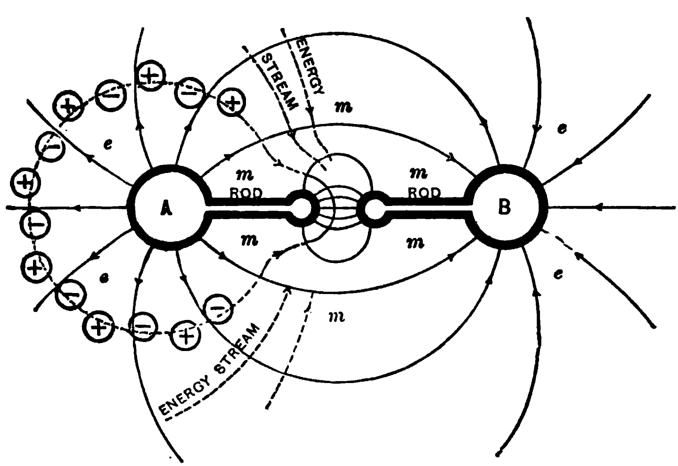

Fig. 184.

(positive on *A*, negative on *B*) until a spark jumps across the air gap, thus establishing a conducting path from *A* to *B* and

causing *A* and *B* to discharge. This discharge, however, is usually oscillatory like the movements of a spring which is pulled to one side and suddenly released, as follows: Consider a chain of geared ether cells which when undistorted lies along the dotted line in Fig. 184, this dotted line being everywhere perpendicular to the lines of force of the electric field. When *A* is positively charged this chain is distorted as shown (in part), but, inasmuch as it is a closed chain, its distortion is fixed, as explained in connection with Fig. 182. When a spark is formed across the air gap, however, a line of slip is established across the distorted chain, and the distortion disappears as explained in Art. 141. What is said of the single chain of ether cells is true of every chain which surrounds *A* or *B*.

If the slip which relieves the distortion of the chain of ether cells takes place with great friction (high electrical resistance of conducting path formed by the spark), the cells near the spark begin turning slowly and the entire energy of the distorted chain is geared into the gap and converted at once into heat. If the slip which relieves the distortion of the chain of ether cells is almost frictionless (low electrical resistance of the conducting path formed by the spark), then the energy of the distorted chain is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time the whole of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted lines in Fig. 184, largely disappears from the regions *ee* and *ee*, and is distributed mainly in the region *mm*. When the chain of ether cells has been relieved from distortion, the rotatory motion of the ether cells in the region *mm* will have reached a maximum, and the cells will continue to rotate because of their momenta, thus producing a reversed distortion of each chain of ether cells. At the same time the energy will stream back from the region *mm* to the regions *ee* and *ee*, being converted again into potential energy of ether distortion, and the balls *A* and *B* will be charged in a reversed sense. This re-

versed distortion of the chains of ether cells is then relieved by a reversed slip (a reversed current in the rods and along spark), and the above described action is repeated over and over again until the original energy of the electrical field has been dissipated.

The oscillatory changes above described take place so rapidly that the portions of the distorted ether which are remote from the oscillator *AB*, Fig. 184, do not follow the changes promptly. This gives rise to electrical waves the nature of which at a distance from the oscillator is explained in a subsequent article.

143. Examples of electric oscillators. — The type of electric oscillator which is described in Art. 142 was devised by Hertz and used by him in his celebrated experimental researches on electric waves in 1887.* An electric oscillator essentially similar to the Hertz oscillator is employed as the sending device in electric-wave telegraphy, wireless telegraphy so-called, as described in Appendix D.

Almost every electric spark discharge is oscillatory in character as may be shown by photographing the spark upon a rapidly moving photographic plate. Thus, a sharp flash of lightning when photographed by means of a rapidly swinging camera generally shows several parallel flashes very close together on the photographic plate. The number of oscillations per second of an electric discharge is, however, generally so great that the sound of the spark cannot be distinguished from a sharp snap or click. According to the principles enunciated in Appendix E, however, it is evident that the number of oscillations per second can be reduced to any desired value by increasing the inductance of the circuit through which the discharge takes place and by increasing the capacity of the condenser in which the charge is stored. Thus, Fig. 185 shows a battery of Leyden jars *JJ* arranged to discharge across an air gap *g* and through a coil of wire *L*. The sound produced by the spark in this case is a high pitch musical tone of very short duration like the ringing sound

* These researches are described in Hertz's book on *Electric Waves*, English translation published by The Macmillan Company.

which is produced by striking an anvil with a hammer. The pitch of the "ringing spark" may be raised by decreasing the number of turns of wire in the coil *L*, or by decreasing the

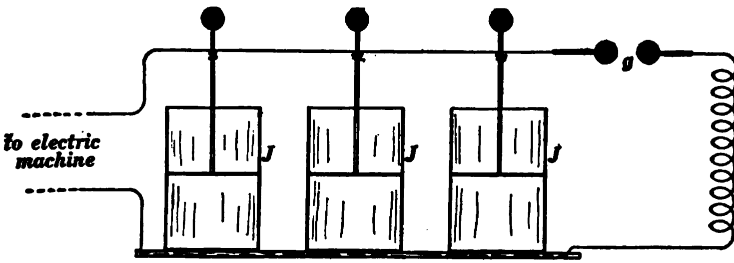

Fig. 185.

number of Leyden jars in the battery *JJ*. The oscillatory character of the spark across the gap *g* may be shown by viewing it in a rotating mirror, and in this way ten or more images of the spark may be seen side by side at each discharge of the battery of Leyden jars.

The oscillatory discharge of a condenser through a coil of wire is utilized in a type of induction coil which is due to Nikola Tesla. A helix *PP*, Fig. 186, of ten or fifteen turns of coarse wire is connected to the terminals *CD* of a charged condenser *AB*, or battery of Leyden jars, with a spark gap in the circuit at *g*. The condenser is connected

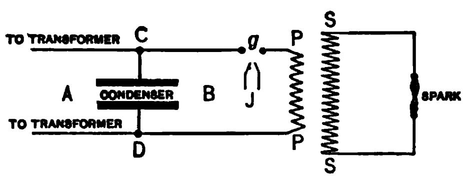

Fig. 186.

to the secondary of a high voltage transformer, each high voltage impulse of the transformer charges the condenser until the air gap at *g* breaks down, and then the condenser charge surges back and forth through the helix *PP* until the energy of the charge is dissipated. A jet of air * issues from a nozzle *J* and blows away the air which has been heated and ionized by

* By using zinc terminals for the spark *g* and by placing two or three very short spark gaps in series the air jet becomes unnecessary.

the spark. Then charge can again accumulate on the condenser until a new discharge takes place. The successive discharges may be as frequent as several thousand per second (a number of successive discharges taking place during each high voltage impulse of the charging transformer), and the oscillations of each discharge may be at the rate of a million or more per second.

A second helix SS of several hundred turns of wire *surrounds* the helix PP (not so shown in the figure), that is, the coils PP and SS constitute the primary and secondary coils of an induction coil. The rapidly oscillating current in PP due to the discharge of the condenser induces very large electromotive forces in SS and produces long sparks between the terminals of SS.

A very striking property of the discharge from SS, which is due to its high frequency, is that it traverses only the surface layers of a conductor and it may therefore be passed through (over) the human body with impunity.

Leyden jars as oscillators and resonators. — Similar circuits may be connected to two Leyden jars so that the oscillations which occur when one Leyden jar discharges through its circuit are in unison with the proper oscillations of the closed circuit of the other jar, so that the inducing action on the circuit of the second jar is cumulative. An instructive experiment is the following: A Leyden jar is connected to a vertical rectangular circuit of wire ww as shown in Fig. 187, and an electric machine repeatedly charges the jar until it discharges across the air gap g and through the circuit ww. This discharge is oscillatory in character and it has a definite frequency. A second jar similar to the first is *short-circuited* by a vertical rectangular wire frame ww as shown in Fig. 188, and placed along side of the arrangement shown in Fig. 187. By adjusting the size of the circuit in Fig. 188, the free period of oscillation of this circuit may be made to coincide with the period of oscillation of the circuit in Fig. 187, and, when this condition is reached, the induced oscillations in the circuit become sufficiently intense to produce a spark across the air gap

a. This experiment illustrates the phenomenon of electric resonance. Each oscillation of the circuit in Fig. 187 induces a slight electromotive force in the circuit of Fig. 188, these succes-

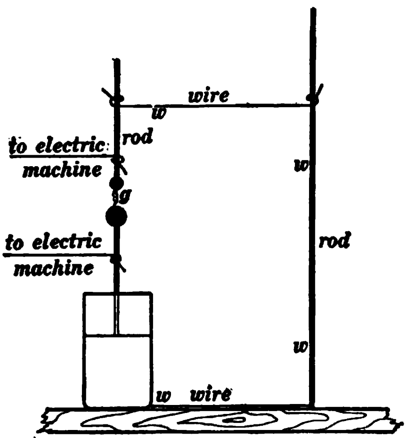

Fig. 187.

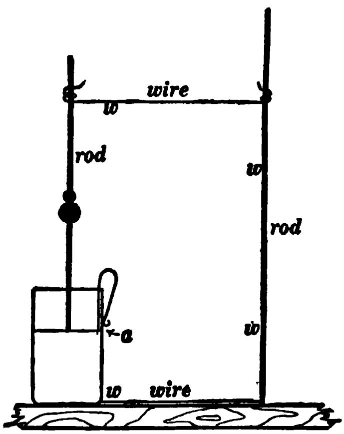

Fig. 188.

sive electromotive forces are in unison with the proper period of oscillation of the circuit in Fig. 188, and therefore their effect is cumulative.

The Tesla induction coil is usually arranged so that the inductance of its primary circuit can be adjusted, thus altering the frequency of the oscillatory discharges through the primary coil. Then by adjusting the inductance of the primary until the frequency of oscillation of the primary circuit is the same as the frequency of oscillation of the secondary circuit, the successive surges of current in the primary coil become cumulative in their effect on the secondary (resonance).

144. Water waves in a canal. — Before attempting to describe electrical waves, it is desirable to consider some of the phenomena presented by water waves. A water wave consists of a moving hill of water, a given particle of water is set in motion when the wave reaches it, and comes immediately to rest after the wave has passed. What supports the hill of water, and what produces the unbalanced force which causes the water to

gain velocity and lose it again during the passage of the wave? *A wave always consists of two elements which travel along together, a local distortion of the medium and a local state of motion of the medium, the forces which are associated with the distortion are the forces which produce the motion; this production of motion involves acceleration and the reaction of the acceleration gives rise to the forces which produce distortion.* The distortion creates the motion and the motion creates the distortion as they both travel along together. The two are mutually dependent.

A consideration of the simplest kind of water waves in a canal, namely, the kind in which the only perceptible motion of the water in the wave is *a uniform horizontal flow*, will serve better than anything else as an introduction to the discussion of electric waves. Consider a canal of rectangular section which is filled to a depth x with still water. Imagine a gate to be moved *slowly* along the canal at velocity v, as shown in Fig. 189. The water next the gate is set in motion, and in being set in motion it heaps up to a definite depth $x + h$; and a *wave of starting* W moves along the canal at a definite velocity V. If the gate is suddenly stopped, the wave of starting W continues to move as before, the water next to the gate, in being stopped, drops to its normal depth x, and a wave of arrest W' moves along the canal as shown in Fig. 190. The elevation h of the water in the wave is supposed to be small.

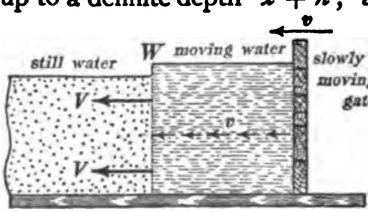

Fig. 189.

The uniformly moving and uniformly elevated body of water A, Fig. 190, constitutes what is called a *complete wave*, or simply a *wave*. The water in front of the wave is continually set in motion at velocity v and raised to the depth $x + h$. The water in the back part of the wave is continually brought to rest and lowered to the normal depth x of the water in the canal. Thus, the state of motion which constitutes the wave A travels

along the canal without changing its character, friction being neglected.

An essential feature of any wave which moves along without changing its shape is that *the kinetic energy is equal to the potential energy in the wave at each point.* Thus, the kinetic energy of the water wave A, Fig. 190, due to the uniform velocity v of

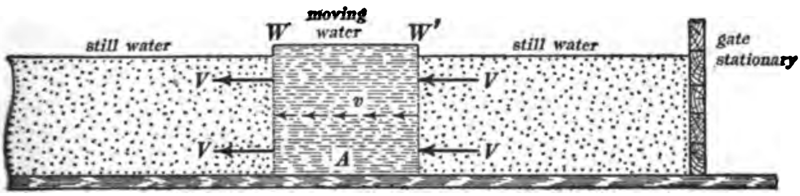

Fig. 190.

the water in the wave is equal to the potential energy due to the elevation h.* When the potential energy in a wave is equal to

*The following derivation of the velocity of a water wave in a canal shows the significance of equality of potential and kinetic energy. This discussion is based upon a slight modification of the conditions shown in Fig. 189, as follows: Water of depth x flows along a canal of rectangular section at a uniform velocity (small) of v centimeters per second. A gate is suddenly closed as shown in Fig. 191; the moving water, in being brought to rest against the gate, heaps up to a depth $x + h$; and a wave of arrest W, Fig. 191, moves along the canal at a definite velocity V. The action involved in Fig. 191 is identical to the action involved in Fig. 189. In fact, Fig. 189 can be converted into Fig. 191, by imagining everything in Fig. 189 to be moving to the right at velocity v. The discussion of Fig. 191 is simpler than the discussion of Fig. 189 because the potential energy is in one portion of the water and the kinetic energy is in another portion, whereas in Fig. 189 the potential energy and the kinetic energy are both in one portion of the water. Let b be the breadth of the canal. Consider a transverse slice of water one centimeter thick. The volume of this slice is bx cubic centimeters and its mass is dbx grams, where d is the density of the water in grams per cubic centimeter. Therefore the kinetic energy of this slice of water when it is moving at a velocity of v centimeters per second is $\frac{1}{2}dbxv^2$. When the wave of arrest W, Fig. 191, reaches the slice of water under consideration, the slice, as it comes to rest, is squeezed together and increased in depth to $x + h$. The slice is decreased in thickness in proportion to its increase in depth, so that its

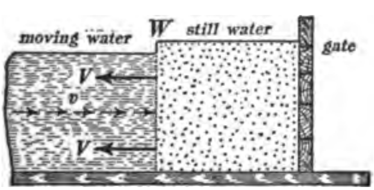

Fig. 191.

the kinetic energy, we have what is called a *pure wave*, and when the potential energy in a wave is not equal to the kinetic energy the wave is called an *impure wave*.

The behavior of an impure wave pulse in a canal may be stated by considering an extreme case of an impure wave as follows: Consider an elevated portion of still water in a canal as shown

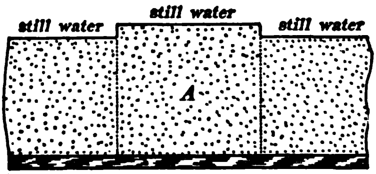

Fig. 192.

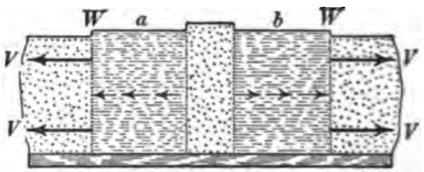

Fig. 193.

thickness is reduced to $x/(x+h)$ or to $(1-h/x)$ of a centimeter, h being very small. Therefore, the decrease of thickness is h/x of a centimeter. The force acting to reduce the thickness of the slice is to be considered as that force which is due to the *increase* of pressure in the water produced by the *increasing depth* h. This increase of pressure is equal to hdg dynes per square centimeter when the slice has reached its greatest depth, so that the average increase of pressure due to increasing depth is $\frac{1}{2}hdg$, which produces over the face of the slice a force equal to $\frac{1}{2}hdg \times bx$, and the product of this force and the decrease of thickness of the slice gives the work done in decreasing its thickness. This work must be equal to the original kinetic energy of the slice, so that

$$\tfrac{1}{2}dbxv^2 = \tfrac{1}{2}dbh^2g$$

or

$$v^2 = \frac{gh^2}{x} \tag{i}$$

Consider the instant t seconds after the closing of the gate in Fig. 191. The wave of arrest W has reached the distance Vt from the gate, and the excess of water that is represented by the raising of the water level $(= Vt \times h \times b$ cubic centimeters) is the amount of water supplied by the flow of the canal in t seconds $(= bxvt$ cubic centimeters). Therefore

$$Vthb = bxvt$$

or

$$V = \frac{xv}{h} \tag{ii}$$

Substituting the value v from equation (i) in equation (ii), we have

$$V = \sqrt{gx} \tag{iii}$$

Therefore the velocity of progression of a wave in a canal is equal to the velocity gained by a body in falling freely through the distance $x/2$.

in Fig. 192. This body of elevated water is an impure wave inasmuch as its velocity of flow v is zero, and therefore its potential energy of elevation cannot be equal to its kinetic energy of flow. Such an elevated portion of still water breaks up into two oppositely moving pure waves, and the initial stage of this process of breaking up is indicated in Fig. 193.

When a wave like A, Fig. 190, travels along a canal, the velocity of flow v is continually decreased by friction, whereas there is no action tending to reduce the elevation h. Therefore *that portion of the elevation* which is in excess of what is required to give a pure wave with what remains of the velocity of flow, behaves exactly like the elevation A in Fig. 192, that is, this excess of elevation breaks up into two pure waves a and b, Fig. 193, the portion a merges with the original wave A and the portion b shoots backwards.

The upper part of Fig. 194 represents, on an exaggerated scale, the elevated portion of water in a pure wave. The velocity

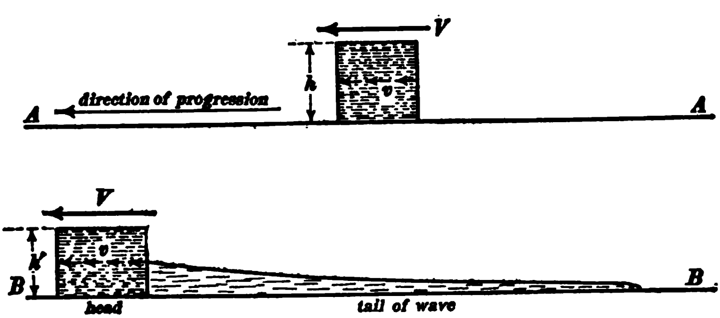

Fig. 194.

of flow v in this wave is continually reduced by friction as the wave travels along the canal, the excess of elevation which is being thus continually left in the wave causes a long drawn-out wave to shoot backwards, and after a time the wave has the form shown in the lower portion of Fig. 194, The head of the wave is greatly reduced in intensity (energy value) partly because of the loss of energy by friction and partly because of the

carrying of energy backwards into the tail of the wave. After a given interval of time the tail of the wave has a total length $2Vt$ where V is the velocity of progression of the wave.

If a canal is filled brimful of water so that the elevation of the water level causes an overflow, or spill, the tendency is for a wave to remain pure, and therefore to be propagated without change of shape, because the elevation is reduced by spill and the velocity of flow v is reduced by friction. This is precisely analogous to the action which takes place on a poorly insulated telephone line and which causes such a telephone limit to transmit speech more distinctly than if it were thoroughly insulated.

145. The electromagnetic wave. — An electromagnetic wave consists of a state of ether distortion and a state of ether motion traveling along together and mutually sustaining each other. The ether distortion is electric field and the ether motion is magnetic field. A layer of electric field unsustained breaks up into two electromagnetic waves just as the elevated portion of water in Fig. 192 breaks up into two water waves.

The action which takes place in an electromagnetic wave may be clearly understood with the help of Maxwell's conception of the electromagnetic field. It is desirable to consider the case of an electric wave which moves along between two wires (or broad sheets of metal) which *bound* the electric wave very much as a speaking tube bounds a sound wave which passes through it.

Figure 195 shows two broad sheets of metal with an electromagnetic wave pulse traveling along between them at velocity V. The fine vertical lines represent the electric field which is towards the top of the page, and the dots represent the lines of force of the magnetic field which is perpendicular to the plane of the paper and directed towards the reader. A single chain of geared cells is shown in the figure, although a complete representation of what takes place in the wave would necessitate the showing of great numbers of horizontal chains of geared ether cells every one of which would be exactly similar to the one shown in Fig. 195. Within the region of the wave the ether

cells are all in uniform rotation as indicated by the small curved arrows, and within the region of the wave the chains of cells are all distorted, positive cells being displaced upwards with respect to the negative cells, as shown in Fig. 195.

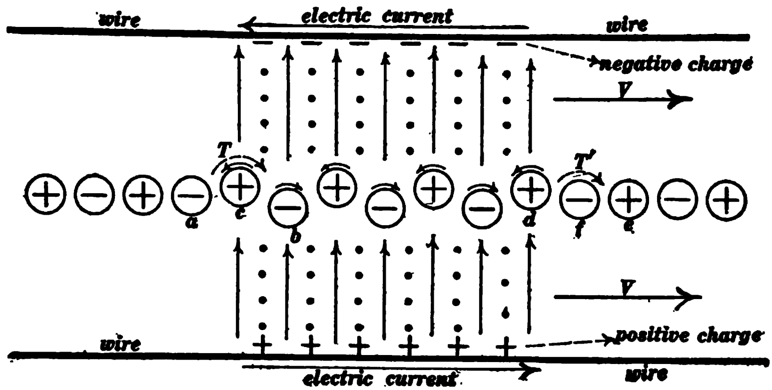

Fig. 195.

Throughout the middle portion of the wave each rotating cell is acted upon by equal and opposite torques by the adjacent cells ahead of it and behind it, as explained in connection with Fig. 175.* Therefore all the cells in the middle portion of the wave continue to rotate at unchanging speed, and the zigzag distortion of the chain of cells remains unchanged in the middle portion of the wave The cell *d*, however, exerts an unbalanced torque upon the cell *f*, as indicated by the dotted arrow T', and this torque quickly sets the cell *f* into rotation. Also the cell *b* exerts an unbalanced torque T upon the cell *c* which quickly stops the rotation of the cell *c*. Thus the combined state of motion and distortion of the ether cells between *c* and *f* travels to the right.

The terminating of the electric lines of force on the wires (or metal sheets) which bound the electric wave constitutes electric

* Figure 195 represents what may be called a *rectangular* electromagnetic wave pulse throughout which the electric field is *uniform* and throughout which the magnetic field is *uniform*.

charges, positive on the lower wire and negative on the upper wire, in Fig. 195. It is evident, furthermore, that the uniform rotation of the ether cells in the region of the wave involves the slipping of the ether cells where they come in contact with the sheets of metal which bound the wave. This slipping constitutes an electric current which flows to the left in the upper wire and to the right in the lower wire in Fig. 195.

Figure 195 represents what is called a rectangular wave pulse. Fig. 196 shows what takes place when a simple train of electromagnetic waves travel along between two broad metal sheets.

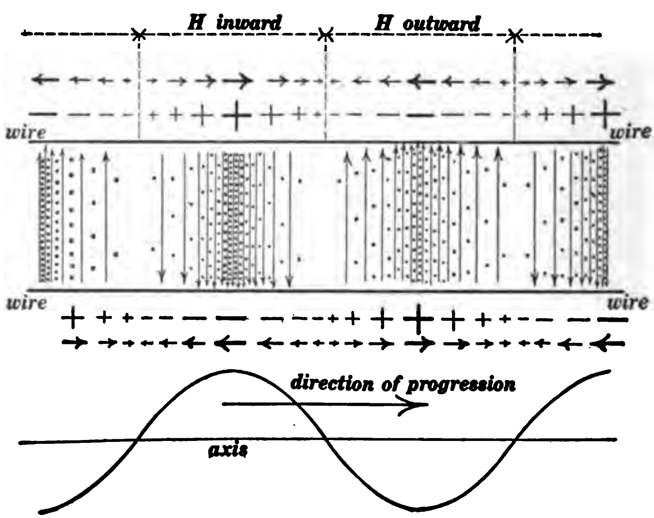

Fig. 196.

*Hertz's experiments with electric waves.** — The *oscillator* used by Hertz consisted of two brass rods A and B with an air gap g, as shown in Fig. 197. These two rods were connected to the terminals of an induction coil as indicated, at each impulse of electromotive force from the induction coil a spark breaks

* These experiments were described originally in *Wiedemann's Annalen*. A very complete description of them may be found in Hertz's book on *Electric Waves*, English translation published by The Macmillan Company.

across the gap g and the discharge surges back and forth along the rods until the energy of the charge is dissipated.

The resonator. — The electric waves were detected in Hertz's original experiments by means of an arrangement similar to the oscillator, but with a shorter spark gap and without connections to an induction coil. This arrangement, which is called the resonator, has the same period of oscillation as the oscillator so that the action upon it of the train of waves from the oscillator is cumulative, causing it to oscillate in sympathy with the oscillator just as one tuning fork vibrates in unison with a similar one which is set vibrating with a hammer blow. The oscillations of the resonator were indicated by minute sparks in its gap g, Fig. 198.

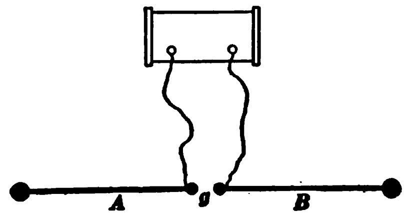

Fig. 197.

The reflectors. — The waves which emanate from the Hertz oscillator are very weak at any considerable distance, and their action upon the resonator may be greatly intensified by the use of parabolic reflectors. The oscillator and the resonator were

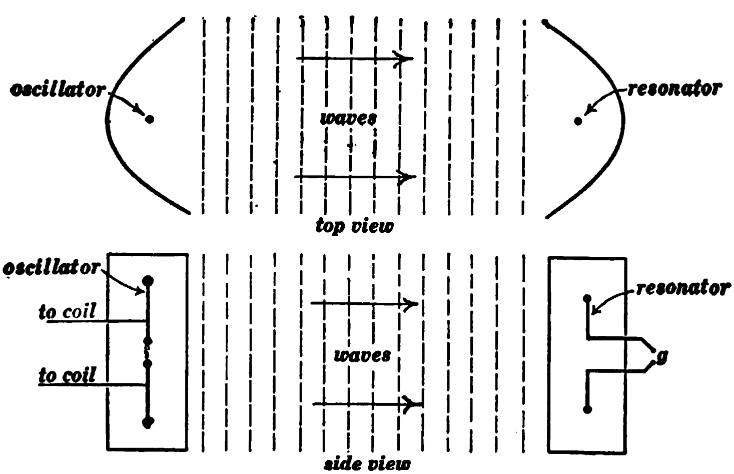

Fig. 198.

placed along the respective focal lines of two parabolic cylinders made of sheet metal, as shown in Fig. 198 and the resonator was arranged so that its spark gap was behind the mirror and thus easily visible.

Reflection of electric waves. — When the oscillator and resonator are arranged as shown in Fig. 198, a very distinct effect of the resonator is produced when the oscillator is active, the waves from the oscillator being concentrated upon the resonator by the action of the two parabolic reflectors.

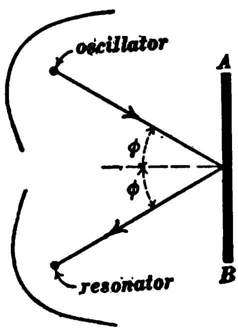

Fig. 199.

When arranged as shown in Fig. 199, AB being a plain sheet of metal, and the angles ϕ being equal, a very distinct effect on the resonator is prodused.

Refraction of electric waves. — When the oscillator and resonator are arranged as shown in Fig. 200, in which PP represents a large prism, of asphaltum or paraffine, a very distinct effect is produced upon the resonator.

Polarization of electric waves. — A frame strung with a grating of fine metal wire acts as a good reflector for the waves from a

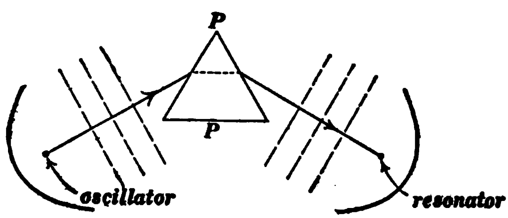

Fig. 200.

Hertz oscillator, when the wires of the grating are parallel to the axis of the oscillator. When the wires of the grating are at right angles to the axis of the oscillator, the waves pass through the grating without perceptible diminution in intensity. Therefore the waves from a Hertz oscillator are plane polarized.

Stationary electric waves. — If the plane waves from the oscil-

lator and its parabolic mirror are allowed to fall perpendicularly upon a plane sheet of metal AB, as shown in Fig. 201, the resonator is not acted upon if it is placed at certain points n, n', n'', and so on, whereas the resonator is acted upon if it is placed at positions intermediate between these points. The

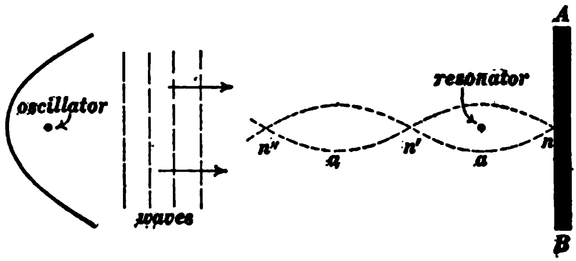

Fig. 201.

reflected waves from AB, Fig. 201, form with the advancing waves a stationary wave train of which nodes are situated at the points n, n', n'', and the antinodes at the points aa.

146. The law of induced electromotive force and its bearing upon electromagnetic wave motion.— Let H be the intensity in gausses of the magnetic field in the region of the wave shown in Fig. 195, let f be the intensity of the electric field in abvolts per centimeter, and let l be the distance across from wire to wire (sheet to sheet). The sidewise motion of the magnetic field at velocity V induces an electromotive force in the region of the wave and this electromotive force in abvolts is given by the equation

$$E = lHV$$

as explained in Art. 64. Therefore the electric field intensity in the wave (E/l) is given by the equation

$$f = HV \tag{78}$$

in which f is expressed in abvolts per centimeter, H is expressed in gausses, and V is expressed in centimeters per second.

Calculation of velocity of progression of the electromagnetic wave. — The intensities of the mutually dependent electric and magnetic fields which constitute a pure electromagnetic wave must satisfy two conditions, namely, (*a*) the magnetic energy per unit volume in the wave must be equal to the electric energy per unit volume in the wave,* and (*b*) the velocity of the wave must be such as to satisfy equation (78), so that the electric field may be wholly sustained by the inducing action of the moving magnetic field.

The magnetic energy in ergs per cubic centimeter in a wave is equal to $H^2/8\pi$ according to equation (27), the intensity H of the magnetic field being expressed in gausses. The electric energy per unit volume in a wave is given by equation (75), in which equation the energy is expressed in joules per cubic centimeter and the electric field intensity is expressed in volts per centimeter. Reducing to c.g.s. units (energy in ergs per cubic centimeter and electric field intensity in abvolts per centimeter) we have

$$f^2/(2B \times 10^9)$$

as the expression for the electric energy in ergs per cubic centimeter. Therefore the first condition above mentioned gives the equation

$$\frac{H^2}{8\pi} = \frac{f^2}{2B \times 10^9} \tag{79}$$

Therefore solving equations (78) and (79) for V, we have

$$V^2 = \frac{B}{4\pi} \times 10^9 \tag{80}$$

but the factor B is equal to 1.131×10^{13}, according to Arts. 91 and 98. Therefore we have

$$V = 2.996 \times 10^{10} \frac{\text{cm.}}{\text{sec.}} \tag{81}$$

The velocity of an electric wave thus calculated is identically

* See footnote to Art. 144.

equal to the velocity of light as determined by direct observation. Therefore the most accurate method for determining the value of the constant B as used in Arts. 91 to 98 is to calculate its value from the observed value of V using equation (81).

The identity of the velocities of electromagnetic waves and of light waves was first pointed out by Maxwell and it is now universally conceded that light waves are electromagnetic waves.

147. Electric wave distortion. — So long as the electric and magnetic field intensities in the wave which is shown in Fig. 195 continue to satisfy equation (79), the electromagnetic wave remains pure and it does not change its shape as it travels along. The effect of the electrical resistance of the two bounding wires (or metal sheets) is to cause a steady decay of the magnetic field, and the effect of imperfect insulation of the material between the bounding wires is to cause a continual decay of the electric field. The continual decay of the magnetic field may be thought of as due to the resistance which opposes the slipping of the rotating ether cells where they are in contact with the bounding wires in Fig. 195, and the continual decay of the electric field is somewhat analogous to the slow disappearance of stress in a stretched piece of rubber which may be supposed to have, in addition to its elastic property, a certain degree of viscosity, like pitch, so as to continually yield under the influence of the stress. When the resistance per unit length of the bounding wires in Fig. 195 bears a certain ratio * to the insulation resistance of the material between unit length of the bounding wires, then the electric and magnetic fields decay in such a way as to continually satisfy equation (79), and the wave progresses without changing its shape. A pair of transmission wires which satisfies this condition constitutes what is called a *distortionless line*. In all ordinary telephone lines the effect of line resistance is greatly in excess of the effect of line leakage, † and therefore an electric wave in being transmitted along

* This relation may be quite easily formulated but an elaborate discussion of wave-distortion is not within the scope of this text.

† Several interesting examples are given by B. S. Cohen in *The Electrician* (London), April 10, 1908.

a telephone line suffers continual distortion because of the rapid decay of magnetic field due to line resistance. The distortion of an electric wave as it travels along a pair of telephone lines is similar in many respects to the distortion of a canal wave as described in Art. 144 and as represented in Fig. 194. Imagine a rectangular electromagnetic wave-pulse to be started at the middle of a long telephone line (two wires of course). Let the small rectangle in the upper part of Fig. 194 represent the initial form of the wave. After the elapse of time the wave changes to the shape shown by *BB*, Fig. 194. The energy in the head of the wave decreases partly because of the RI^2 losses in the line wires and partly because of the shooting of energy back into the tail of the wave.

The transmission of articulate speech over a telephone line depends upon the transmission of characteristic shapes of electric waves. Thus, the shapes of the electric waves necessary to reproduce certain vowel sounds are shown in Fig. 202, and the wave shapes which are necessary to produce consonant sounds are very much more complicated than these. The wave distortion on the line tends to make each elementary portion of a wave spread out as shown in Fig. 194, and if each elementary portion of a complicated wave spreads out in this way the fine details of wave shape are very soon obliterated as the wave travels along.

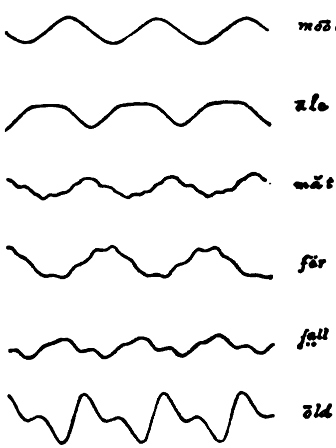

Fig. 202.

It is not desirable to eliminate wave distortion by providing poor insulation between telephone wires because this results in a great reduction in the amount of energy transmitted. The method which is used in practice is to connect small inductance coils in circuit with the line wires at intervals over the whole length of

the line. The effect of these inductance coils is to permit of the satisfying of equation (79) (magnetic energy equal to electric energy) with a very greatly reduced value of current in the line wires so that the RI^2 loss, which is the cause of the wave distortion, is very greatly reduced. A telephone line provided with inductance coils in this way is called a *loaded line*. This arrangement is due to Pupin.

The canal analogue of a loaded telephone line is as follows: Imagine a great number of thin boards to be placed across the canal in the form of diaphragms but free to move with the water in the canal, and imagine these thin boards to be very massive. The effect of these massive boards would be to reduce the velocity v in Fig. 190 and still permit the kinetic energy of the moving water and boards to be equal to the potential energy due to the elevation of the water in the wave. This reduced velocity of flow v would greatly reduce the friction of the water against the sides of the canal and therefore the kinetic energy of the wave would be dissipated much less rapidly than if the water in the canal were not loaded.

The loading of a telephone line is helpful only when the energy loss due to line resistance is much greater than the energy loss due to line leakage (poor insulation). When line leakage (poor insulation) is excessive, the loading of the line tends to increase wave distortion. The explanation of this effect of loading is as follows: The velocity of transmission of the waves along a line is greatly reduced by loading so that a longer time is required for a wave to travel over the line and therefore the wave loses energy by leakage for a longer time.*

PROBLEMS.

143. Ten horse-power is transmitted along a row of gear wheels, the speed of each of which is 1,200 revolutions per minute. The

*The student who wishes to pursue the study of the theory of electric waves should read Heaviside's *Electromagnetic Theory*, Vols. I and II, London, The Electrician Company. The second part of Vol. I, namely, pages 306 to 455, is especially instructive.

diameter of the pitch circle of each gear is 2 feet. Find: (*a*) The tangential force exerted on a given gear wheel by each adjacent wheel, and (*b*) the torque exerted upon a given gear by each adjacent gear and express the result in pound-feet. Ans. (*a*) $21\frac{7}{8}$ pounds. (*b*) $21\frac{7}{8}$ pound-feet.

144. A long wire of which the resistance per centimeter of length is 0.02 ohm carries a current of 3 amperes. (*a*) Find the rate at which energy flows in upon each centimeter of length of this wire in ergs per second. (*b*) Find the intensity of the energy stream at a distance of 15 centimeters from the wire in ergs per second per square centimeter. (*c*) Find the intensity of the electric field parallel to the wire in abvolts per centimeter and find the intensity of the magnetic field in gausses at a distance of 15 centimeters from the wire. (*d*) Find the value of the proportionality factor by which the product of intensities of electric and magnetic fields (at right angles to each other) must be multiplied to give the intensity of an energy stream in ergs per second per square centimeter. Ans. (*a*) 1,800,000 ergs per second. (*b*) 19,100 ergs per second per square centimeter. (*c*) 6,000,000 abvolts per centimeter. (*d*) $1/4\pi$.

145. Consider two line wires in the form of two flat metal ribbons 50 centimeters wide and 3 centimeters apart. At a given point *pp* the electromotive force between the ribbons is 100 volts

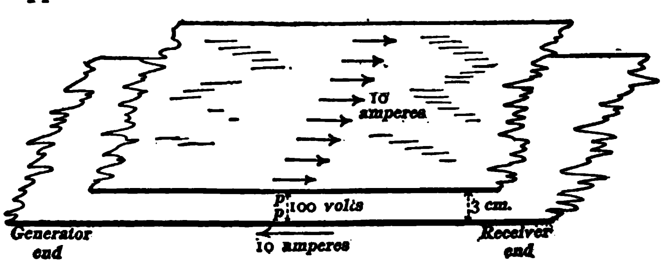

Fig. 203.

and the current in each ribbon is 10 amperes, as shown in Fig. 203. (*a*) Find the rate in ergs per second at which energy flows past the given point *p* from generator towards receiver in

ergs per second, using the ordinary formula $P = EI$. (*b*) The electric field intensity between the points *pp* in Fig. 203 is $33\frac{1}{3}$ volts per centimeter and the magnetic field between the strips is uniform and perpendicular to the plane of the paper. Let the intensity of the magnetic field be H. Express the intensity of the energy stream across *pp* in ergs per square centimeter per second in terms of H and the electric field, intensity using the proportionality factor found in problem 147. Multiply this intensity of the energy stream by the sectional area across which it flows at *pp* and place this result equal to EI (expressed in c.g.s. units of course) and thus find the intensity of the uniform magnetic field between the two ribbons. Ans. (*a*) 10^{10} ergs per second. (*b*) $4\pi/50$ gauss.

146. A water wave travels along a canal in which the normal depth of water is 6 feet, the width of the canal being 12 feet. The wave is 30 feet long and the water in the wave has a uniform velocity of 0.3 foot per second. Find the total energy of the wave counting both potential energy and kinetic energy. Ans. 379.7 foot-pounds.

147. A rectangular electromagnetic wave-pulse is bounded by two broad sheets of metal as shown in Fig. 195. The width of the sheets is 50 centimeters, their distance apart is 3 centimeters and the length of the wave pulse is 100 centimeters. The intensity of the uniform magnetic field in the region of the wave is 10 gausses. Find the total energy of the wave including electric and magnetic energy. Ans. $3{,}000{,}000/8\pi$ ergs.

148. A battery of which the electromotive force is 1,000 volts is connected at a given instant to the two wires of a transmission line. Treating the transmission line as though it consisted of two flat ribbons, make a diagram somewhat similar to Figs. 195 and 196 showing the distribution of current in the bounding metal sheets, the distribution of charge on the bounding metal sheets, and the distribution of electric and magnetic fields in the region between the sheets at an instant t seconds after the battery is connected, Vt being less than the length of the line, where V

19

is the velocity of progression of electric waves along the line. Assume in this problem that the resistance of the bounding metal sheets is negligible.

Note.—A wave of starting travels out from the battery end of the line at velocity V. Ahead of this wave of starting the line is wholly undisturbed. Behind this wave of starting the current in each line has everywhere the same value i (outwards in one line, backwards in the other line), the electromotive force between mains is everywhere equal to 1,000 volts, the electric field intensity between the ribbons has everywhere the same value, and the magnetic field between the ribbons has everywhere the same value H. The electric energy per unit length of the pair of ribbons may be calculated with the help of equations (65*b*) and (62) by considering the ribbons as the two plates of a condenser, the electromotive force between them being 1,000 volts. The intensity of the magnetic field between the ribbons may then be found from the fact that the electric energy must be equal to the magnetic energy, and the current in each ribbon may then be found from the relationship established in problem 147. Calculate the intensity of the electric field, the intensity of the magnetic field, and the current in each ribbon in the region behind the wave of starting in problem 148.

149. The end of the transmission line (pair of ribbons) in problem 148 is short-circuited by zero resistance. Make a diagram showing the distribution of electric and magnetic field, the distribution of charge on the two ribbons and the distribution of current along the two ribbons at an instant after the wave of starting has been reflected from the short-circuited end of the line.

Note.—The student should read Art. 136 of Franklin and MacNutt's Elements of Mechanics in order to be able to understand this problem.

150. The end of the transmission line (pair of ribbons) in problem 148 is open, that is the two ribbons come to an end in air. Make a diagram showing the distribution of electric and magnetic field, the distribution of charge on the two ribbons and the distribution of current along the two ribbons at an instant after the wave of starting has been reflected from the open end of the line.

151. The transmission line specified in problems 148, 149 and 150 is assumed to have zero resistance, and the short-circuit at the end of the line is assumed to have zero resistance in problem 149 so that the current produced by the battery in problem 149 ultimately becomes indefinitely large. Plot a curve showing the growth of current at the battery terminals with lapse of time.

Note.—The battery current starts at a definite value i, as explained in the note to problem 148, and retains this value until the wave of starting travels to the end of the line and back, when the current suddenly increases to the value of $2i$ and so on. The effect of the resistance of the transmission line is too complicated to permit of its being easily taken into account, and therefore the resistance of the transmission line is assumed to be zero in problems 148, 149, 150 and 151.

152. A long train of cars has highly elastic springs in the couplers. Describe the precise manner in which the train gains velocity under a constant pull of the locomotive, ignoring friction.

Note.—The manner of starting of the train is precisely analogous to the manner of setting up a current in the transmission line in problem 149.

CHAPTER X.

ELECTRICAL MEASUREMENTS.

148. Absolute measurements and international standards.— The measurement of an electrical quantity in terms of the mechanical units of length, mass and time directly is called "absolute" electrical measurement. For example, the measurement of current by the Weber electro-dynamometer as explained in Art. 59, is an "absolute" measurement. Absolute electrical measurement requires, in most cases, elaborate apparatus, and, unless extreme precautions are taken, is subject to considerable error. In consequence of this fact a standard of resistance and the electrochemical equivalent of silver have been measured "absolutely" with extreme care and adopted as international standards,* and all ordinary electrical measurements consist in the comparison of the quantity to be measured with these standards.

MEASUREMENT OF CURRENT.

149. Measurement of current by electrolysis.— The electrochemical equivalent of a metal having been determined once for all, the strength of any current may be easily and accurately

*The international standard ampere is defined in Art. 3, and the method by which it was determined is described in Art. 59. The international standard ohm is defined in Art. 52, and the method by which it was determined is described in Art. 152. It is likely that the electromotive force of the Clark standard cell (see Art. 159) will be adopted as an international standard at the next International Electrical Congress. In fact, all practical electrical measurements are now based upon the standard cell and a standard ohm. The use of the silver voltameter is very tedious and the results obtained are less reliable than those which may be obtained with great ease by the use of a standard ohm and a standard cell.

An historical sketch of the international units by Frank A. Wolff is to be found in the *Bulletin of the United States Bureau of Standards*, Vol. I, pages 39–76. The Acts of Congress establishing the legal electrical units for the United States are given on pages 61-65

measured by weighing the metal deposited by the current during an observed interval of time.

An electrolytic cell arranged for the measurement of current by electrolysis is called a *coulombmeter*. Thus we have the silver coulombmeter (which is described on page 21), the copper coulombmeter, and the water coulombmeter. The water coulombmeter consists of an electrolytic cell with platinum electrodes and containing dilute sulphuric acid. It is arranged so that the liberated oxygen and hydrogen may be collected and its volume measured.

150. Measurement of current by the potentiometer and a standard resistance.* — The most convenient method for measuring current accurately in the laboratory is to send the current through a standard resistance and measure the electromotive force across the terminals of the resistance by means of a potentiometer, as explained in Art. 159. This method is convenient because it is very much quicker than the electrolytic method and it is quite accurate because standard resistances are now available which are reliable to within, say, 0.01 of one per cent. and the electromotive force across the resistances can be measured by means of the potentiometer in terms of the accurately-known electromotive force of the standard cell.

151. Direct-reading ammeters. — An ammeter is a galvanometer with a pointer which plays over a scale which is divided and numbered so that the reading of the pointer gives the value of the current directly. The ammeter which is described in Art. 1 consists of a pivoted coil through which the current flows, and a permanent magnet which deflects the coil. This arrangement is essentially similar to the D'Arsonval galvanometer which is described in Art. 61. Another type of ammeter, the electrodynamometer type (see Art. 59), is used generally for alternating-current measurements. It consists of a pivoted coil and a fixed coil connected in series. The current to be measured flows through both coils and the force action between the coils causes the pivoted

* See *Practical Physics*, by Franklin, Crawford and MacNutt, pages 62–74.

coil to be deflected. The plunger type of ammeter is extensively used where cheapness is a prime consideration. In instruments of this type the current flows through a coil of wire which magnetizes and moves a pivoted or suspended piece of soft iron to which the pointer is attached.

Measurement of Resistance.

152. Absolute measurement of resistance. Lorenz's method.— A circular disk of copper *DD*, Fig. 204, is mounted on an axle and driven at a uniform speed of n

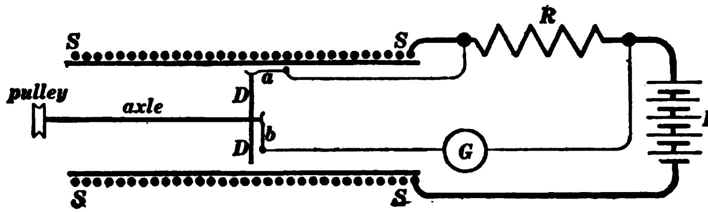

Fig. 204.

revolutions per second. This disk is surrounded by a large coil, or solenoid, *SS* through which a steady current I (the value of which need not be known), flows from a battery B. This current also flows through the resistance R which is to be measured, and an auxiliary circuit containing a sensitive galvanometer G is connected so that the electromotive force which is induced in the rotating disk between the brushes a and b can be balanced against the electromotive force RI across the resistance R, this balance being indicated by zero deflection of the galvanometer. The intensity of the magnetic field in the solenoid *SS* is $H = 4\pi zI$, everything being expressed in c.g.s. units and z being the number of turns of wire per centimeter length of the solenoid. Any given radial filament of the rotating disk cuts $\pi r^2 \times H$ lines of flux during each revolution of the disk, so that the electromotive force induced between the center and the circumference of the disk is equal to $\pi r^2 \times H \times n$ or $\pi r^2 \times 4\pi zI \times n$. When the speed of the disk is increased until the galvanometer gives no deflection, then this induced electromotive force is equal to RI, whence we have

$$RI = \pi r^2 \times 4\pi zI \times n$$

or

$$R = 4\pi^2 r^2 zn$$

153. Resistance boxes.— The measurement of resistance ordinarily consists of the determination of a given resistance in terms of a known resistance. In many cases this measurement is accomplished by adjusting a known resistance until its equal to the resistance to be measured, and a resistance box is an arrangement

by means of which any desired known resistance may be introduced into a circuit. The usual construction of the resistance box is as follows: A series of massive metal blocks are connected by wires whose resistances are 1, 2, 2, 5, 10, 10, 20, 50 ohms, etc., respectively. By means of conical metal plugs which fit snugly between the blocks, the blocks may be connected at pleasure, leaving the resistance between them approximately equal to zero. Figure 206 shows the essential features of this construction.

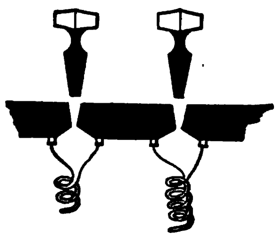

Fig. 205.

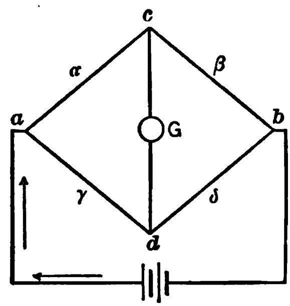

Fig. 206.

154. Measurement of resistance by Wheatstone's bridge.— Wheatstone's bridge consists of a net-work of conductors, as shown in Fig. 206. A battery circuit branches at the points a and b, and the current flows through the four resistances α, β, γ and δ, as shown. A sensitive galvanometer G is connected between the points c and d. When no current flows through the galvanometer the four resistances α, β, γ and δ satisfy the equation

$$\frac{\alpha}{\beta}=\frac{\gamma}{\delta} \tag{82}$$

The method of using this arrangement for the measurement of current is explained in Arts. 155 and 156.

Proof of equation (82).— Let i' be the current flowing through α and β (the same current flows through α and β, since the galvanometer current is zero) and let i'' be the current flowing through γ and δ. Inasmuch as there is no current flowing

through the galvanometer the electromotive force between c and d must be equal to zero. Therefore the electromotive force $\alpha i'$ between a and c is equal to the electromotive force $\gamma i''$ between a and d, that is,

$$\alpha i' = \gamma i'' \tag{i}$$

and similarly we find

$$\beta i' = \delta i'' \tag{ii}$$

Dividing equation (i) by equation (ii), member by member, we have equation (82).

155. Measurement of resistance by the slide wire bridge. — A stretched wire ab, Fig. 207, an unknown resistance α, a

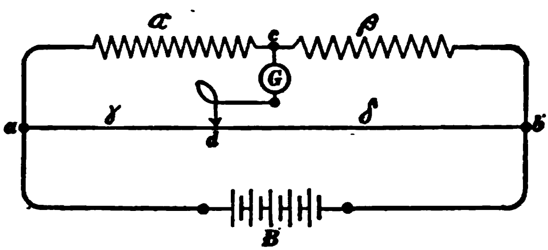

Fig. 207.

known resistance β, and a sensitive galvanometer G are connected as shown to a battery B. The lettering in Fig. 207 corresponds to that in Fig. 206. The sliding contact d is adjusted until the galvanometer gives no deflection and then equation (82) is satisfied, but γ/δ is equal to the ratio of the lengths of the corresponding portions of the wire ab, and it is easily determined by measuring the lengths ad and db. Therefore, β being known, α may be calculated.

156. Measurement of resistance by the box bridge. — The box bridge is a resistance box containing three sets of resistances, β, γ and δ connected as shown in Fig. 208. The dotted lines represent connections outside the box. The portions γ and δ usually have each a 10-ohm, a 100-ohm, and a 1,000-ohm coil,

so that the ratio γ/δ may have a series of values any one of which may be chosen at will. The portion β contains usually 1, 2, 2, and 5 of each units, tens, hundreds, etc., of ohms. An unknown resistance α is connected as shown, the ratio γ/δ is chosen, and the value of β is changed until the galvanometer

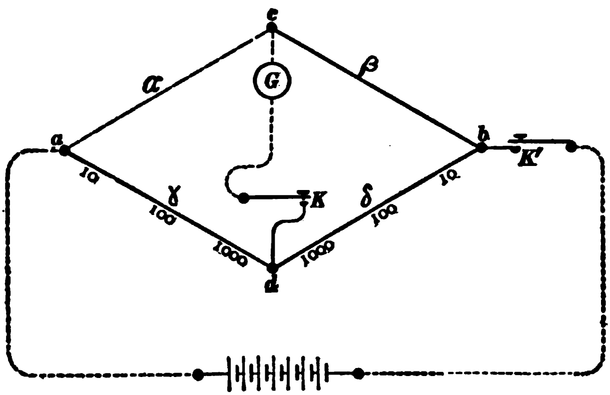

Fig. 208.

gives no deflection, the battery key K' being closed first and the galvanometer key K afterwards.* The value of α is then calculated with the help of equation (82).

157. The measurement of resistance by the ammeter and voltmeter. — In the dynamo testing laboratory, where it is usually inconvenient to use Wheatstone's bridge, resistance is ordinarily measured by means of an ammeter and a voltmeter as follows: A current, which is measured by an ammeter, is sent through the resistance, and the electromotive force between the terminals of the resistance is measured by means of a voltmeter. The value

* If the galvanometer circuit is closed when the battery key K' is closed, a momentary pulse of current may flow through the galvanometer even if equation (82) is satisfied. In order that this momentary pulse of current may not flow, a certain relation must exist between the inductances of the four arms of the bridge α, β, γ and δ. This pulse of current due to inductance is made use of in Maxwell's method of measuring inductance by means of the Wheatstone's bridge. See *Practical Physics*, Franklin, Crawford and MacNutt, Vol. II, pages 129–133.

of the resistance is then found by dividing the electromotive force by the current.

158. Measurement of very high resistances. *Insulation resistance.* — Very large resistances cannot be easily measured by the methods outlined above. Consider, for example, an insulated cable consisting of a core of copper wire surrounded by a layer of rubber and inclosed in a sheath of lead. If the lead sheath is connected to one terminal of a battery and the copper core to the other terminal, a certain amount of current will flow through the insulating layer of rubber, that is to say, the rubber is not a perfect insulator (infinite resistance). Very high resistances are usually determined by measuring, with a sensitive galvanometer, the current I which is forced through the given resistance by a large known electromotive force E. Then according to Ohm's Law the resistance is equal to E/I.*

Example. — One terminal of a 1,000-volt battery is connected through a very sensitive galvanometer to the outside tin-foil coating on a glass jar, and the other terminal of the battery is connected to the inside coating. The current, as indicated by the steady deflection of the galvanometer, is 1.4×10^{-10} amperes. The resistance of the glass between the coatings is therefore equal to 7,100,000 megohms (one megohm is equal to 1,000,000 ohms).

Measurement of Electromotive Force.

159. The potentiometer. — The potentiometer is a device which is now extensively used for the accurate measurement of electromotive force. The essential features of this instrument may be best described by referring to the slide-wire form † of the potentiometer, the essential features of which are shown in Fig. 209.

* Insulators do not conform to Ohm's Law, or, in other words, the current through an insulator is not strictly proportional to the electromotive force. Different values will therefore be obtained for the insulation resistance according to the value of electromotive force used.

† Commercial forms of the potentiometer for accurate electromotive force measurements are described in *Practical Physics* by Franklin, Crawford and MacNutt, Vol. II, pages 66–74.

A bare German silver wire WW is stretched upon a board and connected to a battery B so that an *invariable* current i flows through it. A side circuit, containing a sensitive galvanometer G and a voltaic cell of which the electromotive force e is to be

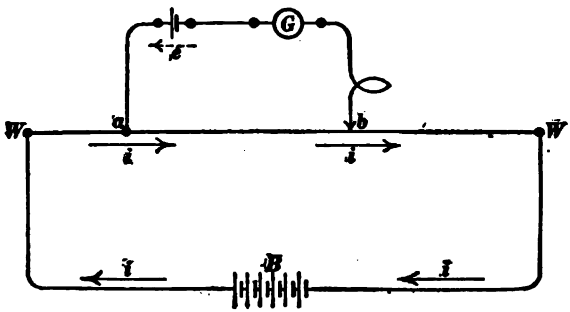

Fig. 209.

measured, is connected to the wire WW by means of two sliding contacts a and b. The sliding contact b is adjusted until the galvanometer gives no deflection, then

$$e = ri \tag{i}$$

where r is the resistance of the portion ab of the German silver wire. The voltaic cell e is replaced by a standard cell of which the electromotive force e' is known, and the sliding contact b is again adjusted until the galvanometer gives no deflection. Then

$$e' = r'i \tag{ii}$$

where r' is the resistance of the portion ab' of the German silver wire. Dividing equation (i) by equation (ii), member by member, we have

$$\frac{e}{e'} = \frac{r}{r'} \tag{iii}$$

The ratio r/r', however, is equal to the ratio of the lengths of the respective portions of the wire WW, and this ratio may there-

fore be determined by measuring these lengths, so that the ratio of the two electromotive forces is then known.

Standard cells. — The Clark standard cell is described on page 16. Its electromotive force in volts at t° C. is given by the equation

$$E = 1.4292 - 0.00123(t - 18) - 0.000007(t - 18)^2$$

The Weston cell is similar in every respect to the Clark cell except that cadmium amalgam and cadmium sulphate are used instead of zinc amalgam and zinc sulphate. The electromotive force in volts of the cadmium cell (with concentrated solution) at t° C. is given by the equation

$$E = 1.0187 - 0.000035(t - 18) - 0.00000065(t - 18)^2$$

Measurement of Power.

160. Measurement of power by means of the ammeter and voltmeter. — The power delivered to an electrical circuit may be calculated from the equation $P = EI$, when the current I in the circuit and the electromotive force E between (across) the terminals of the circuit have been measured. This method is applicable only in the case of direct currents, that is, where the current I and the electromotive force E are steady in value.

161. Measurement by means of the wattmeter. — The wattmeter is a special form of electrodynamometer the connections of which are shown in Fig. 210. A fixed coil of coarse wire B is connected in series with the receiving circuit to which the power to be measured is delivered, and a suspended or pivoted coil A of fine wire is connected across the supply mains in series with a non-inductive resistance R. The total current i which delivered to the receiving circuit flows through the fixed coil B, is a current which is proportional to the supply voltage e flows

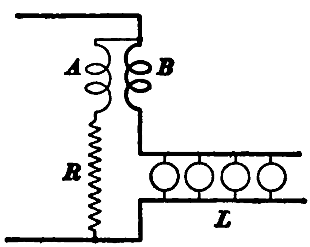

Fig. 210.

through the pivoted coil A (this current is equal to e/R), and the force action between the two coils causes the coil A to move and carry a pointer over a divided scale. The force action between the two coils is proportional to the product of the currents in the respective coils, that is, the force action is proportional to $e/R \times i$ or proportional to ei since R is constant. But ei is the power delivered to the receiving circuit, and therefore, since the force exerted on the pointer is proportional to the delivered power, the scale over which the pointer plays may be divided and numbered so as to indicate watts of power directly. The wattmeter is always used for the measurement of power delivered by an alternator.

Use of the Ballistic Galvanometer.

162. Measurement of electric charge and of magnetic flux by means of the ballistic galvanometer. — (*a*) When a ballistic galvanometer is used to measure the discharge q from a condenser, the charged condenser is connected to the galvanometer terminals, and the throw d of the galvanometer is observed. Then

$$q = kd \tag{i}$$

in which k is a proportionality factor which is called the *reduction factor* of the galvanometer. This reduction factor is generally determined by observing the throw d produced by a known amount of charge q. Thus a condenser of known capacity C may be charged by a known electromotive force E and discharged through the galvanometer, giving $q = EC = kd$ from which k may be calculated.

(*b*) The ballistic galvanometer is frequently used to measure what is called the *impulse value* of the momentary electromotive force which is induced in a coil of wire during the time that the magnetic flux through the coil is changing by a certain amount. Thus, a coil containing Z turns of wire is placed in a magnetic field so that a certain amount of magnetic flux Φ * passes through

* This flux ϕ represents the flux through a mean turn of wire on the coil.

the coil. The coil is connected to a ballistic galvanometer, then quickly removed from the field, and the galvanometer throw d is observed. This throw is proportional to the product $Z\Phi$, so that we may write

$$Z\Phi = k'd \tag{ii}$$

in which k' is a constant *for a given value of the resistance of the galvanometer circuit*, and it is to be determined by observing the throw produced by a known value of $Z\Phi$. If the resistance of the galvanometer circuit is changed, the value of k' is altered.

The value of $Z\Phi$ in the above discussion is the *impulse value* of the electromotive force which is induced in the coil of wire during the time that it is being withdrawn from the magnetic field. Let t be the short interval of time which elapses during the movement of the coil. Then the flux through the coil changes from Φ to zero during t seconds, the average rate of change of flux is Φ/t, the average value of the electromotive force which is induced in the coil is $Z\Phi/t$, and the product of this average electromotive force and the time is equal to $Z\Phi$. *The product of the average value of the electromotive force and the time during which the electromotive force continues to act is called the impulse value of the electromotive force.*

163. Measurement of capacity * — The simplest method of measuring the capacity of a condenser is to charge the condenser by a known electromotive force, discharge it through a ballistic galvanometer of which the reduction factor k is known, and observe the deflection d which is produced. Then $q = kd = CE$ from which C may be calculated.

* The most accurate method for measuring the capacity of a condenser is to use a rapidly rotating commutator-device arranged to charge the condenser a known number of times per second from a battery of known electromotive force and discharge the condenser the same number of times per second through an ordinary galvanometer, the steady deflection of which measures the average value of the current. The most accurate method for determining the ratio of the capacities of two condensers is by means of Wheatstone's bridge, as described in *Practical Physics*, Franklin, Crawford and MacNutt, Vol. 2, page 133. A method for measuring the ratio of the inductances of two coils by means of Wheatstone's bridge is described in *Practical Physics*, Franklin, Crawford and MacNutt, Vol. 2, page 129.

164. Measurement of magnetic flux. — Consider an iron rod through which a certain amount of magnetic flux passes, due, for example, to the magnetizing action of a winding of wire through which a current is flowing. A reversal of this magnetizing current produces a sudden reversal of the magnetic flux Φ through the rod, so that the total *change* of flux (from $+\Phi$ to $-\Phi$) is equal to 2Φ. An auxiliary coil having Z turns of wire is placed upon the iron rod and connected to a ballistic galvanometer, and the throw d of the ballistic galvanometer is observed at the instant of reversal of the magnetizing current. Then we have $2\Phi Z = k'd$, inasmuch as the product of the change of flux 2Φ and the number of turns of wire in the coil gives the impulse value of the electromotive force, and this is equal to $k'd$. The reduction factor k' of the ballistic galvanometer being known,* the value of Φ can be easily calculated.

Measurement of Magnetic Fields.

165. Gauss's method for measuring the horizontal component H' of the earth's magnetic field, and for measuring the magnetic moment of a magnet. — This method involves two independent sets of observations, the first set being made with a certain arrangement of apparatus and the second set being made with a different arrangement of apparatus, as follows:

First arrangement. — A large magnet is suspended horizontally at the place where H' is to be determined, set vibrating about the vertical axis of suspension and the time t of one complete vibration is determined by observation. Then from equation (23) we have

$$\frac{4\pi^2 K}{t^2} = mlH' \tag{i}$$

The moment of inertia K of the magnet is to be determined from the measured dimensions and weight (in grams) of the bar.

* A method for determining the value of k' is described on page 18, Vol. II, *Practical Physics*, Franklin, Crawford and MacNutt.

Second arrangement. — A small magnet ns, Fig. 211, is suspended at the place which was occupied by the large magnet in the first arrangement; this small magnet being free to turn, points in the direction of the magnetic field in which it is placed, that is, in the direction of H'. The large magnet used in the first

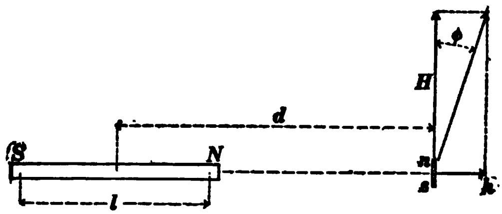

Fig. 211.

arrangement is now placed with its center at a distance d due magnetic east or west of the small magnet ns as shown in Fig. 211. The large magnet then produces at the small magnet a magnetic field h which is at right angles to H', and the small magnet then points in the direction of the resultant of h and H', having turned through the angle ϕ which is observed.

From the diagram, Fig. 211, we have

$$\tan\phi = \frac{h}{H'} \tag{ii}$$

From equation (17) we have $-m/(d - \frac{1}{2}l)^2$ as the expression for the intensity of the magnetic field at ns due to the south pole of the large magnet; and $+m/(d + \frac{1}{2}l)^2$ for the field intensity at ns due to the north pole; so that

$$h = \frac{m}{\left(d - \frac{l}{2}\right)^2} - \frac{m}{\left(d + \frac{l}{2}\right)^2} \tag{iii}$$

This equation may be simplified as follows: Reduce the fractions $m/(d - \frac{1}{2}l)^2$ and $m/(d + \frac{1}{2}l)^2$ to a common denominator. We then have

$$h = 2mld \cdot \frac{1}{\left(d^2 - \frac{l^2}{4}\right)^2}$$

Multiply numerator and denominator of the second member of this equation by $(d^2 + \frac{1}{4}l^2)^2$ and we have

$$h = 2mld. \frac{d^4 + \frac{d^2l^2}{2} + \frac{l^4}{16}}{\left(d^4 - \frac{l^4}{16}\right)^2}$$

In this expression $l^4/16$ may be dropped, since l is small compared to d, and l^4 is very small compared to d^4. Therefore

$$h = \frac{2ml}{d^5}\left(d^2 + \frac{l^2}{2}\right) \qquad \text{(iv)}$$

Substitute this simplified value of h in equation (ii), and we have

$$\tan \phi = \frac{2ml}{H'd^5}\left(d^2 + \frac{l^2}{2}\right) \qquad \text{(v)}$$

The large magnet may now be placed nearer to ns (Fig. 211), say at distance d_1, the corresponding angle of deflection being ϕ_1, and we have

$$\tan \phi_1 = \frac{2ml}{H'd_1^5}\left(d_1^2 + \frac{l^2}{2}\right) \qquad \text{(vi)}$$

The uncertain quantity l, which is the distance between the poles of the large magnet, may be eliminated from equation (v) with the help of equation (vi), giving

$$\frac{ml}{H'} = \frac{d^5 \tan \phi - d_1^5 \tan \phi_1}{2(d^2 - d_1^2)} \qquad \text{(vii)}$$

Observations and calculations. — The quantity t, equation (i), is observed and K is calculated from the measured mass and dimensions of the large magnet, leaving only ml and H' un-

known in equation (i). The quantities d, d_1, ϕ and ϕ_1 in equation (vii) are observed, leaving only ml and H' unknown in (vii). Equations (i) and (vii) then enable the calculation of both ml and H'.

If it is desired to determine the strength of the poles of the large magnet, the quantity l may be approximately measured, and m calculated.

This method* for determining ml and H was devised by Gauss.

166. Measurement of magnetic field intensity by means of the tangent galvanometer. — When the value of the horizontal component of the earth's magnetic field H' is known, the tangent galvanometer may be used to measure the value of the current in amperes or abamperes, as explained in Art. 57. If a known current (measured by a copper coulombmeter, for example) is sent through a tangent galvanometer and the deflection ϕ observed, then the value of H' may be calculated, the number of turns of wire Z and the mean radius r of the coil being known.

167. Measurement of magnetic field intensity by means of the bismuth inductometer. — The bismuth inductometer is a small resistance coil made of fine bismuth wire. Its resistance varies with the intensity of the magnetic field in which it is placed. The relation between resistance and field intensity being once for all determined, the intensity of any field may be found by measuring the resistance of the inductometer when it is placed in the field.

168. Kohlrausch's method for the simultaneous absolute measurement of the horizontal component of the earth's magnetic field and of current. — The coil of a tangent galvanometer is suspended so as to enable the measurement of the torque T with which the earth's horizontal field H' acts upon it. This torque is given

* For fuller discussion of Gauss's method, see A. Gray, *Absolute Measurements in Electricity and Magnetism*, Vol. II, page 69.

by equation (40) in which πZr^2 may be written for A, giving

$$T = \pi Zr^2 IH' \tag{i}$$

At the same time the deflection ϕ of the needle of the galvanometer is observed so that, according to equation (36a), we have

$$I = \frac{rH'}{2\pi Z} \cdot \tan \phi \tag{ii}$$

The mean radius r and number of turns of wire Z in the coil being known, and T and ϕ being observed, these two equations determine the values of both I and H'.

APPENDIX A.

TERRESTRIAL MAGNETISM.*

1. The earth a great magnet. — The tendency of a compass needle to set itself in a particular direction at a given place on the earth was at a very early date attributed to some action of the earth. The famous Dr. Gilbert, Physician in Ordinary to Queen Elizabeth, in his Latin treatise † put forward the important idea that the earth is a great magnet, so that, in the language of

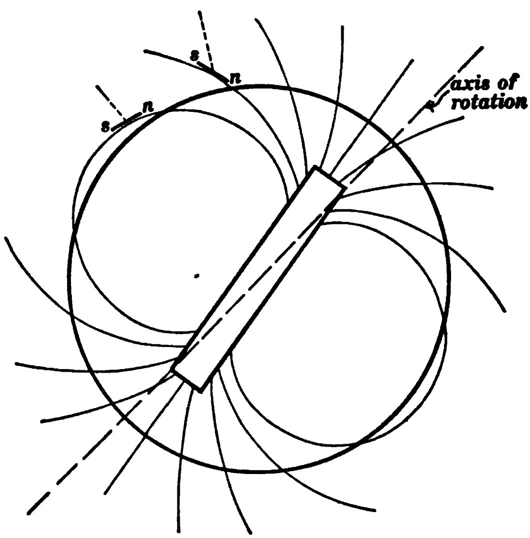

Fig. 1.

Faraday, there exists a magnetic field around the earth. The general character of the earth's magnetic field as to its direction

* A fairly complete discussion of terrestrial magnetism with many important references is given in Gray's *Treatise on Magnetism and Electricity*, Vol. I, pages 58–84, Macmillan and Company, 1898.

† *De Magnete magneticisque corporibus.* Translated into English about 1902.

and intensity at various points on the earth is that which would be produced by a large magnet inside of the earth with its axis slightly inclined to the axis of rotation of the earth, as shown in Fig. 1.

2. The compass. Definition of declination. — The compass needle is a horizontal magnet which is free to turn about a vertical axis. The direction in which such a needle points at a given place on the earth is called the *magnetic meridian* at that place, and the angle between the magnetic meridian and the geographic meridian is called the declination * of the earth's magnetic field at a given place.

3. The dip needle. Definition of inclination. — The needle of a compass is usually weighed at one end to make it lie in a horizontal plane. A steel bar which is magnetized after being accurately balanced on a horizontal pivot constitutes a *dip needle.* When the horizontal pivot of the dip needle is placed at right angles to the magnetic meridian, the needle points in the actual direction of the earth's magnetic field, as shown by the two suspended magnets *ns* and *ns* in Fig. 1, and the angle of inclination of the needle is called the *inclination* or *dip* of the earth's magnetic field at the given place. Figure 2 is a general view of a dip needle, or *dip circle*, as it is usually called.

Fig. 2.

4. Magnetic elements. — The direction and intensity of the earth's magnetic field at a place is completely specified when the declination, the inclination, and the value of the horizontal component

* Sometimes called the *variation* of the compass. This word variation, however, is here used to designate the changes which are continually taking place in the earth's magnetic field.

are given. These three things therefore constitute what are called the *magnetic elements* at a given place.*

5. Magnetic maps. — Figure 3 is a map of the world showing the lines of *equal magnetic declination* for the year 1905. The heavy black curves pass through the regions where the compass needle points true north, and the numbers attached to the fine-

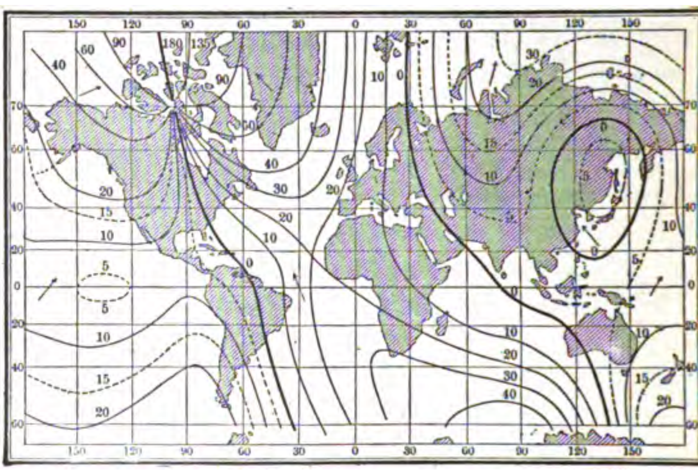

Fig. 3.
Lines of equal magnetic declination.

line curves are the values of the declination. Thus, in England the compass points 20° to the east of north and at San Francisco the compass points about 17° west of north. Figure 4 is a map of the world showing the lines of *equal magnetic dip* or *inclination* for the year 1905. Thus, the dip needle stands in a horizontal position at all places on the heavy curve which is marked zero, the magnetic dip in England is about 70° (north pole of dip needle down), and the dip at Cape Town, South Africa, is about 55° (south pole of dip needle down). Figure 5 is a map of the world showing the lines of *equal horizontal inten-*

* The methods in use in the Magnetic Observatory at Kew, England, for determining the magnetic elements are fully described in Stewart and Gee, *Elementary Practical Physics*, Vol. II, pages 274–313.

sity. Thus, in England the horizontal intensity is about 0.17 gauss, and in Florida the horizontal intensity is about 0.30 gauss.

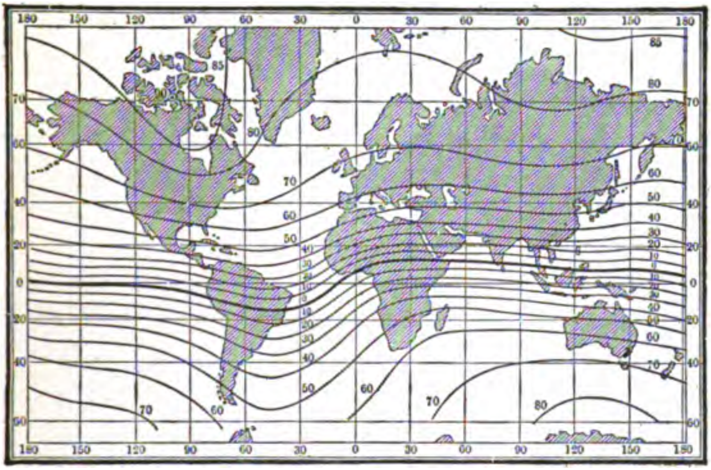

Fig. 4.
Lines of equal magnetic inclination.

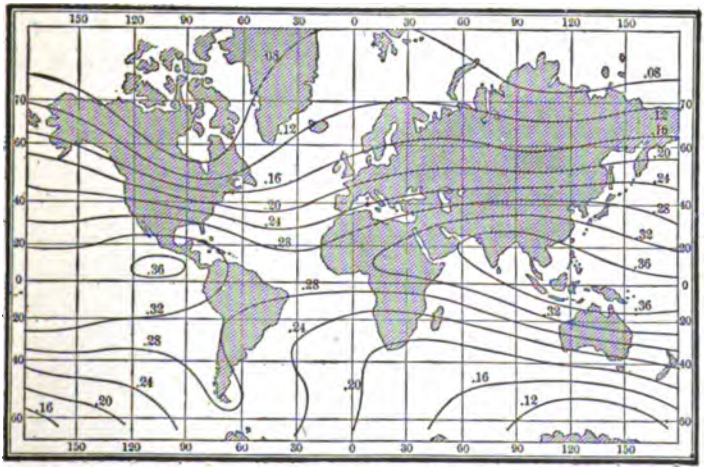

Fig. 5.
Lines of equal horizontal intensity.

6. Variations of the earth's magnetic field.— Each one of the magnetic elements, declination, inclination, and horizontal intensity, is subject to variations of four distinct kinds, as follows:

(*a*) *The diurnal variation.* — Each magnetic element is subject to a daily periodic change. This is called the diurnal variation. Thus, the curves in Figs. 6*a* and 6*b* show the diurnal variation of

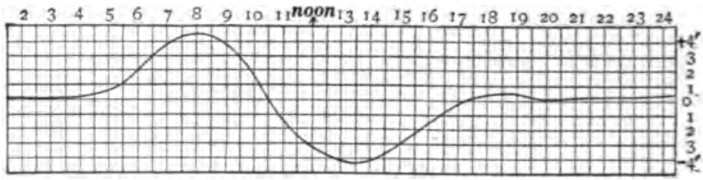

Fig. 6a.

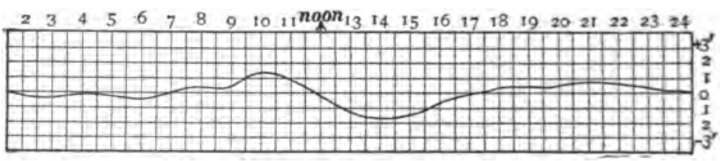

Fig. 6b.

the magnetic declination at the United States Magnetic Observatory at Baldwin, Kansas. Figure 6*a* shows the diurnal variation in mid-summer and Fig. 6*b* shows the diurnal variation in mid-winter.

(*b*) *The annual variation.* — Each magnetic element is subject to an annual periodic change which is called the annual variation.

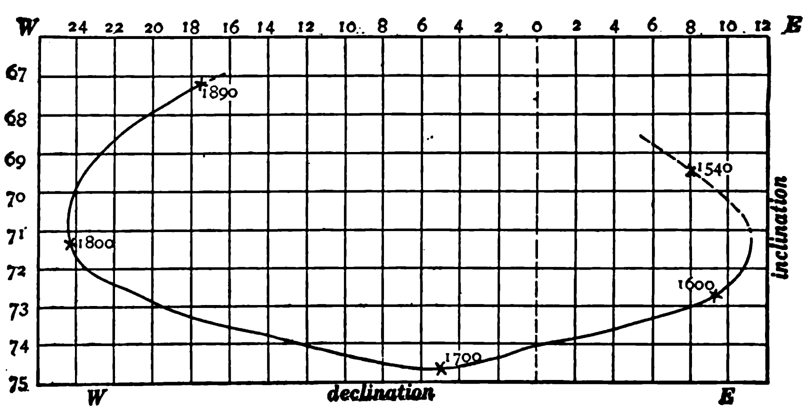

Fig. 7.

(*c*) *The secular variation.* — Each magnetic element is subject to a slow change from year to year. This is called the secular variation. Thus, the curve in Fig. 7 shows the secular variation of the magnetic declination at London from 1540 to 1890.

(*d*) *Magnetic storms.* — Each magnetic element is subject to erratic variations. These erratic variations occur at times of great disturbances in the sun as indicated by sun-spot activity, and also at times of great disturbances in the earth such as volcanic eruptions and earthquakes, and they are called magnetic storms.

APPENDIX B.

SHIP'S MAGNETISM AND THE COMPENSATION OF THE COMPASS.*

7. The ship's compass.— The style of ship's compass which is now almost universally used is that which is due to Lord Kelvin. The card of this compass is shown in Fig. 8. The points of the compass and the circle divisions are printed on a paper ring to which is attached a light rim of aluminum which keeps it in shape. Radial threads connect the ring to a central disk which contains a sapphire cap by which the compass is supported on an iridium point. Eight small magnets of glass-hard steel are tied to the radial threads four on either side of the jewel cap, as shown in the figure. The entire weight of the card, including the magnetic needles, is 170½ grains, and this extreme lightness combined with the relatively large moment of inertia due to the distribution of the mass, insures a long period of free vibration and therefore great steadiness. The lightness of the card also

Fig. 8.

* A good discussion of this subject is given in Gray's *Treatise on Magnetism and Electricity*, Vol. I, pages 85–100, Macmillan and Company, 1898. For full details, the reader is referred to Lord Kelvin's *Instructions for Adjusting the Compass*, to be obtained from James White, of Glasgow. The practice in the United States Navy concerning the matter of compass errors and compass adjustments is given in several small pamphlets which are published by the United States Navy Department, and in a book entitled *A Treatise on Navigation*, by Commander W. C. P. Muir, U. S. Navy, Annapolis, 1906. The practice in the British Navy is given in the *Admiralty Manual of Deviations of the Compass*.

gives a very small frictional resistance at the supporting point. The compass card with its attached needles is supported in a copper bowl which is supported on *gimbals*, so that the compass remains horizontal in spite of the rolling motion of the ship. The complete instrument is supported on a column which contains or supports the compensating devices which are explained later, and the entire arrangement is called the *binnacle*.

When a ship contains no iron or steel the compass points in the direction of the magnetic meridian, and a chart like Fig. 3 enables a navigator to infer the true heading of a ship from an observed reading of the compass. When, however, the ship is made of iron or steel, or, when it carries a cargo of iron or steel, the compass is usually deflected by the magnetism of the ship or of its cargo. In order that a compass may be used for purposes of navigation under such conditions the errors of the compass may be determined by a careful set of observations, or the influence of the ship's magnetism may be compensated, thus reducing the compass errors approximately to zero. The latter method is the one which is usually employed, and in some cases the residual errors which remain on account of incomplete compensation are determined by a careful set of observations and allowed for in the use of the compass.

8. Ship's magnetism. — A ball of iron which is devoid of permanent magnetism, is weakly magnetized by the earth's field. This magnetism, which is not in a fixed direction in the ball, but which is always in the direction of the earth's field however the ball may be held or turned, is called the *temporary magnetism* of the ball, and it is proportional to the intensity of the earth's field. If the ball is elongated like an ellipsoid its temporary magnetism is not in general parallel to the earth's field, and in the case of a long slim iron rod its temporary magnetism is in the direction of its length and proportional to the component of the earth's field which is parallel to it inasmuch as that part of the earth's field which is at right angles to a slim rod produces no perceptible magnetism.

When a ball or rod of iron has a certain amount of permanent magnetism, the effect of the earth's magnetic field upon it is to produce an additional temporary magnetism, that is to say, the magnetism of the ball or rod is the sum of two distinct parts, a *permanent magnetism* and a *temporary magnetism.* Of course the permanent magnetism of a rod may be changed by severe mechanical shocks; the word permanent here refers to that part of the magnetism which does not change as the ball or rod is slowly moved around in the earth's field.

Similarly, an iron ship has a certain amount of *permanent magnetism* which does not change as the ship moves around in the earth's magnetic field and a certain amount of *temporary magnetism* which is due to the magnetizing action of the earth's magnetic field.

9. Compass errors due to permanent magnetism of a ship.—The permanent magnetism of a ship produces at the compass box a magnetic field which is constant in value and fixed in direction with reference to the ship. The horizontal component of this field combines with the horizontal component of the earth's field to give a resultant field in the direction in which the compass needle points. Thus, H' in Fig. 9 represents the horizontal component of the earth's field, P represents the horizontal part of the magnetic field at the compass which is due to the permanent magnetism of the ship, R represents the resultant horizontal field at the compass, and θ represents the compass error due to the ship's permanent magnetism. The field P rotates with the ship and therefore the compass error θ has a series of positive values (to the east) throughout a half revolution of the ship, and a series of negative values (to the west) throughout a half revolution of the ship. Therefore the compass error due to the ship's permanent magnetism is called the *semicircular error.**

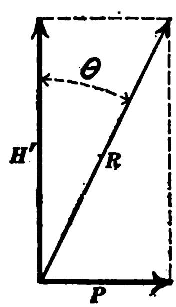

Fig. 9.

*The permanent magnetism of the ship contributes also to the heeling error which is discussed in Art. 14, and the so-called semicircular error is due partly to the temporary magnetism of the ship as explained in Art. 13.

10. The semicircular correctors. — The ideal compensation for the compass errors due to a ship's permanent magnetism would be to place a permanent steel magnet in such a position that it would produce, at the compass box, a magnetic field equal and opposite to the field produced at the compass box by the ship's permanent magnetism. As long as the ship remains on an even keel, however, it is only the horizontal part P, Fig. 9, of the field which is produced at the compass box by the permanent magnetism of the ship, which causes the deflection of the compass. Therefore it is sufficient to neutralize this horizontal field P. For this purpose, one or more horizontal magnets are placed in trays in the pedestal of the binnacle and adjusted until they produce a field at the compass box which is equal and opposite to P. Usually, two such trays are employed, in one of which, magnets are placed parallel to the line of the keel of the ship so as to annul the bow component of P, and in the other of which, magnets are placed at right angles to the line of the keel so as to annul the athwart-ship component of P. These two trays with their permanent magnets are called the *semicircular correctors.*

11. Compass errors due to temporary magnetism of a ship. — An idea of the general character of the compass errors which are due to the temporary magnetism of a ship may be obtained by imagining the ship to be a long slim bar AB, Fig. 10, with a compass box at the point C. The earth's horizontal field H' may be resolved into two components, one parallel to AB and the other at right angles to AB. The component which is at right angles to AB has no perceptible magnetizing action on AB, the component which is parallel to AB causes the end B to become a north pole and the end A to become a south pole, and the magnetic field at C due to these magnet poles is parallel to BA and towards A. The magnetic field at the compass box which is due to the temporary magnetism of the bar AB in Fig. 10 is represented by the arrow T in Fig. 11, the earth's field at the compass box is represented by H', and the arrow R represents the resultant field at the compass box

in the direction of which the compass needle points; therefore the angle ϕ is the compass error. Imagine B, Fig. 10, to represent the bow of the ship, and suppose the ship to start with its bow due north and swing around to the east, the angle a increasing from zero to 360°. A careful consideration of Figs. 10 and 11 will show that the angle ϕ has a series of westerly values

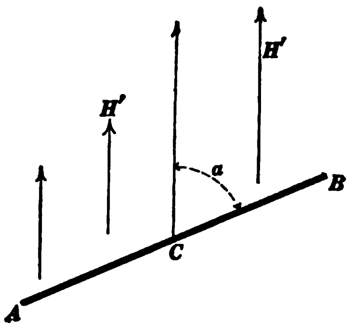

Fig. 10.

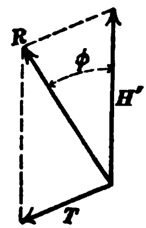

Fig. 11.

throughout the first quadrant (a between zero and 90°), a series of easterly values throughout the second quadrant (a between 90° and 180°), a second series of westerly values throughout the third quadrant (a between 180° and 270°), and a second series of easterly values throughout the fourth quadrant. The compass error due to the temporary magnetism of a ship is therefore called the *quadrantal error.*

When the ship's compass is located on the center line of the ship so that the iron of the ship is symmetrically placed on the two sides of the compass, then the compass error due to the ship's temporary magnetism is zero when the ship heads north, east, south, or west, as may be shown as follows: When the ship heads magnetic north or south, its temporary magnetism is symmetrical as shown in Fig. 12, the magnetic field at the compass due to the temporary magnetism of the ship is therefore due south, and consequently the compass is not deflected. Figure 13 shows a compass box C placed on the center line of a ship of which the dissimilarity of bow and stern is greatly exaggerated.

Imagine the vessel to be made of solid iron and consider the transverse slice of iron which lies between the dotted lines in Fig.

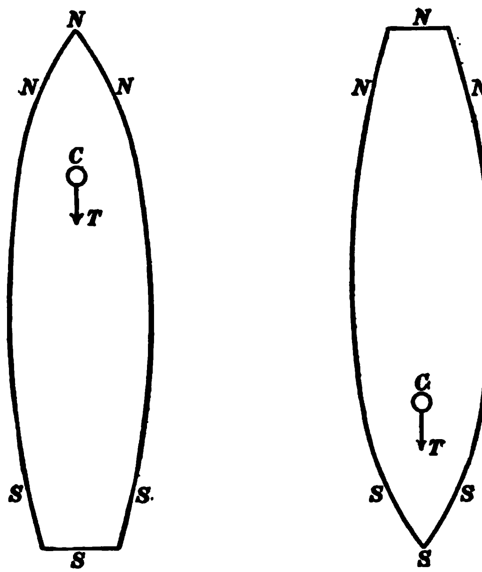

Fig. 12.

13. When the bow points east or west (magnetic) the transverse slice is magnetized as indicated by the letters N and S, and the curved line ff, which represents a line of force due to the poles N and S, shows that the field at C which is produced by the

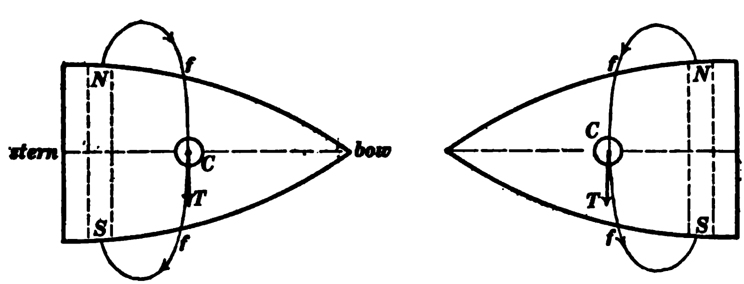

Fig. 13.

magnetism of the transverse slice is towards the south. What is here said concerning a given transverse slice of the ship is true

of every transverse slice, and therefore the field at C due to the transverse magnetization of the entire ship when its bow points east or west (magnetic) is towards the south, and consequently the compass is not deflected.

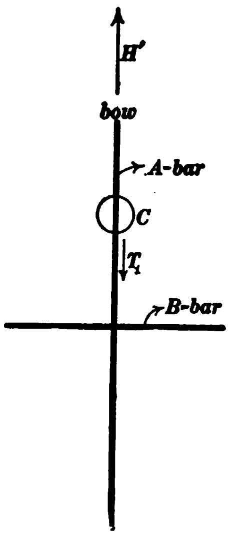

Fig. 14.

12. Compensation of quadrantal error. *Quadrantal correctors.* — From Figs. 12 and 13 it is evident that the temporary magnetism of the ship weakens the field at the compass (T opposite to H' in direction, so that the resultant of T and H' is less than H') when the ship heads north, east, south or west. *If the value of T is the same in Figs. 12 and 13, it can be shown that the temporary magnetism of the ship does not tend to deflect the compass, whatever the direction of the bow of the vessel.*

To prove this proposition, we will assume that the iron of the ship is equivalent to two long slim horizontal bars of iron, one parallel to the ship's keel (the A-bar) and the other at right angles thereto (the B-bar).

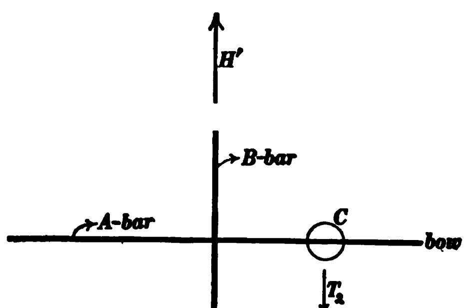

Fig. 15.

When the ship is headed north as shown in Fig. 14, the full value of H' acts to magnetize the A-bar, and the magnetic field T_1,

which is produced at the compass box by the magnetization of the A-bar, is proportional * to H' or equal to k_1H'. When the ship is headed east, as shown in Fig. 15, the full value of H' acts to magnetize the B-bar, and the magnetic field T_2, which is produced at the compass box by the magnetization of the B-bar, is proportional to H' or equal to k_2H'. Therefore, if $T_1 = T_2$, then $k_1 = k_2$. The letter k will be used in what follows for k_1 and k_2.

Consider the ship when it is headed a degrees east of north as shown in Fig. 16. The component of H' which magnetizes the A-bar is $H' \cos a$, and the magnetic field T_a which is produced at C by the magnetization of the A-bar is $k \times H' \cos a$. The component of H' which magnetizes the B-bar is $H' \sin a$, and the magnetic field T_b which is produced at C by the magnetization of the B-bar is $k \times H' \sin a$. The resultant of T_a and T_b is

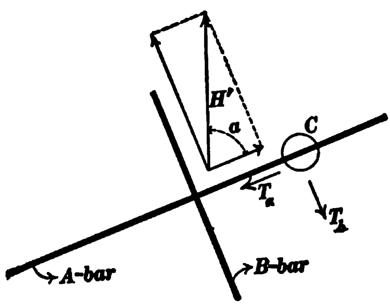

Fig. 16.

$$\sqrt{T_a^2 + T_b^2} = kH' \sqrt{\cos^2 a + \sin^2 a} = kH' \qquad \text{(i)}$$

Therefore the resultant of T_a and T_b is constant in value, and, since $T_a = kH' \times \cos a$ and $T_b = kH' \times \sin a$, it is evident that the resultant of T_a and T_b is always opposite to H' in direction, so that the actual field at the compass box is constant in value and always parallel to H', or, in other words, the compass error due to the temporary magnetism of the ship is zero on all headings of the ship when T_1 in Fig. 14 is equal to T_2 in Fig. 15.

The quadrantal correctors. — The quadrantal error of the ship's compass is eliminated (that is to say, compensated) by means of

* Because the magnetization of the A-bar is proportional to H', and the field T is proportional to the magnetization of the A-bar.

two iron spheres SS which are usually * placed on the two sides of the compass, as shown to an exaggerated scale in Fig. 17. These spheres are called the quadrantal correctors and the practical method of adjusting them is explained in Art. 16. The action of the quadrantal correctors may be understood with the help of Figs. 18*a* and 18*b* as follows: When the line joining the centers of the two spheres SS is parallel to H' as shown in Fig. 18*a*, the magnetic field at the point p is more intense than H'; and when the line joining the centers of the

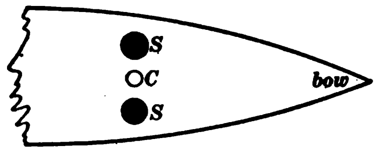

Fig. 17.

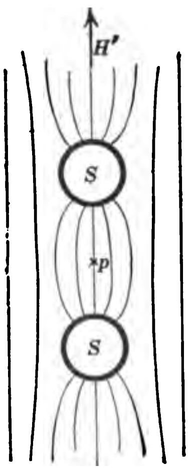

Fig. 18a.

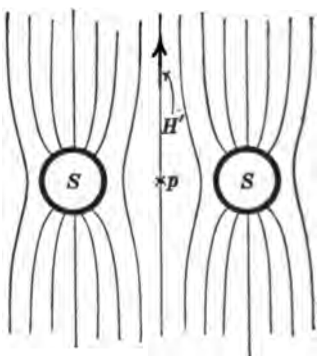

Fig. 18b.

spheres is at right angles to H' as shown in Fig. 18*b*, the magnetic field at p is less intense than H'. Now the weakening of the magnetic field at the compass box by the temporary magnetism of the ship when the ship heads east or west is usually greater than the weakening of the field at the compass box by the temporary magnetism of the ship when the ship heads north or south. That

* In some cases, namely, when the coefficient k_1 is greater than the coefficient k_2, it is necessary to place the quadrantal correctors fore and aft of the compass box.

is to say, T is usually greater in Fig. 13 than it is in Fig. 12, or the coefficient k_2 is usually greater than the coefficient k_1. Therefore by placing the quadrantal correctors in the positions shown in Fig. 17 and moving them closer to or farther away from the compass, the weakening of the magnetic field at the compass by the combined temporary magnetism of ship and correctors, may be made the same with the ship's head north (or south) as with ship's head east (or west), and when this condition is reached the quadrantal error of the compass is eliminated as explained above.

13. Compass error due to the magnetizing action of the vertical component of the earth's magnetic field. — The vertical component V of the earth's field produces a temporary magnetism of all the vertical iron in the ship; this "temporary" magnetism remains *unaltered* as long as V remains unchanged, the ship being supposed to stand on even keel; and therefore the "temporary" magnetism due to V merges with the permanent magnetism of the ship in the production of the semicircular compass error.

The temporary magnetism due to V is distinguishable from the permanent magnetism of the ship, however, because it changes when the ship goes from one port to another where the value of V is different. Thus, if the semicircular error is completely compensated at the home port by means of the semicircular correctors (permanent magnets), then a perceptible amount of semicircular error will appear when the ship sails to a distant port where the value of V is different. In order to overcome this difficulty, that is, in order to compensate the semicircular error so that the compensation may hold good on a long cruise, it is necessary to compensate, by means of the semicircular correctors, only that part of the semicircular error which is due to *permanent magnetism*, the remainder of the semicircular error (which is due to vertical temporary magnetism) being compensated by means of a vertical soft iron rod properly placed near the compass box. The use of this rod was proposed originally by Captain Flinders and it is usually called *Flinders' bar*. The action of Flinders' bar may

be explained as follows: When V changes in value the magnetism of Flinders' bar and the vertical temporary magnetism of the ship change together, and Flinders' bar being once for all adjusted to compensate the effect of the vertical temporary magnetism of the ship, the compensation holds, whatever the value of V may be. Flinders' bar is usually about three inches in diameter and from 6 to 24 inches long, according to the amount of iron in the vessel, and it is usually* placed forward or aft of the binnacle.

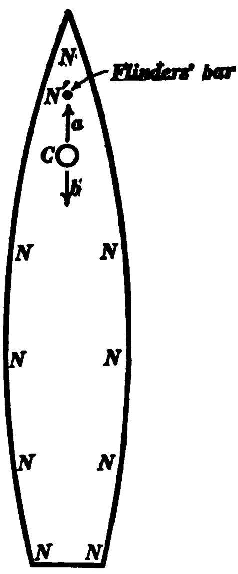

Fig. 19a.

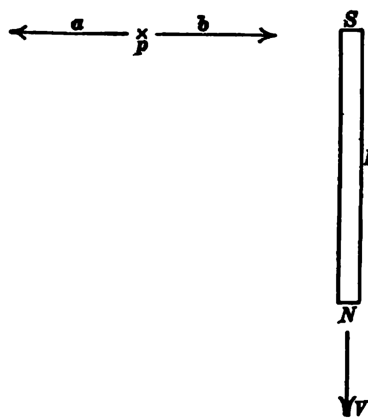

Fig. 19b.

* Figure 19*a* shows the north polarity *NNNN*, etc., on the deck of an iron vessel due to the vertical component of the earth's field. This north polarity is distributed symmetrically with respect to the compass box C (ship's iron being symmetrical with respect to the compass box), and it produces, at the compass, a magnetic field of which the horizontal component is represented by the arrow a which is parallel to the keel. Flinders' bar is placed in the position shown, and its north pole N' (upper end of bar), which is on a level with the compass box, produces at the compass box a field b which is equal and opposite to a. Figure 19*b* shows a side view of Flinders' bar F (the compass box is supposed to be placed at the point p). Flinders' bar is magnetized by the vertical component V of the earth's magnetic field, a is the horizontal part of the field which is produced at the compass box by the vertical

The method of adjusting Flinders' bar is explained in Art. 16.

14. The heeling error. — Let us suppose that the semicircular and quadrantal errors have been completely compensated by means of the semicircular correctors and quadrantal correctors, the ship being all the time on an even keel. Under these conditions a deflection of the compass is produced when the ship rolls, or heels over, at sea. This deflection of the compass is called the *heeling error*, and it is due in part to the variation of the temporary magnetism of the ship which accompanies the change of direction of the earth's magnetic field with reference to the ship's iron as the ship rolls, and in part to the permanent magnetism of the ship, as follows: The horizontal field P, Fig. 9, is annulled by the semicircular correctors, and the vertical component of the field at the compass box which is produced by the permanent magnetism of the ship is left unaltered by the semicircular correctors. By vertical component is here meant that component which is perpendicular to the ship's deck, and which, as the ship rolls, turns out of the true vertical, and has a horizontal component, at the compass, which deflects the compass. In describing the action of the heeling corrector, the heeling error will be assumed to be due entirely to the permanent magnetism of the ship.

Assuming the heeling error to be due entirely to the permanent magnetism of the ship, that is, to be due to the component P' (perpendicular to the deck) of the field which is produced at the compass box by the permanent magnetism of the ship, it is evident that the heeling error is a maximum when the ship heads north or south, and zero when the ship heads east or west; for, when the ship heels over with its head to the east or west, the part of P' which is projected upon a horizontal plane is directed towards the north or south and does not deflect the compass, whereas, when the ship heels over with its head north or south, the part of P' which comes into a horizontal plane is directed towards the east or west and it deflects the compass.

temporary magnetism of the ship's iron, and b is the field produced at the compass box by Flinders' bar.

The *heeling corrector* is a vertical steel magnet placed directly beneath the compass box and adjusted up or down until it produces at the compass box a vertical field which is equal and opposite to P'. The practical method of adjusting the heeling corrector is explained in Art. 16.

15. Compass errors due to magnetic lag. — The temporary magnetism of the ship's iron tends to lag behind the magnetic field which produces it. Thus, after a ship has been standing for some time in one direction the magnetism which is produced by the earth's field does not at once disappear when the ship turns around, but tends to persist. This magnetic lag produces a compass error which is known as Gaussin's error and which cannot be compensated.

16. Directions for adjusting the correctors of a ship's compass. (*a*) *Adjustment of semicircular correctors.* — The quadrantal error is zero with ship's head north, east, south, or west. Therefore any deviation of the compass which exists on these headings is due to the semicircular error. With the ship's head north (magnetic), place one or more athwartship magnets in one of the semicircular-corrector trays and move them up or down until the compass points north. Then head the ship east (magnetic) and place fore and aft magnets in the other semicircular-corrector tray and move them up or down until the compass points north.

(*b*) *Adjustment of quadrantal correctors.* — Having corrected the semicircular deviation of the compass, head the vessel northeast (magnetic) or southeast, southwest, or northwest, and if any deviation of the compass exists, place the quadrantal spheres on the side brackets of the binnacle and move them in or out until the compass reading is correct.

(*c*) *Adjustment of the heeling corrector.* — With the ship headed north or south in a heavy sea, place the heeling-corrector magnet in its tube with its proper end upwards, and raise or lower it until the slow motion of the compass due to the rolling motion of the ship is nearly eliminated. The proper end up of the heeling-

corrector magnet may be inferred as follows: Suppose that the north end of the compass is deflected to the east when the ship rolls to the west. Then it is evident that the perpendicular-to-the-deck component P' of the field which is produced at the compass box by the permanent magnetism of the ship is downwards, because the part of it which is projected into a horizontal plane is to the east when the ship's masts roll to the west. In this case the north end of the heeling-corrector magnet is to be placed upwards so as to produce an upward field at the compass box.

(*d*) *Adjustment of Flinders' bar.* — Having carefully adjusted the semicircular correctors at the home port so as to annul completely the semicircular error, the ship is taken to a distant port and the semicircular error is observed with the ship's head east or west (magnetic). Let this error be represented by ϕ; let V and H' be the vertical and horizontal components of the earth's magnetic field at the home port and let V_1 and H_1' be the vertical and horizontal components of the earth's field at the distant port as determined by observation, or as taken from magnetic charts. The forward (or aft) component of the magnetic field which is produced at the compass box by the vertical temporary magnetism of the ship, is proportional to the vertical component of the earth's field and it is therefore equal to aV at the home port and equal to aV_1 at the distant port. The deviation of the compass which is produced by this field is proportional to its intensity and inversely proportional to the horizontal intensity of the earth's field. Therefore this deviation is equal to bV/H' at the home port and equal to bV_1/H_1' at the distant port, where a and b are proportionality factors. Therefore the observed compass deviation ϕ is equal to $b(V/H' - V_1/H_1')$, and the total compass deviation, ϕ_t, which is due to the vertical temporary magnetism of the ship at the distant port is equal to $\dfrac{V_1/H_1'}{V/H' - V_1/H_1'} \times \phi$. With the ship's head east at the distant port (the condition under which ϕ was observed), put Flinders'

bar into a vertical position in front of, or behind the compass box, and move it towards or away from the compass until the compass is turned through an angle ϕ_t in a direction opposite to ϕ, the angle ϕ_t being reckoned from the deflected position of the compass. Then eliminate the outstanding semicircular error by readjusting the semicircular correctors.

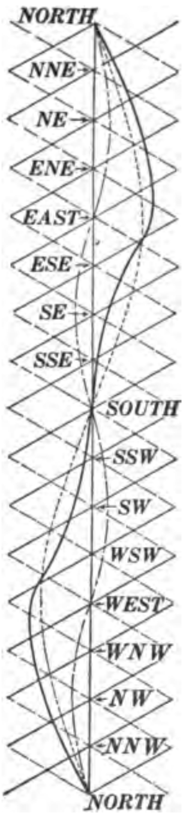

Fig. 20.

17. Napier's diagram.—After the compass correctors have been adjusted so as to approximately compensate the errors of the compass, it is customary, for the purpose of accurate navigation, to determine the residual errors of the compass and allow for them in the use of the compass. The ship is swung round and for successive actual readings of the compass, the compass error is determined by an independent determination of the true magnetic heading of the vessel.* Figure 20 shows Napier's method for representing the compass errors graphically. The successive actual compass readings are laid off along the fine vertical line as an axis, and the compass errors are laid off along the fine dotted lines which are inclined at an angle of 60° to the fine vertical line.

To determine the true magnetic course of the ship from the compass reading, start at the point on the vertical axis which corresponds to the actual compass reading, draw a line parallel to the fine *dotted lines* from the chosen point on the vertical axis to the curve of errors (which is the heavy curve in Fig. 20); from the point so reached on the curve of errors, draw a line parallel to

* The true magnetic heading is determined by a land-mark, if the vessel is in port, or by observations on the sun or stars if the vessel is at sea, the declination of the compass being known for the place of observation.

the fine *full lines*, and the point where this line cuts the vertical axis corresponds to the true magnetic course of the vessel. To determine the compass reading corresponding to a true magnetic course, start from a point on the vertical axis which corresponds to the true magnetic course, travel parallel to the fine *full lines* until the curve of errors is reached and then travel parallel to the fine dotted lines until the desired point on the vertical axis (corresponding to the actual compass reading) is reached.

The heavy curve in Fig. 20 represents the actual compass errors on the old British iron-clad *Achilles*, and the abscissas of the fine sine curves represent the semicircular errors and quadrantal errors, respectively. The maximum value of the semicircular error is 21° 15′, and the maximum value of the quadrantal error is 6° 9′.

PROBLEMS.

1. The semicircular error of a compass on board ship is found to have a maximum value of 20° to the east when the ship heads 36° west of south. Make a sketch of the outline of the deck of the vessel and draw a line on the deck showing the direction of the horizontal component of the magnetic field at the compass box which is due to the permanent magnetism of the ship, find the value of this horizontal component and find the angle between its direction and the direction of the keel, the earth's horizontal field being equal to 0.2 gauss. Ans. (*a*) 0.068 gauss, (*b*) 106° from bow towards port side (left side).

2. What is the value of the semicircular error of the compass when the ship specified in problem 1 heads 20° north of east? Ans. 19° 21′, west of north.

3. The only error of a ship's compass is that which is due to the ship's permanent magnetism, the quadrantal error being compensated. The semicircular error has a value of 6° to the west when the ship's head is true magnetic north and 4° to the west when the ship's head is true magnetic northeast. On what true headings will the error of the compass be zero? Ans. 38 minutes south of east, and 38 minutes north of west.

4. Suppose that the semicircular error of the ship's compass has been completely compensated and suppose that the quadrantal error is observed to be 4° to the west when the ship is headed true northeast. What is the deviation of the compass when the ship heads 30° south of east. Ans. 3° 20′ to the east.

Note. — In this problem treat the ship as one long slim iron bar parallel to the keel.

5. (*a*) A ship is headed true magnetic north (for which position the quadrantal error is zero), and the compass shows a deviation to the east. A permanent magnet is to be placed in an east-west direction (athwartship) underneath the compass box so as to bring the compass to true magnetic north. Which end of the magnet is to be placed to the east? (*b*) The ship is then headed true magnetic east and the compass is observed to have a deviation to the west. A permanent magnet is to be placed in an east-west direction (parallel to the keel) underneath the compass box so as to bring the compass to true magnetic north. Which end of the magnet is to be placed to the east? Ans. (*a*) north end east, (*b*) north end west.

6. The semicircular and quadrantal errors having been compensated the ship is headed magnetic south at sea and the compass is deflected to the west when the ship heels to the east (top of mast moves eastward). Which end of the heeling corrector magnet must be placed upwards in order to eliminate the heeling error? Ans. North end up.

APPENDIX C.

MISCELLANEOUS PHENOMENA.

18. Thermo-electricity.* *Seebeck's discovery.* — In 1821 Seebeck found that an electric current is produced in a circuit of two metals when one of the junctions of the two metals is warmer than the other. Seebeck used the arrangement shown in Fig. 21. The ends of a bent bar of copper were soldered to the ends of a rod of bismuth, a magnetic needle was pivoted between the bars as shown in the figure, and one of the junctions was heated by a spirit lamp. The existence of current is indicated by the deflection of the magnetic needle, and the direction of the current which is produced is shown by the arrows in Fig. 21. An arrangement such as is shown in Fig. 21 is called a *thermo-element.*

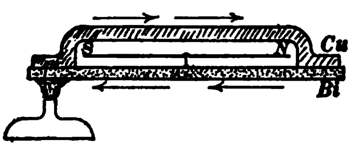

Fig. 21.

The thermopile. — The electromotive force of a single thermo-element seldom exceeds a few thousandths of a volt, even when the two junctions are at widely different temperatures. A number of thermo-elements may, however, be connected in series, as in Fig. 22, in which *AAAA* are bars of one metal and *BBBB* are bars of another metal. Junctions 1, 3, 5 and 7 are heated, while junctions 2, 4 and 6 are kept cool, or *vice versa.*

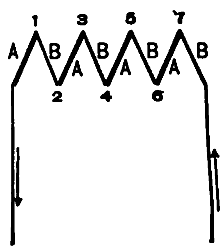

Fig. 22.

The thermo-element used as a pyrometer.† — When one junction of a thermo-element is kept at a constant standard temperature,

* A very good discussion of Thermo-electricity is given in *Magnetism and Electricity for Students* by H. E. Hadley, pages 359–382, Macmillan and Company, 1906.

† A pyrometer is a thermometer for measuring very high temperatures.

the electromotive force of the element is a function of the temperature T of the other junction, and, if the electromotive force of the element is determined once for all for a series of values of T, then any unknown temperature may be determined by observing the electromotive force of the thermo-element when one of its junctions is at the standard temperature and the other is at the temperature which is to be measured.

The electromotive force of a thermo-element can * be represented with a fair degree of accuracy by the equation

$$e = a + bT + cT^2 \tag{i}$$

when one junction of the element is kept at a fixed standard temperature, a, b and c being constants. Therefore in order to use a thermo-element as a pyrometer, it is sufficient to measure the electromotive force e of the element for three chosen known values of T. The thermo-element which has proved most satisfactory for use as a pyrometer is one employing pure platinum and an alloy of platinum and rhodium.

The Peltier effect.—In 1834 Peltier discovered that heat (independently of the heat generated in accordance with Joule's Law, Art. 12, Chapter II) is generated or absorbed at a junction of two metals when a current flows across the junction, that is, heat is generated when the current flows in one direction and absorbed when the direction of the current is reversed ; the generation of heat being shown by an increase of temperature of the junction, and the absorption of heat being shown by a cooling of the junction. For strong currents this Peltier effect is masked by the heat that is generated on account of electrical resistance, for the rate of generation of heat by the Peltier effect is proportional to the current, while the rate of generation of heat on account of resistance is proportional to the square of the current. The Peltier effect is most easily shown as follows : A current from a voltaic cell is sent through a thermopile. This current heats one set of junctions and cools the other set. The thermopile is then

* See *Magnetism and Electricity for Students*, H. E. Hadley, pages 361–367.

disconnected from the voltaic cell and connected to a galvanometer and the difference in temperature of the two sets of junctions is shown by the deflection of the galvanometer.

The Thomson effect. — When a liquid, like water, flows along a pipe which is not at a uniform temperature, the liquid always absorbs heat from the pipe at each point when it flows in the direction of increasing temperature of pipe, and the liquid always gives out heat to the pipe at each point when it flows in the direction of decreasing temperature. Lord Kelvin (then Sir William Thomson) discovered in 1851 that an electric current may either absorb or give out heat at each point in a wire when the temperature of the wire is not uniform. If the electric current absorbs heat at each point of a wire when it flows along a wire in the direction of increasing temperature, the Thomson effect is considered to be positive. If the electric current gives out heat at each point when it flows in the direction of increasing temperature, the Thomson effect is considered to be negative.

19. Pyro-electricity.*— A peculiar property of a crystal of tourmaline after its temperature had been increased or decreased was noted by Daumius in 1707. The crystal had the property of attracting small particles of ashes. Aepinus in 1756 recognized this property of a tourmaline crystal as an electrical phenomenon, and he was able to show that the two ends of a tourmaline crystal become oppositely charged when the temperature of the crystal is changed. Very extensive experimental studies of the production of electric charges on the surface of crystals by changes of temperature were carried out by Hankel, beginning in 1839. Hankel found that the property of becoming charged by a change of temperature is common to all crystals, although hemihedral crystal forms show the effect more strikingly. A method for demonstrating this so-called *pyro-electric property* of crystals is to place a mixture of finely-powdered sulphur and red lead in a fine cotton sieve and dust it upon the crystal after the temperature of the crystal has been changed.

* See Wiedemann, *Die Lehre von der Elektricität*, Vol. II, pages 316–340.

The effect of the cotton sieve is to give a negative charge to the sulphur particles and a positive charge to the red lead particles, so that the sulphur particles cling to the positively charged parts of the crystal and the red lead particles cling to the negatively charged parts of the crystal.

*Piezo-electricity.** — In 1880 it was found by J. and P. Curie that many kinds of crystals become electrically charged when they are subjected to pressure. This effect is produced in hemihedral crystal forms when a crystal plate with its faces at right angles to the hemihedral axis is compressed between two metal plates. The effect is to charge the two metal plates oppositely.

20. Magnetic rotation of the plane of polarization of light. — Faraday † discovered in 1846 that a transparent substance such as glass or carbon bi-sulphide rotates the plane of polarization of light when it is placed in the magnetic field and when the light is passed through it in the direction of the magnetic lines of force.

21. The Hall effect.‡ — When a conductor through which an electric current is flowing is placed in a magnetic field, the conductor is acted upon by a force which pushes it sidewise as explained in Chapter IV. Ordinarily this force does not alter the distribution of current in the conductor, that is to say, the current is not pushed to one side of the conductor. E. H. Hall discovered in 1880, however, that the current is pushed to one side of the conductor to a very slight extent in some metals, especially in bismuth. This peculiar effect is satisfactorily explained by the electron theory of metallic conduction (see Lodge's *Electrons*, pages 106–109).

22. The Kerr effect. § — One of the most universally applicable principles in physics is the principle of superposition, so-called,

* See Wiedemann, *Die Lehre von der Elektricität*, Vol. II, pages 341–346, and Vol. IV, pages 1280–1284.

† See Faraday's *Experimental Researches*, Series 19, 1846. A description of Faraday's experiments and of later experiments along the same line is given in Wiedemann, *Die Lehre von der Elektricität*, Vol. III, pages 907–968.

‡ See Wiedemann, *Die Lehre von der Elektricität*, Vol. III, pages 192–194.

§ See Wiedemann, *Die Lehre von der Elektricität*, Vol. II, pages 126–136.

which in its most general form may be stated as follows: Given a cause which produces an effect which is proportional to it. Then two such causes acting together produce an effect which is the sum of the effects which they would produce if they acted separately, and the total effect may be divided into two parts which correspond to the two parts of the cause, or in other words, each cause produces the same effect that it would produce if it were acting by itself. One of the best examples of this principle is that light passes from two windows, for example, *through the same region* to the eyes of two observers and each observer sees his window distinctly, that is to say, the light travels through the given region from each window exactly as if it were traveling through the region alone. This principle of superposition is quite accurately true in most of the phenomena of the electromagnetic field. It was discovered, however, by Kerr, in 1875 that an isotropic transparent substance such as glass or oil becomes doubly refracting when subjected to a strong electric field.

23. The Zeeman effect.* — About 1900 it was predicted by Lorenz and experimentally verified by Zeeman, that the light emitted by a hot vapor is altered in a peculiar way when the vapor is placed in an intense magnetic field. The character of this alteration when the emitted light travels parallel to the lines of force of the magnetic field is as follows: Imagine an atom to consist of a positively charged nucleus with one or more nega-

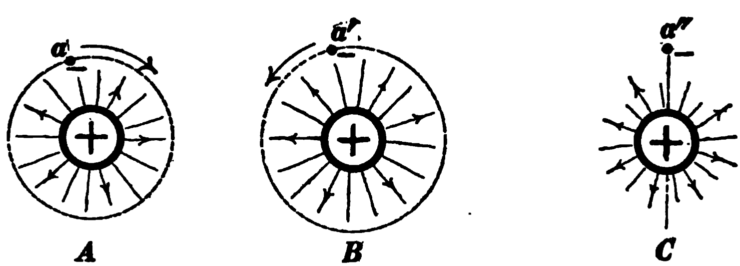

Fig. 23.

* See Lodge's *Electrons*, pages 109–115.

tively charged satellites revolving round the nucleus as represented in Fig. 23. Imagine the region in Fig. 23 to be a magnetic field directed towards the reader. The effect of such a magnetic field would be to push outwards on the satellite *a*, thus increasing its periodic time of revolution, whereas the effect would be to push inwards on the satellite *a'*, thus decreasing its periodic time of revolution. Now the hypothesis which has been used in the discussion of the Zeeman effect is that a given line of the spectrum of the hot vapor is due to the rotation of a certain satellite in the atom, at a certain speed. The plane of the orbit of this particular satellite has every possible orientation in the different atoms of the vapor as shown by *A*, *B* and *C*, Fig. 23, and, when the vapor is not in a magnetic field, the periodic time of rotation of the satellite *a* is the same for all the atoms. When, however, the vapor is in the magnetic field, the periodic time of the satellite *a* is increased, the periodic time of satellite *a'* is decreased, and the periodic time of satellite *a''*, the plane of whose orbit is parallel to the magnetic field, is unaltered. Therefore, instead of one single spectrum line corresponding to the given satellite, there will be three lines, one in the original position and one on each side of the original position.

24. Lippmann's electrometer.* — A pool of mercury underneath an electrolyte, such as dilute sulphuric acid, can of course be used as an electrode of an electrolytic cell. When this is done the surface tension of the mercury is altered, the change of surface tension being approximately proportional to the polarization electromotive force (electromotive force between the metal and the electrolyte). This change of surface tension of mercury by electrolytic polarization may be demonstrated by the change in level of a mercury column in a capillary tube when the surface of the mercury column is polarized. This effect was discovered about 1870 and it was employed by Lippmann in the construction of a capillary electrometer in which the movement of a mercury

* See Wiedemann, *Die Lehre von der Elektricität*, Vol. II, pages 708–720, for a full discussion of the polarization of mercury.

column in a capillary tube is used as an indicator of electromotive force.

25. Electric osmosis. * — A U-tube *AB*, Fig. 24, is filled with water and provided with two platinum electrodes, and an electric current is sent through the cell in the direction of the arrows. The bend of the tube is filled with fine sand. Under these conditions the water is found to rise in the arm *B* and fall in the arm *A*, or, in other words, the current causes the water to diffuse through the sand from *A* to *B*. This forced diffusion of a liquid through a porous diaphragm is called *electric osmosis*. It was discovered in 1807 by Reuss. This effect is greatly reduced when a good conducting liquid, such as an acid or salt solution, is used instead of water. In 1835 Becquerel discovered that fine particles of clay or other material suspended in water are caused to travel in one direction or the other when an electric current is sent through the water.

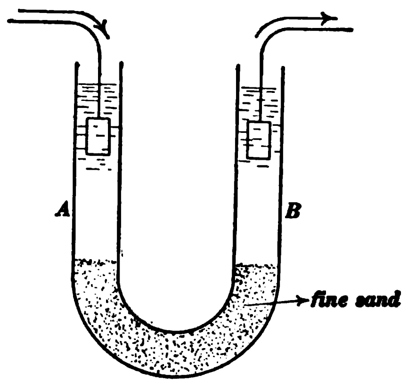

Fig. 24.

26. The change of electrical resistance of selenium by illumination.† — Willoughby Smith, in 1873, discovered that the electrical resistance of metallic selenium is reduced to one half or one third of its normal value when the selenium is exposed to brilliant sunlight.

27. Atmospheric electricity. — It was shown by Benjamin Franklin about 1760 that the lightning discharge is identical in its nature to the ordinary electric spark. Very little was learned after Franklin's time concerning the cause of atmospheric elec-

* See Wiedemann, *Die Lehre von der Elektricität*, Vol. II, pages 166–195.
† See Wiedemann, *Die Lehre von der Elektricität*, Vol. I, pages 547–551.

tricity until about 1896 when the electron theory had been developed. The present theory of atmospheric electricity as developed chiefly by Wilson, of Cambridge, England, is as follows: When moist air is cooled, the water vapor always becomes supersaturated unless there are nuclei present upon which the water vapor can condense. It has been experimentally demonstrated that both positive and negative ions can serve as condensation nuclei, and that a lower degree of super-saturation is required to cause the negative ions to act as condensation nuclei than is required to cause the positive ions to act as condensation nuclei. The upper regions of the atmosphere where the ultra-violet rays of the sun's light are very intense, are strongly ionized, and, when the water vapor in these upper regions becomes super-saturated by cooling, the negative ions, becoming loaded by the condensation of moisture, fall towards the earth leaving the upper regions of the atmosphere positively electrified. The great intensity of the electric phenomena of the atmosphere during the summer time is probably due to the fact that during the summer the condensation of moisture takes place at very great altitudes where the ionization of the atmosphere is very great, whereas during the winter time most of the condensation which takes place occurs at very much lower altitudes where the atmosphere is not strongly ionized.

Lightning protection. — The use of the lightning arrester for protecting electrical machinery is described in Chapter VI. The use of the lightning rod for the protection of buildings against damage by lightning is due to Benjamin Franklin. A lightning rod is simply a good conductor leading as directly as possible from a point above a building to a good ground connection in moist earth. A house which is not guarded by a lightning rod may not be damaged, and, in many cases, houses which are guarded, are severely damaged, but statistics show that the number of casualties is very greatly reduced by the use of lightning rods. There is therefore no question as to the usefulness of the lightning rod. Information concerning lightning rods may be obtained from Sir Oliver Lodge's book *Lightning Conductors and Lightning Guards*, Whitaker & Co., London, 1892.

APPENDIX D.

MISCELLANEOUS PRACTICAL APPLICATIONS.*

28. The Morse telegraph is an arrangement for signalling between distant stations as follows: An insulated wire leads from one station to the other and back. The ground is generally used instead of a return wire. An electric current from a battery or other source is sent intermittently through this circuit by operating at one station a *key* which makes and breaks the circuit. This current excites an electromagnet at the other station, and the armature of this electromagnet makes a graphical record on a moving strip of paper, or produces sound signals which are interpreted by the operator at the receiving station.

Relays and sounders. — A fairly strong electric current is required to operate the instrument which produces the signals at a telegraph receiving station, and it is not desirable to send so strong a current over a long line because of the great number of voltaic cells that would be required. This difficulty is obviated by the use of the relay. The current in the line flows through

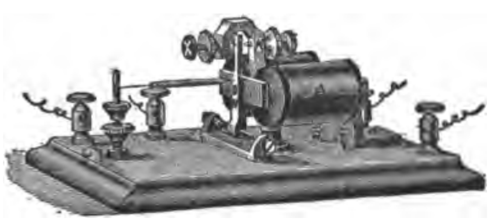

Fig. 25.

many turns of fine wire which are wound upon an electromagnet at the receiving station. This magnet actuates a very light lever and this lever is arranged to open and close what is called a *local*

* Many of the practical applications of electricity and magnetism have been described in the foregoing chapters.

circuit as it moves back and forth between stops. Figure 25 is a view of such an instrument, which is called a relay. The local circuit which is opened and closed by the relay contains a battery which supplies the large current that is required for the operation of the instrument which produces the sound signals. This instrument is called a sounder. It consists of an electromagnet,

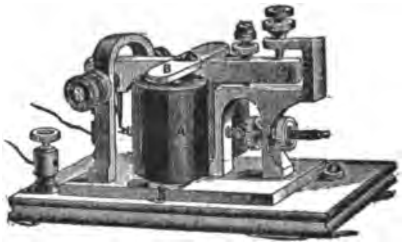

Fig. 26.

Fig. 27.

which is wound with moderately coarse wire and which actuates a massive lever and produces audible signals as it moves back and forth between stops. Figure 26 shows the ordinary telegraph sounder. Figure 27 shows an ordinary telegraph key.

29. The polarized relay. — The ordinary relay which is shown in Fig. 25 responds to a make-and-break key. By using the proper tension on the spring which pulls the lever back (see Fig. 25), the lever of the ordinary relay may be made to respond to *increase and decrease* of current, whereas a quick reversal of current may not affect the instrument, inasmuch as the lever may not have time to move perceptibly while the current is passing through zero value.

The *polarized relay* is so constructed as to respond to *reversals* of current, but not to respond to *increase and decrease* of current. An electromagnet NN_1, Fig. 28*a*, is mounted, as shown, upon one pole of a U-shaped permanent magnet. A light iron lever a, Fig. 28*b*, pivoted at p, passes through a slot in the south pole SS of the permanent magnet, between the poles NN_1 of the electromagnet, and plays between the stops p' and p''. This lever a is magnetized inasmuch as it bridges over from the

south pole SS of the permanent magnet to the soft iron cores NN_1 which stand upon the north pole of the permanent magnet. When a current flows in a certain direction through the coils of the electromagnet NN_1 one of its poles, N_1, for example, becomes a strong north pole and attracts the lever *a*. When the current is reversed, the other pole N becomes a strong north pole and attracts the lever *a*. Thus, the lever *a* is pulled towards N_1 or towards N according to the direction of the current which flows through the coils of the instrument, and a local circuit connected, as shown in Fig. 28*b*, may thus be opened and closed at will by repeated reversals of the current through the winding of the electromagnet NN_1.

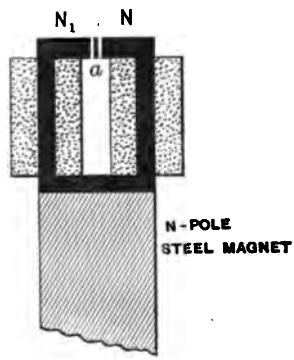

Fig. 28a.

The ordinary relay is usually called the *neutral* relay to distinguish it from the polarized relay.

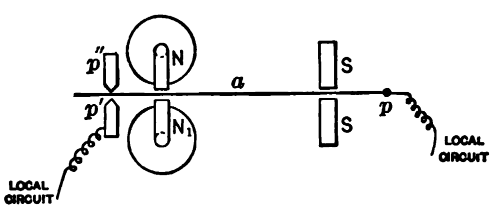

Fig. 28b.

30. Diplex telegraphy. — The sending of two messages in the same direction over one line wire simultaneously is known as *diplex telegraphy*. This is accomplished as follows: At the sending station are two keys. One of these keys is arranged to vary the strength of the current in the line (never actually breaking the circuit) by throwing a number of voltaic cells in and out of circuit as it is operated. The other key is arranged to reverse the direction of the line current as it is operated, the line current being in one direction while this key is down, and in the other

direction while it is up. At the receiving station a neutral relay and a polarized relay are connected in circuit with the line. The neutral relay responds to the key which varies the strength of the line current, and the polarized relay responds to the key which reverses the line current.

31. Duplex telegraphy.—The sending of two messages in opposite directions over one line wire simultaneously is known as *duplex telegraphy*. This is accomplished as follows: Fig. 29 represents the arrangement of apparatus at one station. An exactly similar arrangement is installed at the other station. Let c be the total resistance of the line through the distant station to the ground. Then the resistances a, b, c and d form a Wheatstone's bridge. When these resistances are so adjusted that $a/b = c/d$, then the key at the home station may be pressed without sending a current through the home relay. When the key at the home station is pressed, however, current flows over the line to the other station, and it is easily seen from the figure that a line current coming to a station divides, and flows in part through the relay at that station. Therefore the relay at each station responds to the movements of the key at the other station.

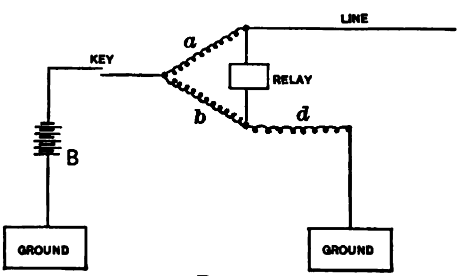

Fig. 29.

32. Quadruplex telegraphy.—The sending of two messages each way over one line wire simultaneously is known as *quadruplex telegraphy*. This is accomplished by combining the arrangements for diplex and duplex telegraphy. The single key represented in Fig. 29 is replaced by two keys, one for reversing the current and the other for altering its strength; and the single relay is replaced by two relays, one a neutral relay and the other a polarized relay. With this arrangement the polarized relay at

each station responds to the reversing key at the other station, and the neutral relay at each station responds to the key at the other station which alters the strength of the current.

33. The printing telegraph is an arrangement by means of which a simple form of typewriter is operated at a distant station from a keyboard at a sending station. A simple form of printing telegraph is as follows: * Twenty-six equidistant pins are arranged in a helical row around a long metal cylinder. This cylinder is rotated by a small electric motor or by clockwork, and above the cylinder is a bank of twenty-six lettered keys so arranged that when a key is depressed, one of the pins comes against it and the cylinder is stopped in a certain position; the next key would stop the cylinder $\frac{1}{26}$ of a revolution farther on, and so on. Attached to the rotating cylinder is a device for reversing an electric current fifty-two times for each revolution of the cylinder. This repeatedly reversed electric current passes over the telegraph line and through two electromagnets at the receiving station. One of these electromagnets is like a neutral relay with a heavy lever, and the other is like a polarized relay with a light lever which oscillates with the rapid reversals of current and actuates an escapement which turns a type wheel with the twenty-six letters arranged round its periphery. This type wheel is thus turned step by step, keeping pace with the rotating cylinder at the sending station.

When the cylinder at the sending station is stopped by depressing a key, the A-key, for example, the current-reversing device stops also, a steady current flows over the line, the tongue of the polarized relay stops oscillating, the type wheel stops, and

* When a person is thoroughly familiar with the elements which enter into the construction of a machine, that is, when a person is familiar with shafts and wheels, and with simple devices like switches for opening and closing electric circuits and for reversing connections, a more easily intelligible description of a complicated machine can be made without illustrative diagrams and drawings than can be made with the help of diagrams and drawings. In fact, it is confusing under the specified conditions to have recourse, even, to a working model of a complicated machine, when the object in view is to impart a clear idea of its fundamental features.

the steady current excites the neutral relay, the lever of which pushes a strip of paper against the type wheel and prints the letter A. When the key at the sending station is raised, the current reversals begin again, the type wheel at the receiving station starts, and at the same time the lever of the neutral relay falls back and actuates a device which moves the strip of paper a step forward for the printing of the next letter.

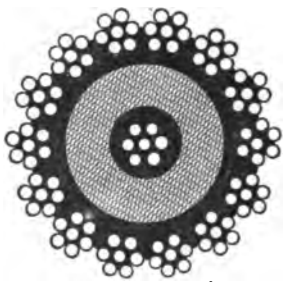

Fig. 30.

34. Submarine telegraphy. — Figure 30 shows a full-size sectional view of a submarine telegraph cable. The conductor at the center consists of a number of strands of copper wire. Surrounding this is a layer of gutta percha, and the whole is protected by a covering of tarred hemp and steel wire.

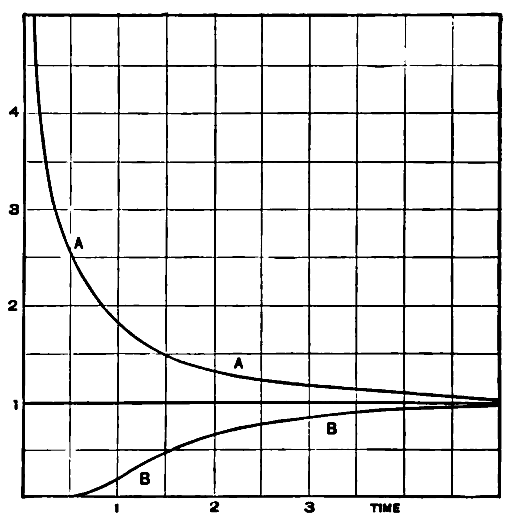

Fig. 31.

The conductor and metal sheath of the cable, together with the intervening insulator, constitute a condenser of large electrostatic capacity. The effect of this large electrostatic capacity is

as follows: At the instant a battery is connected to a cable a very large current begins to flow into the cable. Most of this current goes to charge the cable, and, as the cable becomes charged, the entering current falls off in value, settling finally to a steady value which is determined by the resistance of a cable. The ordinates of curve A in Fig. 31 show the successive values of the current which enters a cable from a battery. At the distant end of the cable an infinitesimal current begins almost at the instant the battery is connected at the sending station, and, as the cable becomes charged, this current rises in value until it reaches a steady value very nearly equal to the steady value of the entering current. The curve B, Fig. 31, shows the growth of current at the distant end of a cable when a battery is connected to the near end. When the battery is disconnected the current which enters the cable ceases at once, and the current at the distant end drops slowly to zero as the accumulated charge flows out of the cable.

Distortion of current pulses by a cable. — The curve a, Fig. 32, shows the character of the current pulse which enters a cable when

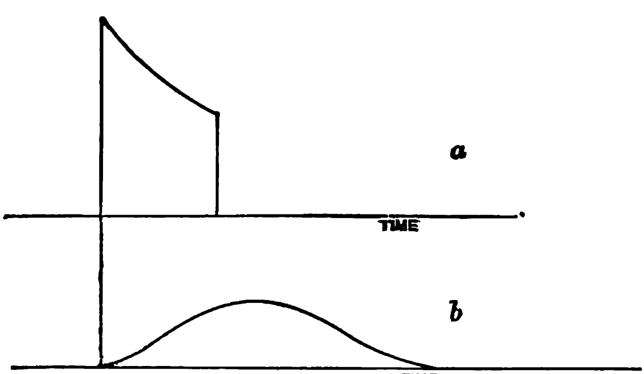

Fig. 32.

a battery is momentarily connected to the cable, and the curve b shows the character of the current pulse which flows out at the distant end of the cable. The action of a cable in thus alter-

ing the character of a current pulse is called distortion. Land lines distort current pulses to some extent, and the distortion seriously impairs the distinctness of telephonic transmission if the land line is fairly long (see Art. 147).

The curve *a*, Fig. 33, represents four short current pulses

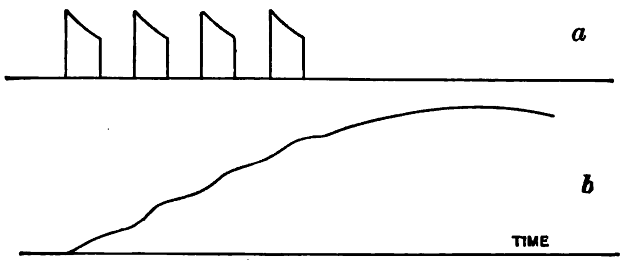

Fig. 33.

sent into a cable at one end, and the curve *b* represents the resultant pulse of current which flows out of the cable at the other end. *The receiving instrument in submarine telegraphy is a galvanometer which is arranged to trace the resultant current curve at the receiving end of the cable*, and the separate current pulses that are sent into the cable at the sending end are inferred from the slight kinks in the curve which is traced by the receiving instrument.

The distortion of electric current pulses by a submarine cable is analogous to the distortion of pulses of water current by a long thin-walled rubber tube.

35. The syphon recorder is the receiving instrument used in submarine telegraphy. It consists of a D'Arsonval-type galvanometer, the moving coil of which is attached by means of a fine thread to a syphon of very fine glass tube. This syphon takes ink from a small reservoir and traces an ink line upon a moving paper ribbon. When the galvanometer coil is quiet a straight line is traced upon the moving paper. When signals are being received the varying current which flows through the galvanometer coil causes the coil to move and the glass tube traces a wavy line upon the moving paper. It is necessary for the

syphon to move sidewise with the utmost freedom, and therefore the tip of the syphon cannot be allowed to rest against the moving paper. This difficulty was overcome in the early form of the syphon recorder * by keeping the ink reservoir and syphon highly charged with electricity by means of an influence machine, thus causing the ink to issue from the tip of the syphon in the form of a fine jet. In the present form of the recorder the syphon is

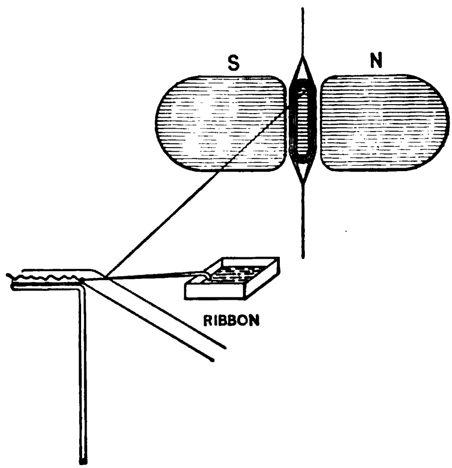

Fig. 34.

kept vibrating rapidly against the paper so as to trace a finely dotted line as the paper moves while at the same time the sidewise motion of the syphon is not hindered by friction. The essential features of the syphon recorder are shown in Fig. 34.

* The syphon recorder was devised by Lord Kelvin, who contributed more, perhaps, to the development of transatlantic telegraphy than any other man. In an article by Professor W. E. Ayrton, which appeared in the London *Times* shortly after Lord Kelvin's death (reprinted in *Popular Science Monthly* for March, 1908), much interesting information is given concerning what Kelvin did for submarine telegraphy. "When signals through the 1858 Atlantic cable became weak, and a message from the President to our Queen took thirty hours in transmission although containing only 150 words, and which would need only three or four minutes to transmit through any one of our good Atlantic cables of to-day, the only remedy of those who looked down upon the theories of the young Glasgow professor was to use Whitehouse's "thunder pump," a magneto-electric machine which produced a sudden large electromotive force when the armature of the permanent magnet was jerked off the poles of the

36. The telephone consists of a thin sheet-iron diaphragm D, Fig. 35, which is very near to one end of a steel magnet M with a winding of fine insulated wire C.

The action of the telephone as a transmitter.—When the telephone first came into use, the same instrument was used as trans-

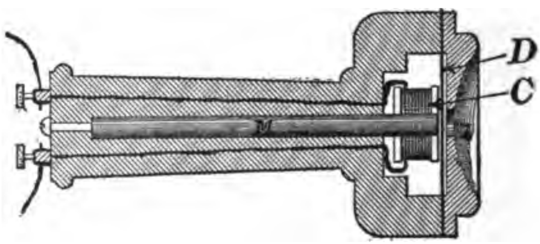

Fig. 35.

mitter and receiver, being moved alternately from mouth to ear of the speaker. The action of a telephone as a transmitter is as follows: The coil C being near the end of the magnet M, only a portion of the magnetic flux from M passes through the coil. When the diaphragm moves nearer to the end of the magnet, a greater portion of the magnetic flux from the magnet passes through C, and when the diaphragm moves farther away from the magnet, a smaller portion of the magnetic flux from the magnet passes through C. Thus, as the diaphragm D vibrates, the magnetic flux through the coil C increases and decreases. This pulsation of the flux through the coil C induces an electromotive force in the coil, and this induced electromotive force produces a current in the coil and in any circuit to which the coil is connected. This induced current flows in one direction while the diaphragm is moving towards the magnet, and in the other direction while the diaphragm is moving away from the magnet.

The action of a telephone as a receiver.—If a current passes through the coil C first in one direction and then in the other

magnet. But these shocks only sent sparks through the gutta-percha insulating coating and hurried the poor cable to its doom, so that even the three words per minute which would have been the utmost limit of speed possible had this cable been entirely uninjured, were replaced by absolute silence."

direction, the magnet M will be alternately weakened and strengthened, the force with which the magnet attracts the diaphragm will vary accordingly, and the diaphragm will be caused to move to and fro in unison with the reversals of current.

Consider two telephones, A and B, connected in circuit. A sound strikes the diaphragm of telephone A and causes the diaphragm to vibrate. Telephone A acts as a transmitter, and telephone B acts as a receiver, as explained above, and the diaphragm of telephone B is caused to vibrate in a manner exactly similar to the vibrations of the diaphragm of telephone A, and thus the diaphragm of telephone B reproduces the original sound.

37. The carbon transmitter. — The alternating current which is produced by a telephone acting as a transmitter is very weak even when the transmitter telephone is exposed to a loud sound. The carbon transmitter is an arrangement by means of which a vibrating diaphragm may control a strong battery current and

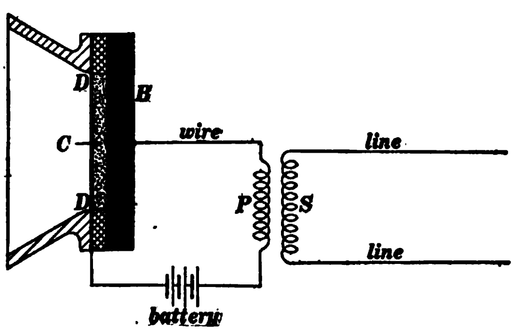

Fig. 36.

cause a strong induced current to surge back and forth through the telephone line in unison with the movements of the diaphragm. The arrangement of the carbon transmitter is shown in Fig. 36. The current from a battery passes through the primary P of a small induction coil and through a mass of granular carbon C which lies between a carbon block B and a diaphragm DD. The electrical resistance of the granular carbon varies with the

pressure exerted upon it by the vibrating diaphragm, this causes the battery current to fluctuate, the fluctuating battery current induces an alternating current in the secondary S of the induction coil and this alternating current passes over the line and actuates the receiver telephone at the distant station.

38. Wireless telegraphy. * — The intensity of the magnetic field in the neighborhood of an isolated magnet pole decreases as the square of the distance increases, and the intensity of the magnetic field at considerable distances from a complete magnet (having two opposite poles) decreases as the cube of the distance increases. The energy of a magnetic field is proportional to the square of the field intensity and therefore the energy of the magnetic field in the neighborhood of an isolated pole decreases as the fourth power of the distance increases, and the energy of the magnetic field at considerable distances from a complete magnet decreases as the sixth power of the distance increases. The same laws of decrease of the energy apply in the case of the electric field due to an isolated charge and in the case of the electric field due to a doublet consisting of two opposite charges near together, respectively. In the case of wave motion of any kind which spreads out uniformly in all directions from a source, the energy falls off as the square of the distance increases. Therefore an enormously greater amount of energy can be brought into action at great distances from a source of disturbance by wave motion than by actions which produce a steady distribution of field. This remarkable property of wave motion is illustrated by the familiar fact that an audible effect may be produced upon the ear of a distant person by the wave disturbance in the air which is produced by a vibratory motion, whereas an extremely violent but steady circulation of air produced, for example, by a powerful fan-blower, does not lead to any perceptible energy manifestations at moderately great distances from the blower. In consequence of the very great energy manifestations

* This article describes the simple original arrangement which is due to Marconi.

at a distance due to *wave motion* as compared with the extremely small energy manifestations at a distance due to *steady actions*, it may be said that the only feasible method * of signalling at moderately great distances is by means of wave motion.

The use of the vocal organs for producing sound waves and of the auditory organs for perceiving them at a distance, constitutes the most familiar example of "wireless" signalling. The term *wireless telegraphy* is applied particularly to the use of an electric oscillator for producing electric waves and an electric resonator or detector of any kind for perceiving the waves at a

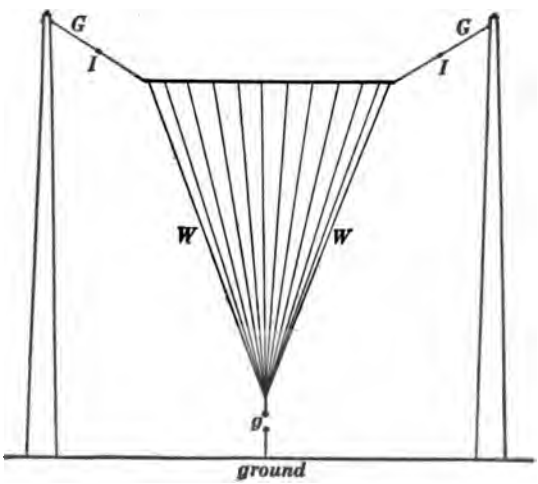

Fig. 37.

distance. This system of electric wave signalling is due to Marconi.

The sending apparatus or oscillator. — A charged body, an expanse of metal, is suspended in the air so as to be thoroughly insulated from the earth. This body of metal is usually made of many strands of wire *WW*, Fig. 37, which are supported by guy wires *GG* from poles, as shown, the guy wires being

* Where the energy is not transmitted along a well-defined path like a wire, or a pipe, or a string.

provided with insulating links *II*. The body of metal *WW*, which is separated from the ground by a short air gap *g*, is connected to one terminal of a high voltage induction coil, the other terminal of which is connected to the earth, the body of metal *WW* is charged until the air gap *g* breaks down, the discharge which takes place is oscillatory in character as explained in Chapter IX, and electric waves pass out in all directions from *WW*.

The receiving antenna. — A long vertical wire is suspended by insulating supports at the receiving station and connected to earth through a device which is called a detector. The passage of the electric waves causes electric charge to surge up and down in this vertical wire or antenna, and the weak alternating current thus produced actuates the detector and produces the signal at the receiving station. The detector which was used in the earlier days of wireless telegraphy was the coherer of Branly. The essential parts of this coherer are shown in Fig. 38. It consists of two short brass rods between which is a loose mass of metal filings. This coherer is connected across the air gap of the receiving antenna or resonator as shown in Fig. 39, in which *S* is an ordinary telegraph sounder. An auxiliary device, not shown in the figure, is used to keep the metal filings vibrating slightly. Under these conditions the filings do not conduct the battery current to any

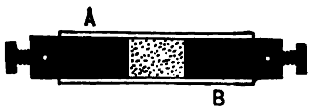

Fig. 38.

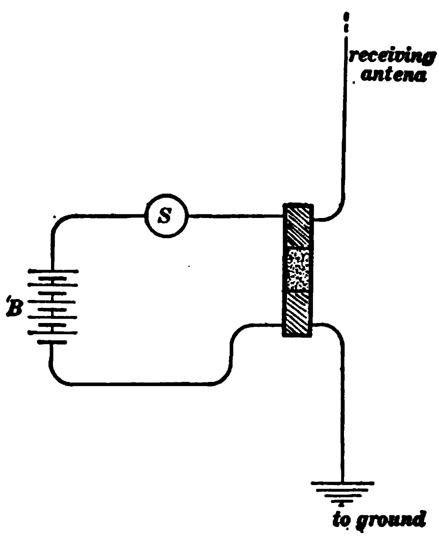

Fig. 39.

perceptible extent. When, however, electric waves act upon the antenna, the slight current which is produced in the antenna is forced through the filings and produces what seems to be a welding together of the particles of the filings at the points of contact. At any rate, as long as a slight amount of current is forced through the filings from the antenna, the filings form a good conducting path for the battery current and the sounder is excited, but, the moment the electric waves cease, the vibratory motion of the metal filings causes them to become detached from each other and the battery current ceases.

Figure 40 shows the trend of the electric lines of force in the electric waves as they approach the receiving antenna. The

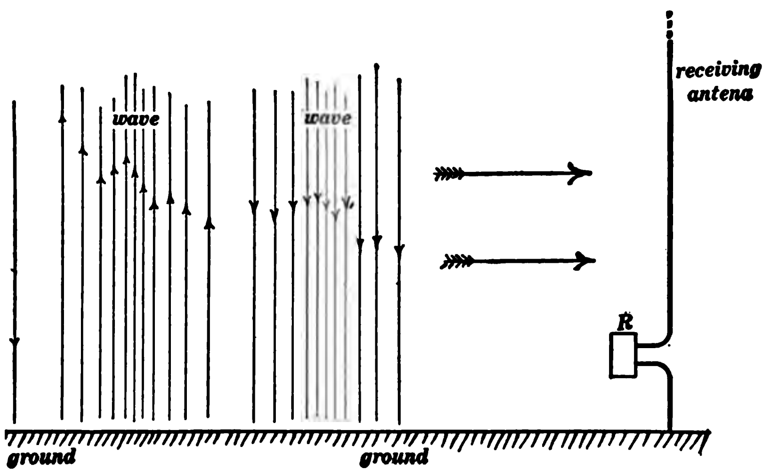

Fig. 40.

magnetic lines of force are horizontal and perpendicular to the plane of the paper.

39. Electric lighting. — One of the most extended applications of the electric current is in the production of artificial illumination. This is usually accomplished by the heating to incandescence of a high resistance portion of a circuit, by the electric current. The high resistance portion of the circuit, together with its mounting, is called an electric lamp. Two types of electric lamp are in gen-

eral use, namely, the glow lamp, or incandescent lamp, and the arc lamp.

The glow lamp consists of a fine filament or wire of highly refractory material which is enclosed in a glass bulb from which the air is exhausted. In the older type of glow lamp the filament is made of charred vegetable material upon which a dense deposit of carbon is formed by heating it in the vapor of gasoline. The heating is accomplished by sending an electric current through the filament. The carbon-filament glow lamp consumes from three to four watts for each candle of light emitted. Thus, a 16-candle carbon filament lamp consumes about 55 watts.

Recently several varieties of metal-filament glow lamps have been placed on the market. The earliest of these was the osmium lamp, the filament of which is made of metallic osmium which is sufficiently refractory to stand a temperature high enough to emit one candle of light with a consumption of about 1½ watts. The scarcity of metallic osmium, however, was a serious obstacle in the way of extensive use of the osmium lamp. The next metal filament lamp to be placed on the market was the tantalum lamp, in which the filament consists of a wire of metallic tantalum. In the tungsten lamp, which is now coming into extensive use in Europe and America, the filament consists of metallic tungsten. The carbon filament lamp consumes from 3 to 4 watts for each candle of light emitted, the osmium lamp consumes about 1½ watts per candle of light emitted, the tantalum lamp consumes about 2 watts per candle, and the tungsten lamp consumes about 1¼ watts per candle. The greatest difficulty with the metal filament lamps is that the filament must be excessively fine to give a low candle power lamp with the standard voltages now in use for lighting purposes (110 volts and 220 volts), because of the low specific resistance of metals as compared with carbon. This difficulty is greatly enhanced in the case of the tungsten lamp by the excessive brittleness of the material.

The arc lamp. — When an electric arc is formed between carbon points as described in Chapter VIII, the carbon points become

intensely heated and give off a very brilliant light. The arc lamp is a mechanism for automatically moving two carbon rods so that a steady electric arc may be maintained between the ends of the rod. There is a great variety of arc lamp mechanisms but the following description will serve to give an idea of their action: The current comes into the lamp and divides as shown in Fig. 41. A very small portion of the current flows through a shunt coil *B* without passing through the arc, and the remainder flows through the coil *A* and thence through the arc. An iron rod *AB*, passing loosely into the two coils *A* and *B*, is carried upon one end of a lever which is pivoted at the point *p*. The other end of this lever is provided with a clutch *c* through which a smooth brass rod *bb* passes. This clutch *c* is so constructed that it releases the rod *bb* when the iron rod *AB* is raised, thus allowing the carbons to come together. Each of the coils *A* and *B* acts to pull the rod *AB* into itself, and a spring which is attached to the lever is so adjusted that when the arc is burning properly the combined action of this spring and the two coils *A* and *B* holds the lever in such a position that the clutch grips the brass rod *bb*. As the arc continues to burn, the carbons are slowly consumed, causing the gap between the carbon tips to widen. This increases the resistance of the arc and causes a greater portion of the current to flow through the shunt coil *B* which pulls up on the iron rod *AB*, moves the lever, releases the clutch, and allows the rod *bb* to fall slightly, thus bringing the carbons again to the proper position.

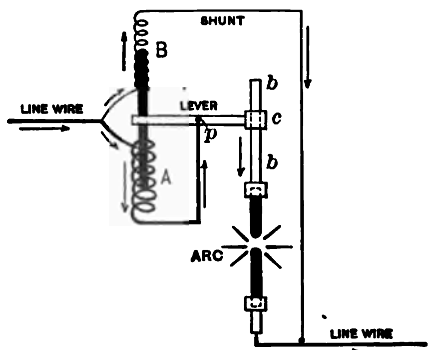

Fig. 41.

A variety of arc lamps have been developed in which the light is emitted by the arc itself. Thus we have the so-called *flaming-*

arc lamp in which the carbon rods are impregnated with metallic salts, the vapors of which give an intensely luminous arc. Another form of arc lamp in which the arc itself is intensely luminous is the *magnetite arc lamp* in which the arc is formed between a rod of compressed titanium carbide and iron oxide (the cathode) and a rod of copper (the anode). The result is the vaporization of the iron oxide and the production of an intensely luminous arc.

40. The electrolytic interrupter (Wehnelt). — The primary circuit of an induction coil is usually interrupted by a vibrating reed or spring which makes and breaks contact between two platinum points. Wehnelt discovered that the sudden generation of oxygen on a small platinum anode in dilute sulphuric acid causes an abrupt stoppage of the electric current. This effect is utilized in the electrolytic interrupter as follows: A glass jar *CC*, Fig. 42, is filled with dilute sulphuric acid and provided with two

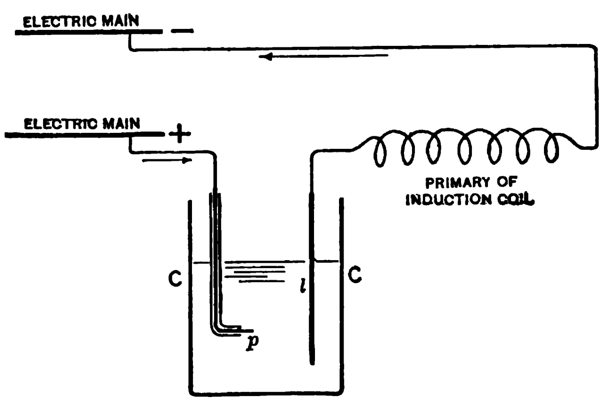

Fig. 42.

electrodes *p* and *l*. The anode *p* is a tip of platinum wire projecting from a glass tube, and the cathode *l* is a large plate of lead. The electromotive force between the mains, which must be 30 volts or more, causes a sudden rush of current through the cell *CC* and through the primary of an induction coil. This rush of current generates a layer of oxygen over the platinum

tip which stops the current abruptly. The layer of oxygen then collects as a bubble and rises, leaving the platinum tip again in contact with the acid when another rush of current takes place, and so on. From 200 to 1,500 interruptions per second may be produced by this arrangement according to the size of the platinum tip, the inductance of the circuit and the value of the electromotive force.

41. Electric welding. *Thomson's process.* — The two metal rods to be welded are connected to the terminals of an electric generator and brought into contact with each other. The current, flowing across the relatively high resistance contact, heats the ends of the rods to the melting temperature, the rods are then pushed slightly together and the weld is complete. Alternating current is generally used in this welding process; a transformer takes current at high voltage from ordinary supply mains and delivers a very large current at very low voltage to the rods to be welded.

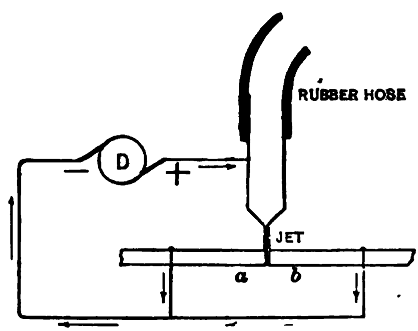

Fig. 43.

The wet process. — When a direct-current generator having an electromotive force of from 200 to 500 volts is connected to an electrolytic cell with small cathode, the cathode becomes intensely heated. This effect is utilized for welding as follows: The two rods *a* and *b*, Fig. 43, which are to be welded are connected to the negative terminal of the dynamo *D*. The positive terminal of the dynamo is connected to a metal nozzle from which a jet of salt water issues. This jet impinges upon the ends of the two rods and quickly fuses them together.

APPENDIX E.

MECHANICAL AND ELECTRICAL ANALOGIES.

The mechanical analogies which are pointed out in Art. 62 of Chapter V, in Chapter VI, in Arts. 89 and 93 of Chapter VII, and in Arts. 106, 107 and 108 of Chapter VIII are here collected together for convenience ot reference, and the mechanical analogies of electrical oscillations are added :

$x = vt$ (1) in which x is the distance traveled in t seconds by a body moving at velocity v.	$\phi = \omega t$ (2) in which ϕ is the angle turned in t seconds by a body turning at angular velocity ω.	$q = it$ (3) in which q is the electric charge which in t seconds flows through a circuit carrying a current i.
$W = Fx$ (4) in which W is the work done by a force F in pulling a body through the distance x.	$W = T\phi$ (5) in which W is the work done by a torque T in turning a body through the angle ϕ.	$W = Eq$ (6) in which W is the work done by an electromotive force E in pushing a charge q through a circuit.
$P = Fv$ (7) in which P is the power developed by a force F acting upon a body moving at velocity v.	$P = T\omega$ (8) in which P is the power developed by a torque T acting on a body turning at angular velocity ω.	$P = Ei$ (9) in which P is the power developed by an electromotive force E in pushing a current i through a circuit.
$W = \frac{1}{2}mv^2$ (10) in which W is the kinetic energy of a mass m moving at velocity v.	$W = \frac{1}{2}K\omega^2$ (11) in which W is the kinetic energy of a wheel of moment of inertia K turning at angular velocity ω.	$W = \frac{1}{2}Li^2$ (12) in which W is the kinetic energy of a coil of inductance L carrying a current i.
$F = m\dfrac{dv}{dt}$ (13) in which F is the force required to cause the velocity of a body of mass m to increase at the rate $\dfrac{dv}{dt}$.	$T = K\dfrac{d\omega}{dt}$ (14) in which T is the torque required to cause the angular velocity of a wheel of moment of inertia K to increase at the rate $\dfrac{d\omega}{dt}$.	$E = L\dfrac{di}{dt}$ (15) in which E is the electromotive force required to cause a current in a coil of inductance L to increase at the rate $\dfrac{di}{dt}$.
$x = aF$ (16)	$\phi = bT$ (17)	$q = CE$ (18)
$\dfrac{4\pi^2 m}{\tau^2} = \dfrac{1}{a}$ (19)	$\dfrac{4\pi^2 K}{\tau^2} = \dfrac{1}{b}$ (20)	$\dfrac{4\pi^2 L}{\tau^2} = \dfrac{1}{C}$ (21)

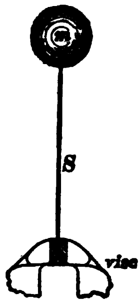

Fig. a.

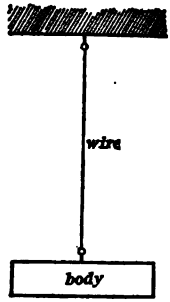

Fig. b.

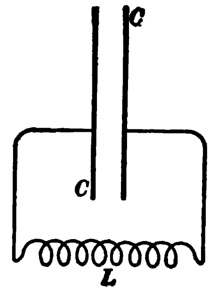

Fig. c.

A body of mass m is supported by a flat spring S, clamped in a vise as shown in Fig. a. A force F pushing sidewise on m moves it a distance x, which is proportional to F, according to equation (16). When started the body m will continue to vibrate back and forth and the period τ of its vibrations is determined by equation (19).

A body of moment of inertia K is hung by a wire as shown in Fig. b. A torque T acting on the body will turn the body and twist the wire through an angle ϕ, which is proportional to T, according to equation (17). When started, the body will vibrate about the wire as an axis and the period τ of its vibrations is determined by equation (20).

A condenser C is connected to the terminals of a coil of inductance L as shown in Fig. c. An electromotive force E acting anywhere in the circuit pushes into the condenser a charge q, which is proportional to E, according to equation (18). When started the electric charge will surge back and forth through the coil, constituting what is called an oscillatory current and the period of one oscillation is determined by equation (21).

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